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SYMMETRY IN MATHEMATICAL TOYS

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Abstract: *Mathematical toys can be very instructive, particularly if they display certain unusual types of symmetry. The present talk will illustrate this theme with some examples: first from a seminal movie, "Some Mathematic Toys," made in 1963, where a toy consisting of tetrahedral and octahedral blocks, contained in thin-walled boxes of the same two shapes will be shown. A simple technique for constructing models of polyhedra, either self-reflexible or unselfreflexible (cheiral), out of galvanized wire will be demonstrated. A recent toy (RHOMBO) consisting of "golden rhombahedra" of two classical shapes, constructed so as to hold together magnetically, will be demonstrated, with its use for explaining the structure and the growth of quasi-crystals.*

1 INTRODUCTION

Intelligent people of all ages can derive great pleasure from contemplating and playing with mathematical toys, and such toys may be of great help in the teaching of mathematics. Instruction is best accompanied by pleasure.A mathematical toy is to be distinguished from a puzzle, which requires ingenuity for its solution. A puzzle is hard work, a toy is fun.Examples of symmetry in mathematical toys will be shown first in a movie: "Some Mathematical Toys," (Longuet-Higgins, 1963). Then we discuss the use of symmetry in making polyhedron models. Finally we demonstrate a new toy (RHOMBO) consisting of magnetic blocks in the form of golden rhombohedra.

2 EXAMPLES FROM AN EARLY FILM

2.1 Uniform tessellations

From sets of regular polygons one can construct uniform tessellations, in which the same arrangement of polygons occurs at each vertex. The example shown is unselfreflexible or cheiral, that is to say it differs from its mirror image. Uniform tessellations can also be constructed from 12-sided coins.

2.2 Matt's Bricks

In three dimensions the commonest building blocks are cubical or rectangular, but other types are more attractive. A toy, "Matt's Bricks" (designed and made by the author in 1952 for Matthew Hinton, as a present on his sixth birthday), consists of coloured tetrahedra and octahedra in similarly shaped boxes. The blocks can easily be made from painted cardboard.

2.3 Stellations of regular polyhedra

Other examples of other polyhedra constructed in this way include the five regular polyhedra and their stellations, some of which are cheiral.

2.4 Compound polyhedra

Symmetric combinations of the regular polyhedra include the "2-dodecahedra" and "2 icosahedra" (which have cubic symmetry); "3-cubes," "3-octahedra" and "6-tetrahedra;" "4-cubes," "4-octahedra" and "8-tetrahedra," and the well-known "5-cubes," "5-octahedra" and "10-tetrahedra."

2.5 Wire models, and their construction

There is another method of construction which is essentially simpler. The only material used is garden wire, and the only tool is a pair of pliers. A cuboctahedron, for example, is constructed from 4 hexagonal rings, which hold together through their springiness. Such models have been made of all the uniform polyhedra. Examples include the "snub" uniform polyhedra, some of which are unselfreflexible.

2.6 Flexible models: degrees of freedom

An interesting and delightful toy consists of a set of wooden rods and flexible (plastic) joints. With this one can construct polyhedra which are either "just stiff," or which have a certain number of degrees of freedom. After removing 6 of the edges of a stiff icosahedron in a cubically symmetric way, the structure can be made to expand into a cuboctahedron or to collapse into an octahedron or even further into a flat triangle. Further, one can construct a flexible ring of 6 or more tetrahedra which can be used to demonstrate a generalization of Euler's theorem on the number of faces, edges and vertices of any convex polyhedron.

2.7 A model of DNA

From the toy "Playsticks" one can construct a double helix with cross-links, which illustrates the structure of DNA, the molecule that carries most of the genetic information in living cells.

3 SYMMETRY IN MODEL-MAKING

Symmetrical subgroups are very useful in the construction of models of polyhedra. This is illustrated by a wire model of G_{60}, one of the stellations of the triacontahedron (see Longuet-Higgins, 2001)

4 A TOY USING "GOLDEN" RHOMBOHEDRA

Lastly we demonstrate a new toy, "RHOMBO", consisting of blocks in the form of golden rhombohedra. From these can be made many structures with varying types of symmetry, including the "Kepler Ball," whose outer surface is a triacontahedron, and the "Kepler Star," whose surface is one of the stellations of the triacontahedron. Handling the blocks is facilitated by making each block so as to attract each other block magnetically. The blocks can be used to illustrate the structure of quasi-crystals, and to show how a quasi-crystal can grow by the addition of successive thin layers of atoms; see Longuet-Higgins (2002). Each additional layer is no more than 4 blocks thick.

References

Coxeter, H. S. M., Longuet-Higgins, M. S., and Miller, J. C. P. (1954) Uniform polyhedra, *Phil. Trans. R. Soc. London,* A 246, 401-450.

Longuet-Higgins, M. S. (1963) *Some Mathematical Toys* [film], Aberdeen, Scotland, U.K.: British Association Meeting.

Longuet-Higgins, M. S. (2001) On the use of symmetry for constructing polyhedron models, pp. 11-18, In: *Symmetry 2000*, eds. I. Hargittai and T. Laurent, London, U.K.: Portland Press, 800 pp.

Longuet-Higgins, M. S. (2002) Nested icosahedral shells, or How to grow a quasi-crystal, *Math. Intelligencer*, 25, 25-43.