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SIMPLE GOLDEN SECTION EXAMPLES

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More information is found in another Tihany Congress contribution, entitled "Simple-minded golden section examples"; see Huylebrouck 2004.

Abstract: *While some "applications" of the golden number $\phi = 1.618...$ can be called "simple-minded", the present text shows how the (equally simple) quadratic equation for the golden section easily permits the "discovery" of more (amusing) golden section applications. It is hoped that the focus on simple and simple-minded examples of the "divine" proportion may encourage an "earthly" discussion.*

1 INTRODUCTION

Following Euclid's "rule", the golden number (or golden section, mean, or ratio, or divine proportion, etc.) arises "when a line segment of length x (>1) is divided into two pieces of lengths 1 and x-1, such that the whole length is to 1, as 1 is to the remaining piece, x-1". This produces the equation $x^2 - x - 1 = 0$, of which $\phi = (1+\sqrt{5})/2 \approx 1.618...$ is the positive solution. Note that $\phi^2 - \phi - 1 = 0$ implies $\phi - 1 = 1/\phi \approx 0.618...$, which is a solution of $x^2 + x - 1 = 0$. Recent discoveries of a pentagonal structure in quasi-crystals and DNA-structure, justified the introduction of a "golden angle" of 36°, as $sin(2\times36°)/sin(36°)=\phi$. While the pentagonal property may be the reason for the occurrence of the golden number in elementary geometry, population growth models may appeal on the number since $\phi^2 = \phi + 1$, implying that the arithmetical progression 1, ϕ, $1+\phi$, ... is a geometrical one as well. Furthermore, the ratio is ϕ, and since the proportion of consecutive Fibonacci terms 1, 1, 2, 3, 5, 8, ... tends to ϕ as well, the occurrence of ϕ in many unpretentious proportions of 2 to 3, 3 to 5 etcetera is not surprising. Nevertheless, when the context is different from artistic proportions or population growth, ϕ's apparition may still come as a surprise. For instance, when gears allow to reach the consecutive speeds v_1, v_2, v_3, ... it seems natural to require that this increase in velocity uses the same proportion, while technical cog wheels constructions may lead to an addition property of these speeds (see, Huylebrouck, 2002). Thus, the v_1, v_2, v_3, ... should be arithmetical and geometrical, that is, $v_2 = \phi v_1$, and $v_3 = \phi v_2 = v_1 + v_2$ (see Fig. 1).

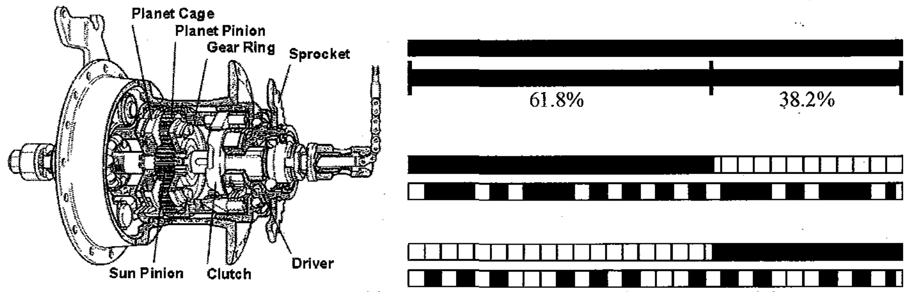

Fig. 6 : A cogwheel for adding golden gears, and the Euclidian line segment subdivision.

2 MORE SIMPLE BUT TRUE EXAMPLES

Though it should become better and better known in these recent times that many occurrences of the golden section must be considered with the greatest precaution (see Huylebrouck, 2004), some still hold on to their faith in the "divine" proportion. To Underwood Dudley, the simplicity of the golden proportion may be a reason for its alleged frequent occurrence (see Dudley, 2001): *"It is an instance of Richard Guy's Law of Small Numbers, that there are not enough small integers available for the many tasks assigned to them. The origin of* ϕ *is* $x^2 = x + 1$*, a quadratic equation with very small integer coefficients. [...] If* ϕ *were a root of* $25x^2 = 26x + 24$*, then I would be surprised to encounter it in more than one place, but as it is I am no more astonished than I am that there are three dimensions, three ships of Christopher Columbus, three subjects in the trivium, three degrees of burns [...]. Three comes up a lot, and so does* $x^2 = x + 1$*."* So, let us take up the challenge, and create more "golden" examples. We start with Euclid's subdivision of a line, tear the pieces apart, and scramble the remaining white parts slightly (arbitrarily would have been better, but then the drawings may be harder to track). These subdivisions will be interpreted as transparent pixels, as holes, as free time on an agenda, or as nonsense in a scientific paper.

3 GOLDEN GRAY

We start by stretching the Euclidian line subdivision, with 61.8% black and 38.2% black, to form squares. If the layer with 38.2% black is put on top of an identical layer with 38.2% black lines (twisted over 90° here for sake of clarity), the golden section property implies that the result is again the first 61.8% black layer. This set-up is applied for a rotating disk, covered for 50% with a layer with 61.8% transparent pixels and for another 50% of a double layer, thus containing only 38.2% transparent pixels. Since ½.61.8% + ½.38.2% = 50%, the result will be a 50% gray. Here, a 50% gray is supposed to contain exactly 50% black and 50% white pixels, but for more technical considerations about gray, "pixels", and colors on rotating disks, we refer to Huylebrouck (2000-2002).

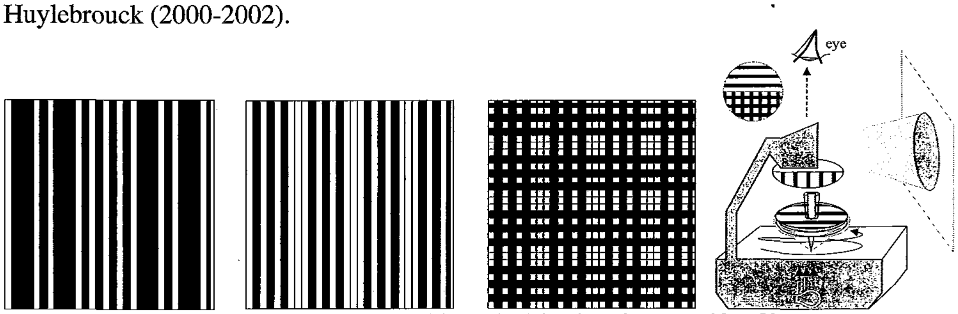

Fig. 7 : A rotating disk covered for ½ by each of the above layers, yields a 50% gray.

4 GOLDEN TASTE

Now we interpret the white squares of the lines constructed following Euclid's rule as the holes on a salt pot. Suppose a salt pot uses 2 overlapping layers of holes, as is the case in models regulating the amount of salt by slipping one layer over the other. A meal without salt can be quoted "0", while the maximum amount of salt one can support will be attributed 1. We imagine the corresponding number of holes for the salt pot can be measured and that opening half of all holes corresponds to an ideal "50% salty sensation". A gastronomical question now is how to distribute salt on a dish so that a double dose of salt falls on French fries and a single dose on a steak, together yielding an average 50% sensation. The answer is again the golden number: open 61.8% of the salt potholes for the fries, and 32.8% of the holes for the steak.

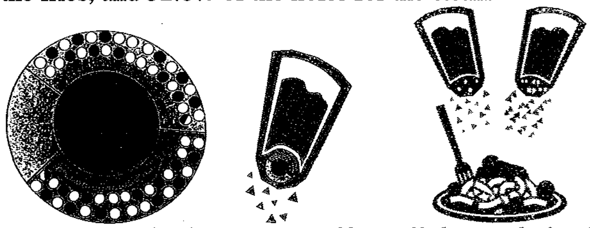

Fig. 8 : Some salt pots can be regulated moving a second layer of holes over the first (ideally with more non-overlapping holes). Right: 61.8% of salt on fries, and 38.2% on a steak is a 50% sensation.

5 GOLDEN SCHEDULE

We now imagine the white spots or holes on the golden segment subdivision correspond to empty spaces on a schedule. For instance, consider a two-months calendar, of a total of 60 days, to keep the numerical aspects simple. For half the time, a single schedule is followed: $x\%$ of time will not be allocated to work and free for a hobby. For another half, a double schedule will be followed, of $x^2\%$ of time not attributed to work. What should be the ideal schedule so that the single and double schedules together yield a 50% agenda? Again, x = 61.8%: if someone works for 38.2% = 1- 0.618 of the time from January 1st to January 30th, and with a double intensity at 61.8% = 1- 0.618^2 of his time from January 30th to February 29th, he will have gotten half of his time available for his hobby.

January 2004						
Su	Mo	Tu	We	Th	Fr	Sa
				1	~~2~~	~~3~~
~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17
~~18~~	19	~~20~~	~~21~~	~~22~~	23	~~24~~
25	~~26~~	~~27~~	~~28~~	29	~~30~~	31
7:○	15:◑	21:●	29:◐			

February 2004						
Su	Mo	Tu	We	Th	Fr	Sa
~~1~~	2	~~3~~	4	~~5~~	6	7
8	~~9~~	10	~~11~~	12	~~13~~	14
15	16	~~17~~	18	~~19~~	20	~~21~~
22	23	24	~~25~~	26	~~27~~	28
~~*29*~~						
6:○	13:◑	20:●	28:◐			

Fig. 9 : Crossing out 18.5 days from 01-1 to 01-30 (a single slash is half a day), but only 11.5 days from 01-30 to 02-29, creates a single and a double schedule, with an average of 30 days.

Scheduling conference participations is a golden section problem too. Start by writing a paper summarizing golden section facts, checking $x\%$ of the proposed facts, and still containing $(1-x)\%$ of eventual nonsense. Send this version to half of the conferences spamming you with invitations. Next, double-check $x\%$ of the alleged true facts, and propose this second version to the other half of meetings you happen to know of. The border line of 50% of chance you may participate to a no-nonsense conference is reached if x=61.8%, as illustrated by the present conference contribution.

References

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