

PYTHAGOREAN APPROACH TO PROBLEMS OF PERIODICITY IN FERMIONIC SYSTEM

DMITRY WEISE

Name: Dmitry Weise, Physician (b. Moscow, U.R.S.S., 1956).

Address: Belovejskaya, Complex 39, Building 2, Suite 133, 121353 Moscow, Russia.

E-mail: dweise@gol.ru, *Phone:* ++7 (095)448-13-34

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Abstract: A Pythagorean approach to numerical sequences in both chemical and nuclear physics has allowed us to show geometrical analogies based on figurate numbers (three-dimensional forms of Mendeleyev's periodic table and packing models for nuclei). Pascal's triangle is used to deduce analytical equations, from which magic numbers through 12,360 for the atoms and through 21,400 for the nuclei are.

1 INTRODUCTION

Structural similarities exist between atomic nuclei and other fermionic system such as metal clusters. In particular both of these systems exhibit specific magic numbers.

1.1 Figurate numbers, gnomon, and Pascal's triangle

Figurate or *polygonal numbers* appeared in 15th-century arithmetic books, and were probably known to ancient Chinese; but they were of particular interest to ancient Greek

mathematicians. To the Pythagoreans (c. 500 BC), numbers were of paramount significance: they believed everything could be explained by numbers, and numbers were invested with specific characteristics and "personalities". Among the properties of numbers the Pythagoreans endowed them with "shapes"

The connotation of the term *gnomon* is that originally given by Hero of Alexandria, namely, 'A Gnomon is that form that, when added to some form, results in a new form similar to the original. *Pascal's triangle* is an arrangement of numbers such that each number is the sum of the two immediately above it in the previous row. Very interesting figurate entries can be found in the columns and rows of the table: *triangular* and *tetrahedral numbers*. There is also the sequence of *Fibonacci numbers*, 1, 1, 2, 3, 5, 8, 13, *Fibonacci numbers* occur in various natural patterns, including living beings, such as plants. Thus, Pascal's triangle appears as a cognitive bridge between microcosm and wildlife.

2 PERIODICITY OF ATOMIC PROPERTIES

In the atomic shell model, the shells are filled with electrons in order of increasing energy until they fill a closed shell, producing the inert core of noble gases. The atomic magic numbers 2, 10, 18, 36, 54, 86, correspond to the total number of electrons in filled shells.

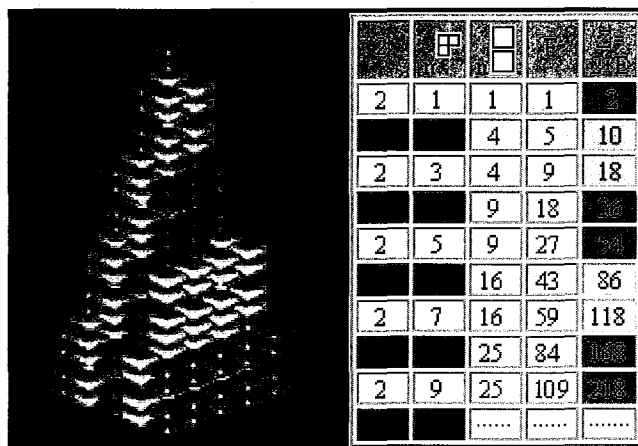


Figure 1: On the left – figurate number 218 – 3D Mendeleyev's periodic system. Two monochromatic building blocks posed above one another in each layer correspond to an electron orbital. On the right – Pascal's triangle modification "A".

The manipulations performed with Pascal's triangle and figurate numbers result in a 3D model for Mendeleyev's periodic system, to which one may give the interpretation

submitted in Figure 1. The figurate-numerical approach yields an algebraic expression for the atomic numbers of inert gases:

$$Z = [(-1)^n (3n + 6) + 2n^3 + 12n^2 + 25n - 6] / 12, \quad (1)$$

where $n = 1, 2, 3, \dots$ is the period number.

3 PERIODICITY OF NUCLEAR PROPERTIES

Since Pascal's triangle has proven successful in determining atomic magic numbers, the question arises as to whether it can also help to identify nuclear magic numbers – can it provide a geometrical image and an analytical formula just as for atoms?

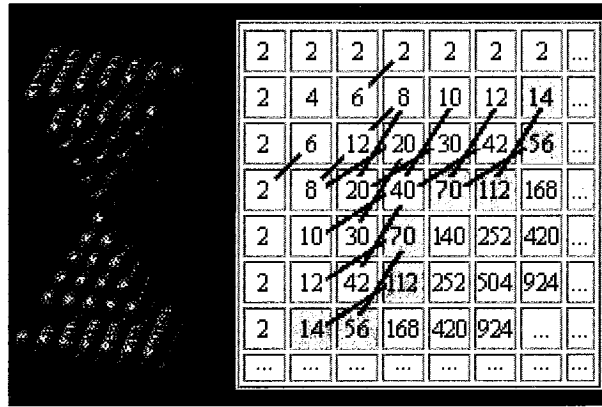


Figure 2: On the left – figurate number 126. On the right – Pascal's triangle modification "B". The values of all numbers correspond to doubling the numbers in the conventional Pascal's triangle. The segments of oblique lines bridge the numbers whose sum is equal to the next magic number (2, 8, 20, 28, 50, 82, and 126).

The magic numbers for nuclei are: 2, 8, 20, 28, 50, 82, 126, corresponding to the total numbers of protons and neutrons in filled nuclear shells. Nuclei with magic numbers of protons and neutrons are unusually stable. A formula valid for all the nuclear magic numbers is:

$$MN_m = k * (m^2 - m) + (m^3 + 5m)/3, \quad (2)$$

where $m = 1, 2, 3, \dots$ with $k = 1$ if $m=1, 2, 3$; $k = 0$ if $m > 3$.

4 PERIODICITY OF CLUSTERS PROPERTIES

The Pythagorean approach can be applied to the discovery of magic numbers in the analysis of stability properties of clusters formed by inert-gas or alkali-metal atoms. Let's consider application of the Pythagorean approach for the analysis of one of magic numbers sequences: 7, 29, 66, 118 and 185. The method of finite differences can sometimes be used to guess a formula $f(n)$ (but not to prove it). Application of Pascal's triangle means use of a method of finite differences. In terms of Pythagorean school it is possible to tell: if the method of finite differences is applicable, each difference can be considered as *gnomon* for an above row.

Table 1 - Pascal's triangle modification "C".

	7	7	...
15	22	29	...
15	37	66	...
15	52	118	...
15	67	185	...
...	...	...	...

$$\text{Explicit form: } C_n = (15n^2 - n) / 2, \quad \text{where } n = 1, 2, 3, \dots \quad (3)$$

CONCLUSION

In this paper we have shown an attempt to couple modern atomic theories with an ancient scientific guiding principle. In particular, a parallel between atomic and nuclear shells and such a fundamental concept of the Pythagorean school known as the *gnomon* is carried out.

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