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EXTENSION PROCESSES

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Abstract: *With extension processes we associate a partial structure and an extension process, a way of adding new elements to the already existing ones and/or a way for systematically modifying the already existing ones. In many cases the partial structure and the extension process define the final structure e.g. symmetries, incidences, types of subsets etc., but in other cases the result is ambiguous, there may be more than one non-isomorphic resulting structures. From an abstract point of view the question arises what kind of basic properties of partial structures, extension processes and extended structures can be identified. Some basic questions to be considered: does an extension process always define the final structure, or does the final structure depend on the partial structure as well? When can different extension processes lead to the same final structure? If a final structure is given, what kind of partial structures and extension processes can lead to it? Can we always find canonical partial structures? The lecture will provide examples and certain known partial results from geometry and biology. We will take a look at free geometries, look at homology and analogy in biological systems.*

1 EXTENSION PROCESSES IN GEOMETRY

By an extension process we understand a partial structure associated with an extension, that is a pair (A, F), where A is the partial structure, and F is the way we add new elements. An extension process may be discrete or continuous, according as the elements added are finite or infinite. In some cases elements may be added element-wise. We limit our study to extension processes where a limit (final) structure exists. In the element-wise extensions the process can be considered to be an ordered set, with inclusion as the relation among elements of the set. So an extension would look like the following: $A=A_0 < A_1 < \ldots\ldots < A_n < \ldots\ldots < (A,F)$

One very simple example of such an extension process is **free extension** in free geometries.

2 FREE EXTENSIONS

Here a partial structure is a set of points and lines, such that:

1. There is at most one line through any two points.
2. There is at most one common point on any two lines.

This is a partial projective plane. If in addition we require this partial structure to contain a rectangle (four points no three of which are on one line) than we get a non-degenerate partial projective plane, otherwise the structure is degenerate.

The extension is defined step-wise for *A*:

1. Add intersection points.
2. Add lines to every two points.

According to this extension we add to A_0 the intersection points of all the lines to get A_1, then we add lines (defined by two points) to A_1 to get A_2, and so on.
It can be proven that such an extension leads to so-called free projective planes, which do not contain Desargue-configurations, except possibly for the partial structure at the outset. It turns out that free projective planes always have unique canonical representations, which may be represented by the simple structure below:

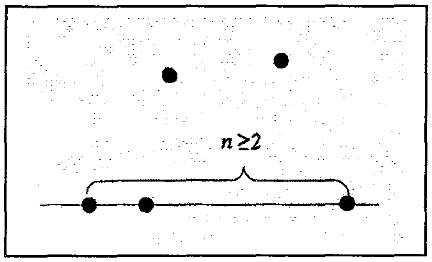

Figure 1: Canonical partial free projective plane

3 EXTENSION PROCESSES IN BIOLOGY

In nature evolution itself may be thought of as an extension process. Here a partial structure may be any living organism on which evolution acts (certain conditions like: hot and dry weather, limited food supply, space limitations etc.), and the final structure is the adapted organism to the given conditions. In this sense anatomy (form) and behaviour may play key roles, as the organism will try to adapt in every possible way. The extension may best be considered as continuous, although if we are rigorous it is still discrete. (There are a finite number of substances changing on the atomic level). In this case there is always an outcome. If we consider offspring then we get a cascade of extended structures, which may nevertheless be considered as subsequent extensions. Below is an experiment showing the development of a constrained plant. Here the extension process is the constraint presented by the net to which the plant adapts.

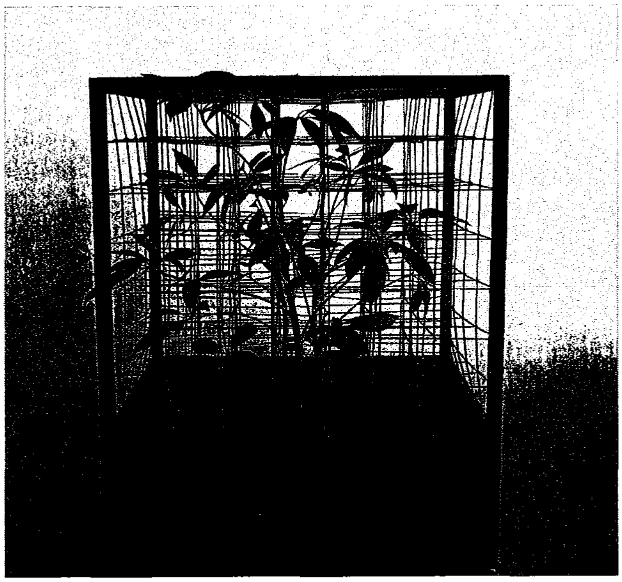

Figure 2: A plant evolving under the constraint of a net

4 HOMOLOGY AND ANALOGY

There are numerous examples for homology. In fact all vertebrates can be considered as the result of evolution from the same organism type. Even though the extension process may have been similar different species evolved from this ancestor. In many cases the differences can be linked to different extension processes.

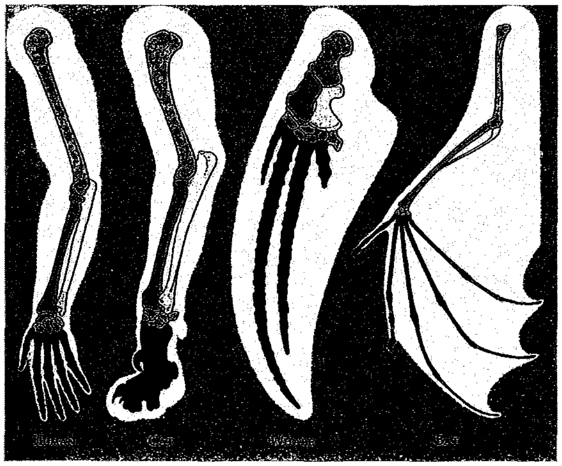

Figure 3: Homology

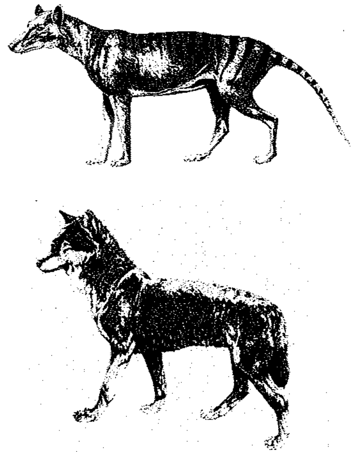

Figure 4: Analogy

There are many examples in nature for homology. Here two distinct partial structures lead to similar forms and function. The extension processes may be very similar, and yet clearly different. Examples of such extensions would be the wolf and the marsupial wolf, which evolved independently from two distinct ancestors, and yet show striking similarities in size, anatomy and behaviour.

References

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