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2-MANIFOLDS SURFACES AND SPACE PARTITIONS

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Abstract: *The issue of space partitioning underlies the architectural planning and design of buildings, structures and spaces intended for human activity.*
This thesis explores the phenomenon of periodic dual spaces, the periodic three-dimensional networks that represent their inner structure and the partition between them.
It is assumed that every dual-pair of networks, often referred to as complementary or reciprocal pairs, can be separated and partitioned by a continuous smooth hyperbolic surface.
The thesis is focused on the unique phenomenon of identical dual spaces and the related network pairs' and the hyperbolic surface-partitions, separating them and thus dividing the entire space into two identical (complementary) sub-spaces.
The adopted approach implies investigation of order and organization of these spaces and related parameters, their inner and overall symmetry structure and the nature of the 2-manifold partitions, dividing between them.
An additional central goal was to conduct a systematic exhaustive search of possible (thus defined) surface-partitions, in order to establish their range of existence and to facilitate their classification. The study described in this thesis comprises three stages.

1 TOPOLOGICAL ATTRIBUTES OF SMOOTH 2-MANIFOLD WHICH DIVIDE THE SPACE INTO TWO IDENTICAL SUBSPACES

The first stage consisted of studying identical dual spaces, their properties and parameters trying to develop insights into their nature and order.
The tunnel-like periodic spaces, represented by networks, are defined within the Euclidean three-dimensional space, and are composed of periodic cells. This property indicates the relation between the network-pairs and the surface separating them and the symmetry groups acting on this space.
The smallest repetitive cell that is derived from a periodical space using the symmetry operations of the symmetry group that acts on this space is called "Elementary Periodic Region" (E.P.R.). The E.P.R. contains complete representation of all the phenomena taking place within the "periodic complex" and particularly, representation of the whole periodic space, its symmetry group, the two complementary (identical) subspaces, the self dual lattice-pairs characterizing them, as well as the surface partition in between.
At this stage of the research, the topological properties of smooth 2-manifolds in general, and of smooth 2-mainfolds which divide the space between two dual networks in particular, were explored.

2 THE METHOD OF SEARCHING AND CLASSIFYING OF 2-MANIFOLD

The main properties of these 2-manifolds are (Figure 1):

a. They are smooth, periodical and hyperbolic, and exist in the three-dimensional space.
b. They divide the space into two identical subspaces, that graphically represent two interwoven and non-intersecting tunnel-networks. The two complementary tunnel systems are identical in volume and shape, and their tunnel axes form two identical, three-dimensional space networks.
c. 2-fold rotation axes contained within the 2-manifold rotate one subspace into the identical complimentary subspace, forming in it a periodic (three-dimensional) space network, referred to as a "2-fold network".

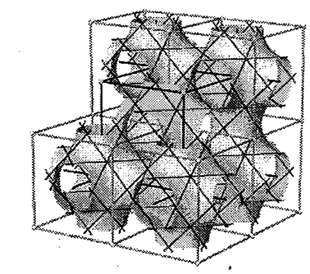

Figure 1

An unequivocal distinction was made between minimal hyperbolic surfaces that can be physically realized by dipping a defined perimeter in soap solution (henceforth "soap membrane networks") and minimal surfaces physically non-solvable through a dipping process.
In the second stage of the research, a method for the enumeration and classification of the 2-manifolds that divide the space into two identical subspaces and identification of the axes of the identical dual tunnel networks was developed. The method of

enumerating the 2-manifolds was based on their topological properties, that were studied in the previous stage.

It is noteworthy that the issue of periodic minimal hyperbolic 2-manifolds that divide the space into two identical subspaces were investigated in the past with special regard to the first category of minimal surfaces realizable with a dipping process and related networks. Seven surfaces corresponding to these characteristics were found. The second category, that of periodic minimal surfaces which cannot be physically realized by dipping the perimeter in the soap solution, was investigated in connection with the division of the space into two identical subspaces in a very preliminary manner. It was clear even then that numerous identical networks and a smooth hyperbolic partition dividing them could be put through every E.P.R. containing 2-fold axes. No attempt has been made to date either to construct and characterize these surfaces or to define the range of their existence and to provide their classification.

The existence and periodicity of the 2-manifolds that divide the space into two identical subspaces in the Euclidean three-dimensional space indicate the link between these 2-manifolds and the symmetry groups operating within this space.

The "atomistic" conception of space suggests the existence of an E.P.R. that represent all the properties of the complex. Finding all the E.P.R.s, which represent identical-dual spaces and the 2-manifold in between them, leads to discovery of the 2-manifolds.

The number of the different E.P.R.s is finite due to the fact that the number of symmetry groups, which operate in the Euclidean space is finite.

The method for the enumeration and classification of the 2-manifolds consists of several consecutive steps described below; each step narrows down the scrutinized area and bring us closer to the final goal.

a. Locating all the E.P.R.s derived from the symmetry groups.
b. Reducing the list of the relevant E.P.R.s to those containing 2-fold axes, capable of rotating the E.P.R. into itself.
c. Obtaining the 2-fold networks by repetitive duplication of the E.P.R.s found in stage "b".
d. Locating periodic cells enclosed in the 2-fold networks (more than one is likely), which may enclose a periodic unit of the 2-manifold, and conducting exhaustive search to the possible 2-manifold units (Figure 2).
e. Duplicating the periodic cells with the enclosed periodic unit of the 2-manyfolds, until a large enough section of the surface, enabling identification of the self-dual tunnel networks is obtained.
f. Topological enumeration of each of the 2-manyfolds, according to their respective tunnel network.

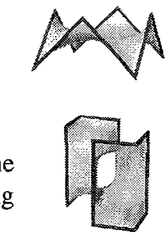

Figure 2

3 THE PROCESS OF SEARCHING AND CLASSIFYING OF THE REQUIRED 2-MANIFOLD

In the third stage, by applying the previously developed method a process of the actual identification of the self-dual spaces was carried out.

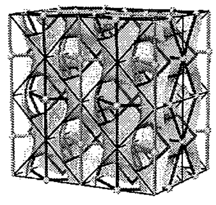

Figure 3

Finding the 2-fold networks is supposed to be exhausted at this stage. The method of locating the 2-fold networks is based on identifying the E.P.R.s that may, as stated above, contain 2-fold axis that rotates them into themselves. It is not a combinatorial method, and hence we may have missed some of the "2-fold networks" in our search.
At this stage fourteen different "2-fold networks" were found.
Among those twelve were found to contain seventeen different closed periodic cells that may enclose a periodic unit of a 2-manifold.
With regard to the first category, duplicating these closed cells – "the soap membrane networks" – led to the discovery of eight topologically different 2-manifolds so far.
Among them, a new, unknown to date, 2-manifold was discovered, which was designated "The Cubic Diamond 2-Manifold." (Figure 3)
Among the seventeen different closed periodic cells, mentioned above, there are six closed cells that consist of a "split perimeter". These "split perimeters" can enclose a "periodic 2-manifold unit" of the second category surfaces, i.e. they can enclose in the periodic closed perimeter cell a successive smooth hyperbolic "periodic 2-manifold unit", which is topologically different from the minimal "periodic 2-manifold unit" previously discovered.
Thus a new class of 2-manyfolds, designated "The Multiple-Sleeved Class" was found (Figure 4). This class contains infinite number of topological different 2-manifolds.

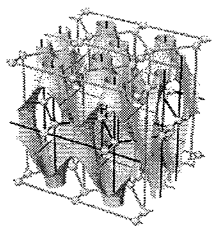

Figure 4

This indicates that the tunnel axes of these 2-manifolds represent pairs of identical-dual networks that differ from each other.
The proposed method of identifying the 2-manifolds that divide between pairs of self-dual networks, led to the discovery of new 2-manifolds in addition to the seven 2-manifolds that two of them were first discovered by Hermann Amandus Schwarz (1843-1921). The 2-manifolds were looked for in representative groups, relating to symmetry groups, the number of which is finite. This does not necessarily mean that the number of 2-manifolds and/or the number of self-dual pairs of networks are also finite.

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