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VISUALIZING SYMMETRY OF KNOTS BY USING PROGRAM *LinKnot*

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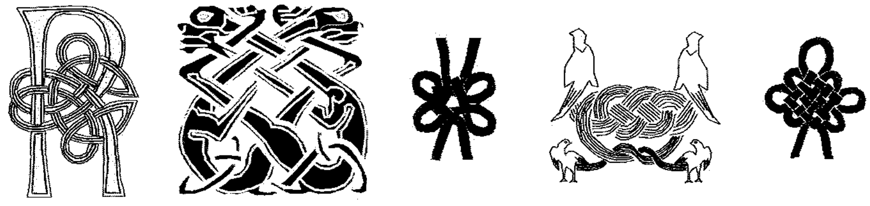
Abstract: *Some of the features of the Mathematica-based program* ***LinKnot*** *written by the authors are used in visualizing symmetry properties of knots and links. By exporting images from* ***LinKnot*** *to* ***KnotPlot,*** *it is possible to produce animations illustrating amphicheirality or periodicity. The most symmetrical projections of knots and links, as well as torus knots and links, can be used as a basis of symmetrical knot-art works.*

1 INTRODUCTION

Every knot or link (*KL*) inputted in *LinKnot* by its Conway code can be visualized directly as a *Mathematica* graphic by using the function ***ShowKnotfromPdata***, shown in ***OpenGL*** by ***ShowKnotbyOpengl***. *KL* can be exported to ***JavaView***, or to the program ***KnotPlot*** created by R. Sharein, that gives a possibility to relax knots and links, produce their 3*D* images and animations. In this article, we will consider visualizations of knot-theoretical properties discussed in the paper "Discovering symmetry of knots by using program *LinKnot*".
Decorative knots have been used from prehistory till our days as a basis of different artworks, so they still can be an inspiration for artists and artisans. Fig. 1 shows two Celtic knots and Chinese decorative knots, and Fig. 2 shows an "impossible trefoil" created by the first author.



(a) (b)

Figure 1

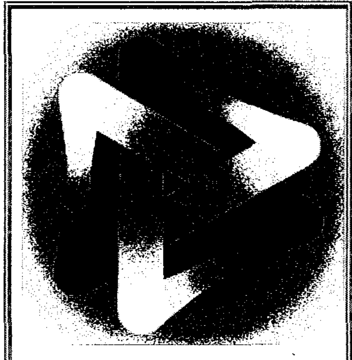
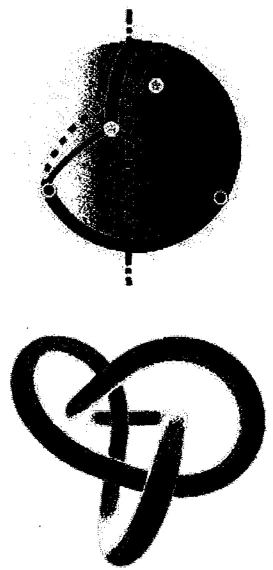


Figure 2

2 VISUALIZING AMPHICHEIRALITY

In the case of rational knots, all amphicheiral knots are derived from the same source: from the figure-eight knot 2 2 (or 4_1 in the classic notation). The amphicheirality of the figure-eight knot can be visualized by an animation consisting from a series of ambient isotopies transforming a "left" figure-eight knot to the "right" (Fig. 3). For the knot 2 2, the graph symmetry group is G = [2+, 4], and the knot symmetry group G' = [2+, 4+] is generated by the rotational reflection, with the axis defined by the midpoints of colored (i.e., double) edges of the tetrahedron (Fig.4). Considering the sign of the vertices, it is a rotational antireflection. Its effect is preserved in all rational knots with an even number of crossings that have a symmetrical Conway symbol. As the result, we obtain a series of centro-antisymmetrical graphs corresponding to rational knots (Fig. 5).

Figure 3



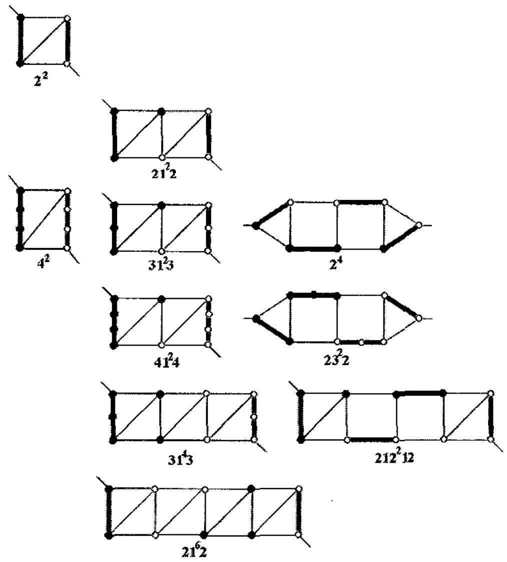


Figure 4

Figure 5

3 VISUALIZING PERIODICITY

Beautiful knot-art images can be produced from periodic knots, like basic polyhedra. A series of *KL*s corresponding to antiprismatic basic polyhedra (2n)*, beginning from Borromean rings 6* and containing 8*, 10*, 12*, *etc.*, is illustrated in Fig. 6. The result of the relaxation of a periodic link 10*2 1.2 1 0.2 1.2 1 0.2 1.2 1 0.2 1.2 1 0.2 1.2 1 0 is illustrated in Fig. 7.

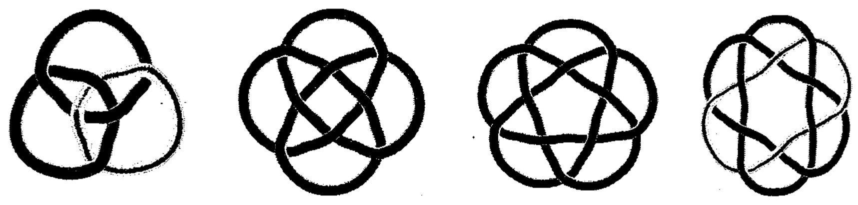


Figure 6

Figure 7

References

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