

GEODESIC DOME UNDER THE PAW OF AN ORIENTAL LION

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Abstract: *Geodesic domes, in a broader sense, are spherical domes composed of smaller units where the spherical surface is subdivided into triangles not too different from each other. The paper deals with such structures in Oriental culture. An earlier version of this paper was given, together with Koji Miyazaki, at IASS 2004: International Symposium on Shell and Spatial Structures from Models to Realization (September 20-24, 2004, Montpellier, France).*

Travelling in China and Taiwan, we can see stylized lion (*fu-dog*) statues in front of temples, palaces and public buildings, sometimes restaurants and homes. There are such statues also in Japan where the lion is called *koma-inu* (*koma* means Korea and *inu* a dog). These lions keep guard at the gate or entrance door and protect the building. They are in pairs. If we face the building, then the lion on the left is a female, while on the

right is a male. Very often, the left lion with her left paw holds down a baby lion, and the right lion has his right paw on a ball (Figure 1a). This arrangement is due to the Yin-Yang theory where opposite qualities, often complementary to each other, are in pairs such as left and right, female and male, ruler and compass, square and circle, earth and heaven, moon and sun, and so on (Miyazaki, 1986). Sometimes, lions on both sides have a ball. In Japan, it occurs that foxes replace lions, and the male fox keeps the ball in his mouth.

The lion is a symbol of power, but the meaning of the ball under the paw of the male lion is not clear enough. Some people think that the ball symbolizes the globe or the world, but others think that it represents a god. In many cases, the lions' ball looks like a winded thread decorated with embroidery and bows (Figure 1b). In Japan, such decorated spherical artifacts are called *temari*, and in the past, a *temari* was worshipped as an image of a god. At lion statues in Taiwan, the size of balls sometimes is so big that the lion climbs on it with two paws.

Decoration of the balls has a great variety, and very often its geometry is analogous to that of geodesic domes. The aim of this paper is to survey the different ball patterns from the point of view of structural morphology.

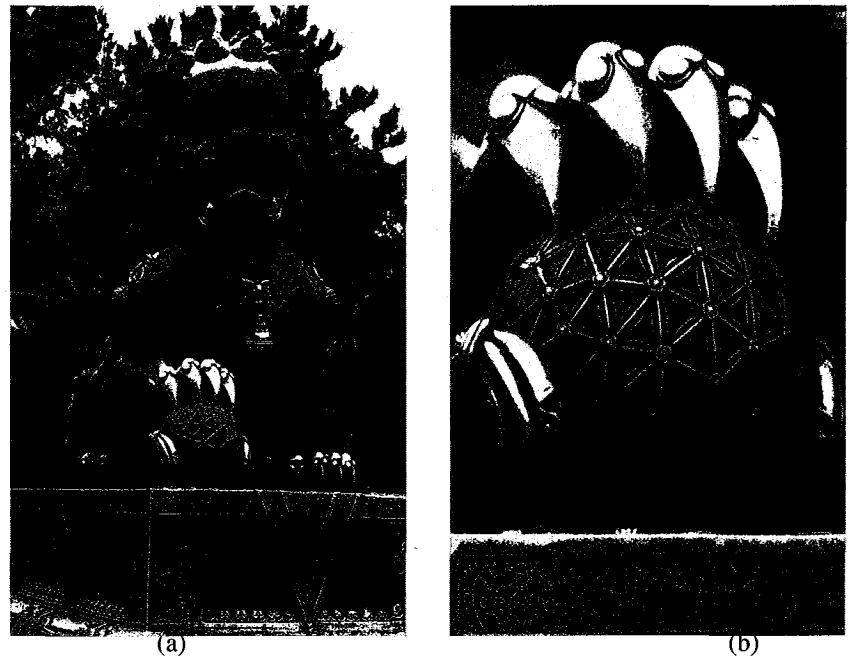


Figure 1. (a) Chinese lion in the Summer Palace, Beijing. Eighteenth or nineteenth century. (b) His ball showing a complete geodesic sphere. Geodesic domes in a broader sense are spherical domes composed of smaller units where the spherical surface is subdivided into triangles not too different from each other

(Tarnai, 1996). If the vertices of the triangles are considered as the basis of the construction, then four classes of geodesic domes can be defined. Interestingly enough, most lions' balls also belong to these four classes.

(a) *Spherical polyhedra with triangular faces*

The spherical triangular network directly serves as a base of the geodesic dome where the structure is composed of bars running along the sides of the triangles. A well-known classical example of these structures is the external layer of the US pavilion at the 1967 Montreal Expo designed by R. Buckminster Fuller and Shoji Sadao. Similar to this type of geodesic domes, the lion's ball in Figure 1 also provides a triangular subdivision of the spherical surface. It shows clearly the well-known fact that it is impossible to make a triangular subdivision using only 6-valent vertices. We need some vertices with valency less than six. And indeed, we can identify some 5-valent vertices in Figure 1b.

(b) *Spherical polyhedra with trivalent vertices*

The edge network of this sort of geodesic domes is the topological dual of a spherical triangular network. The structure is usually composed of hexagons and pentagons. The internal layer of the US pavilion at the Montreal Expo has this property. Pavlov (1987) has designed a number of domes of this type. Among the lion statues, for instance, the gold-plated male lion in front of the Gate of Heavenly Purity in the Forbidden City, Beijing has a ball belonging to this class. This is a remarkable example, and because of fullerenes, it is of interest to chemists (Hargittai, 1995).

(c) *Packing of circles on a sphere*

If in a spherical triangulation, the vertices of the triangles are considered as centres of non-overlapping equal circles, then a packing of equal circles is obtained on the sphere. Fuller's *fly's eye* domes are based on circle packings where the equal circles appear as circular openings on the dome (Tarnai, 1996). The idea of *fly's eye* domes comes from the American sculptor Kenneth Snelson's artistic atom model which looks like Chinese nested spherical shell carvings, where each layer encased by the others has equal circular holes [K. Snelson's personal communication to T.T.]. We found lion's balls with circle packing decorations mainly in Taiwan. Sometimes, the circles are drawn on the surface of the sphere, sometimes they appear as circular holes similar to that in the nested shell carvings. There are examples where the circles are at the vertices of a regular icosahedron or a rhombododecahedron, or are arranged along parallel small circles of the sphere.

(d) *Covering the sphere with circles*

If in (c), the radius of the circles is big enough, then we can obtain a system of circles covering the sphere without gaps. In this case a geodesic dome can be composed of equal spherical caps. Such a scaly dome was designed by Pavlov (1987). At lion's balls, this kind of decoration is the most common. In Japan, a covering system of equal circles fitted to a square lattice in the plane is called *shippo*. If this system is intended to be

used on the sphere, then the arrangement will be distorted or the size of the circles will not be equal any more. There are known many examples of both cases.

(e) *Other configurations*

There are some additional patterns on the lion's balls which are not directly related to geodesic domes: "melon" configuration defined by meridians, "bamboo framework" where the strips are running along longitudinal and latitudinal circles, and probably many more we are not yet familiar with.

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