ARCHITECTURE, MATHEMATICS, AND A "SYMMETRIC LINK" BETWEEN THEM.

(From the Atomium building to the Mat@mium project)

DÉNES NAGY

Dedicated to

- János Bolyai, the co-discoverer of non-Euclidean geometry, who was born 200 years ago in 1802, and

- Victor Horta, the pioneer of Art Nouveau, who designed the Tassel House, his landmark building, 110 years ago in 1892.

1 Architecture and mathematics? (from the dome of the Eskimos to the St. Paul Cathedral)

There is an intricate relationship between architecture and mathematics since prehistoric times. Making an artificial dwelling obviously required some sort of planning and inspired geometry-related ideas. We may call this knowledge as proto-mathematics or ethnomathematics. Gradually, some people developed special skills in such questions, and they were able to initiate more complex designs. Such an interaction between proto-architecture and proto-mathematics led to the development of both sides and to various ingenious ideas. For example, the Eskimos invented the structure of the dome in the form of '*igloo,* their typical dwelling, and widely used it perhaps earlier than Roman architects. Turning to African culture, the studies of Paulus Gerdes demonstrated the importance of various mathematics-related ideas in design, while Ron Eglash pointed out the appearance of self-similarity and even fractal geometry in traditional architecture. We may believe that some of these geometrical structures are very old and kept "invariant" by generations of craftsmen. We also should pay a special attention to wooden buildings that were developed in many regions in different forms. Although the ancient versions of these buildings do not survive, the lashing technique used in Oceania and the art of joinery mastered in China, Korea, and Japan give an insight into the possible methods that were used for fixing these structures. Both of these are associated with mathematical problems, including knot theory and solid geometry. Turning to the architecture with stone blocks, the art of cutting stones and fitting them together, often without using binding materials, required a high-level of geometrical accuracy. This craft is called *stereotomy*, but in mathematics we may characterize it as constructing 3-dimensional tilings. "Geometry" (*geo + metron*, i.e., land-measuring) was obviously necessary for marking out the foundation of a new pyramid, palace, or temple, and, during the completion of the buildings, further mathematical ideas were used. The Rhind Papyrus, for example, includes a mathematical problem that is associated with the angle of a pyramid. Around the 4th century B.C., the birth of deductive mathematics with axioms, theorems, and proofs marked a split between mathematics and architecture. In Euclid's *Elements* (*Stoicheia*, perhaps around 300 B.C.) there is no reference to architectural problems, this is a work of "pure" mathematics. In Vitruvius's *Ten Books on Architecture* (*De architectura libri decem*, 1st century A.D.) we see some mathematics, but there is no reference to Euclid. I even suggested speaking about Euclidean (or deductive) mathematics vs. Vitruvian (or craftsman's) mathematics (c. f. *Mathematics and Design 98*, San Sebastian, 1998, pp. 17-25). However, the two types of mathematics were not totally separate. For example:

- a part of the so-called 15th book of Euclid was written, with great probability, by a pupil of Isidorus of Miletus, the architect of the Church of St. Sophia in Constantinople (now Hagia Sophia or Aya Sofya in Istanbul), in the 6th century A.D.,

- the builders of the Milan cathedral consulted with the mathematician Gabriele Stornaloco, whose sketch and calculations survive form the late 14th century,

- the mathematician Al-Kashi included problems related to architecture in his mathematical treatise in the 15th century, and

- major figures of the Renaissance dealt with both architecture and mathematics.

In most cases, however the master masons or the architects constructed their buildings without mathematical assistance and used their own skills. Interestingly, they "solved" higher-degree equations and "simulated" complicated curves and surfaces without the utilization of advanced mathematics and long before the age of computers. Their usual secret was a set of simple geometric rules and the method of trial-and-error. Indeed, many Gothic churches collapsed several times before the master masons determined the suitable shape of the structural elements. In some cases, geometric scale models helped the work, including the problems of constructions and stability. A famous example was the brick and plaster model of the Church of San Petronio in Bologna in the 14th century. Note, however, that the questions of strength cannot be studied in scale models, as Galileo pointed it out in 1638. He illustrated the problem by the different shapes of leg-bones of small and big animals. Instead of geometric similarity, we see much thicker bones in the case of large animals. Specifically, the mechanical strength of a structural element varies with the cross-sectional area, hence with the square of its linear dimensions, while the volume varies with the cube of its linear dimensions (square-cube law). Perhaps masons also used an original method of "simulating" the optimal curves and surfaces by mechanical analogies. For example, by hanging a cord between two points we get a curve that can be used, after inverting it up-side down, as the shape of an arch. Although we do not know too much about such methods in the Middle Ages, perhaps these were kept secret by the free masons, we have an insight into the importance of hanging experiments from later periods. Here we may mention Sir Christopher Wren's design of the St. Paul's Cathedral in London, and more recently Antonio Gaudi's workshop at the Sagrada Familia in Barcelona. With the names of Galileo Galilei and Sir Christopher Wren, we also reached a point where applied science started to replace the Vitruvian mathematics and the medieval traditions in architecture. Galileo's referred to work contributed to the birth of structural design, and Wren was originally a professor of astronomy and made important contributions to mathematics, including the rectification of the cycloid ("On finding a straight line equal to that of a cycloid and the parts of thereof", *Philosophical Transactions of the Royal Society*, November 1673). Characteristically, the second edition of Newton's *Principia* (1713) mentioned Wren, together with Wallis and Huyghens, as the leading geometers of his age, while the general public recognized him as the person who rebuilt many churches after the Great Fire of London in 1666. However, the specialization of mathematics made less probable the "personal union" of the work of the architect and of the mathematician. The rapid developments led to separate fields, which are manifested in Departments of Mathematics and Schools of Architecture at the universities. These institutions rarely cooperated, although the "Euclidean distance" between them on the campuses became smaller than that one in the Hellenistic age.

2 From the "handcuff" of historic styles to the "freedom" of form in Art Nouveau (from a gardener's contribution to an architectural revolution in Belgium)

Before turning to the modern relationships between architecture and mathematics, we should make an outlook to some interesting developments in the history of design and architecture. In the 18th and 19th centuries, most architects still focused on the revitalization of some historic styles and the decorative art also followed this tendency. However, the political changes that were associated with the French Revolution in 1789 and later with Napoleon's empire opened the door for new ideas in art and architecture and made Paris the center of influence for a long period. Interestingly, a revolutionary movement in French architecture did not follow the events, but became a forerunner. Shortly before the revolution, Étienne-Louis Boullée's and Claude-Nicolas Ledoux's presented their utopian designs, and later their student Jean-Nicolas-Louis Durand introduced his composition principle based on the combination of basic geometric units. Although most of their designs, including Boullée's enormous spherical building, the Cenotaph for Sir Isaac Newton (1784), were never executed, their works and geometrical ideas became influential for later generations via Durand's textbooks. Their fresh approach, however, did not prevent the majority of architects from continuing the application of historic styles. It is true, that this historicism was not always a way of simply looking backwards. Eugène-Emmanuel Viollet-le-Duc studied the Gothic construction principles as an example for "rational architecture" and his goal was to create a modern parallel of it by using iron. As a prolific writer, he devoted a ten-volume dictionary to this topic (Paris, 1854-1868). He was also a leading, although controversial, figure of restoration of ancient monuments and wrote a monograph on the geodesic and geologic "construction" of the Mont-Blanc and its glaciers. In the mid-19th century, another and more radical inspiration came from "outsiders" who introduced a new structural rationality into architecture and design. The rapid development of industrial production provided new materials and new possibilities. The gardener Sir Joseph Paxton's iron-and-glass Crystal Palace for the Great Exhibition in London in 1851, which was also marked by the 1851-foot length of the building, contributed to the birth of a new aesthetics. Later the engineer Alexandre-Gustave Eiffel continued this process. He designed a 300-meter-high wrought iron Tower for the World's Fair in Paris in 1889. (The earlier invention of the elevator was also essential for the Eiffel Tower, while the utilization of steel was, surprisingly, missed.) The changing focus of these "expos" is also characteristic. While the central part of the Crystal Palace was a gigantic hothouse for a horticultural exhibition with a hidden drainage system that utilized the hollow cast-iron pillars, the later Paris World's Fairs gave more emphasis to machines and industrial products. The "machine age", however, gained not only support, but also a strong opposition with social criticism: the Arts and Crafts Movement, which was initiated by John Ruskin and William Morris in England, promoted the value of handicrafts.

Figure 1 Victor Horta's Tassel House, Brussels (1892-93).

This tradition was also continued in the "New Art" or *Art Nouveau*, which suddenly appeared in architecture, first in Belgium in the 1890s. Its innovator, the architect Victor Horta, ignored the traditional styles and introduced a new form-language for architecture, which is dominated by curves, organic motifs, and the interplay between symmetry and asymmetry. His Tassel House in Brussels (1892-93) was a *Gesamtkunstwerk*, the synthesis of the arts, where all details, from architecture to internal decorations, from metalwork to furniture, were carefully executed according to his plans. Horta's building is a celebration of the freedom of form, the imaginative use of geometry, and the harmony among various fields of art, as well as the promotion of the use of iron as structural material. Remember that Viollet-le-Duc's also suggested the latter. Horta made this move in a period when even the machines were often decorated with Greek columns or Gothic arches. This "stylization", however, became immediately nonsense and the designers of Art Nouveau were able to concentrate on the optimal form of objects, which is required by their function. Horta was an architect who became involved in the design of almost everything inside the building, while another Belgian artist, Henry van de Velde took the opposite direction from paintings to design and architecture. This was the time when the role of Paris as the "global center" of new art movements was challenged: an interesting "anti-globalization" and "de-centralization" was marked by pioneering achievements in architecture and design in Belgium and many other countries. In Germany, Velde moved there in 1897, the new style was first called as "Veldean" (*Veldische*) or "Belgian" (*Belgische*) and later simply Youth-Style, *Jugendstil*, which reflected not only the young age of the artists, but also the related interest of the Munich-based magazine *Jugend*. In Austria and Hungary, however, Secession-style or simply Secession (*Sezessionstil* in German, *Szecesszió* in Hungarian) became the conventional expression. There were about a dozen different names for very similar movements. Some of these new centers were directly related, while others worked independently, from Catalonia (Antonio Gaudi) to Hungary (Ödön Lechner). In Art Nouveau, the main inspiration of a new form-language came for organic nature, especially from botany. Samuel Bing, who opened his gallery and shop *L'Art nouveau* in Paris, claimed that nature is "the infallible code of all laws of beauty" (in the journal *The Craftsman*, Vol. 5, October 1903). Some artists published botanical and horticultural articles (Emile Gallé, Eugène Grasset), while Ernst Haeckel, a leading biologist with a special interest in evolution and morphology, released his beautifully illustrated album *Art-Forms of Nature* (*Kunstformen der Nature*, Leipzig, 1899-1904). Note that in the late 18th century, Johann Wolfgang von Goethe, beyond his literary and political activities, was also involved in morphological studies and wrote an essay on the spiral tendency in vegetation In the 19th century, there were important mathematical-biological works on spirals in nature, including the discovery of the spiral phyllotaxis by German and French scholars in the early 1830s. It is a tendency that the number of left and right spirals on the surface pinecones, sunflowers, and cacti are neighboring Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, ...; each term is the sum of the previous two and the ratios of the neighboring numbers approximate the value of the golden section ($a/b = b/(a + b) = 0.618$) Sir Theodore Andrea Cook's books *Spiral in Nature and Art* (London, 1903) and *The Curves of Life* (New York, 1914), and, especially, Sir D'Arcy W. Thompson's monograph *On Growth and Form* (Cambridge, 1917) attracted a special attention. The latter was written originally for biologists, but influenced rather designers and geometers of the forthcoming generations. Last, but not least, Art Nouveau was also inspired by the "discovery" of Japanese art, following the end of seclusion of that country in the late 19th century, and by a growing interest in objects of art from Africa, Asia, and Oceania that were collected by enthusiastic expeditions and displayed in museums. Japanese architecture offered many interesting features for designers: the use of organic motifs, the striking geometric simplicity, the delicate interplay of the inside and the outside of the building, and the modular system of units. The half-size model of the Katsura Imperial Villa was exhibited at the World Exhibition in Chicago in 1893 and attracted a great attention among visitors, including the American architect Frank Lloyd Wright. The influence of Japanese art had two levels: partly reinforced an interest in floral motifs, partly contributed to a new level of "organic art" where the nature of the materials and geometric simplicity dominates without depicting flowers and other motifs. Later we shall return to Wright and the importance of his "organic simplicity", which was also inspired by his reading of Herbert Spencer's biological-philosophical works. In Europe, Rudolf Steiner, originally a Goethe-scholar and a supporter of Theosophy (ancient "divine wisdom" in East and West), later the founder of Anthroposophy, initiated a form of organic architecture and an interdisciplinary educational movement. (From the 1960s, the Hungarian Organic Architecture, the schools of György Csete and of Imre Makovecz, gave a new emphasis to organic forms and materials, as well as to Hungarian folk art.)

3 Industrial design and geometric abstractions with "style" in the period of war and piece (from the *Werkbund* in Germany to *verhouding* in the Netherlands, as well as *beelding* instead of bleeding)

The popularity of new *objects d'art*, from furniture to wallpaper, from doors to lamps, was growing, but their prizes remained high. The obvious question was the following: do you need the expensive object made by handicraft or the cheap industrial product that lacks the artistic quality? Since many people preferred the advantages of both, some artists initiated a compromise, first by making multiple copies of the same object, and later by cooperating with the industry to mass-produce it. Van de Velde, in his 1897 paper "A chapter on the design and construction of modern furniture", explained his principle of "systematically avoiding designing anything that cannot be *mass-produced*" (in the Berlin journal *Pan*, Vol. 3, pp. 260-264, 1897, English trans. in Tim Benton at al., *Form and Function*, London, 1975, pp. 17-19). Indeed, a new field was born, which is called, by a later term, industrial design. One of its early representatives was Peter Behrens, a German painter turned designer and architect, who become the artistic advisor of the large electric company AEG in 1907. Interestingly, Behrens' office employed Mies van der Rohe, Walter Gropius, and Le Corbusier, three architects who later played crucial role in the 20th century developments. The cooperation between artists and engineers also required some exact mathematical methods in order to describe the parameters of the artist's original design and to organize its industrial multiplication. The keyword was "types" and even the mass-produced furniture was named as "type-furniture" (*Typenmöbel*). The central figure of integrating art and technique, as well as modernizing the system of art education in Germany, was Hermann Muthesius. Earlier he studied the arts and crafts movement in England as a diplomat, wrote a book about *The English House* (*Das englische Haus*, Berlin, 1904-05), and returned to work in the Prussian ministry. In 1907, he founded the German *Werkbund* (Work-Federation). This organization, which institutionalized the cooperation among architects, artists, engineers, and industrialists, contributed to the extensive standardization and mechanization. Sadly, this was utilized not only by the civil sphere, but also by the military industry.

The new freedom that Art Nouveau gave to designers, together with the influence of Japanese and African art, paved the way to a more aggressive geometric interest in art in the early 20th century. This was first manifested in Cubist paintings and Futurist sculptures, then in abstract compositions where the figurative motifs totally disappeared and simple geometric figures dominated (Kandinsky, Malevich). This period became the age of manifestoes and the foundation of new artistic centers in Italy (Futurism), Russia (Rayonism, Suprematism, and the early form of Constructivism), Hungary (Activism), and the Netherlands (Neo-Plasticism, Elementarism). The number of *–isms* rapidly grew and started to approximate the number of significant artistic groups. In architecture, the preference for more simple geometric form replaced the Art Nouveau's extensive usage of floral motifs. The Austrian architect Adolf Loos, who had a great influence on the next generation of architects (including André Lurçat, Eric Mendelsohn, Richard Neutra, and Raymond Schindler), did not hesitate to speak about *Ornament and Crime,* which is actually the title of his book *(Ornament und Verbrechen*, 1908). We should not forget, however, the debt of modern architecture to Art Nouveau and its revolution against historic styles. The tensions before and during World Word I (1914-1918) led to various contradictory feelings with illusions and disappointments, enthusiasm and nihilism, but also to the search for new ideas in small groups. Shortly before the war, the artists of the Futurist group had a great hope in technology They also liked to refer to mathematics, at least metaphorically, as Filippo Tommaso Marinetti's *Manifesto of Geometrical and Mechanical Splendour, and the Sensibility of Numbers* (1914) demonstrates, while Antonio Sant'Ellia predicted in his designs many aspect of the "new city". As a numerical obsession, they published the majority of their manifestoes on the 11th day of the actual month. Their interest in military technology, however, led to a tragic direction: the leaders of the movement volunteered to serve in a motorcycle-battalion during the war and Boccioni and Sant'Ellia were killed. The disillusionment with the war and the military machine also marked the end of the movement (although Marinetti remained active as a writer and later joined the fascist leader Mussolini).

The case of the Netherlands needs a special attention. Since the country remained neutral in the war and the only military goal was to defend its borders, as well as to help the occupied Belgium economically, the developments in Dutch architecture and design were not interrupted. In the late 19th century, the architect Jan Hessel de Groot and some of his colleagues generated a special interest in mathematical aesthetics, especially in proportional systems. Perhaps this was associated with the idea of "rational architecture" that Viollet-le-Duc initiated earlier. De Groot summarized his views in many papers and books; here we refer just to one of his later work, a monograph on *Form-Harmony* in Dutch (*Vormharmonie*, Amsterdam, 1912). For him the importance of proportion was to create a whole, "to make from, say, twenty forms, one form". J. L. M. Lauweriks suggested combining the cosmic significance of mathematical proportions and the ethics of socialism. In Dutch both terms, the international *proportie* and the native *verhouding*, can be used in the sense of proportion, although the latter also means "relationship", including a love affair and the relationship with God. Perhaps this was suitable for Lauweriks, who had a "love affair" with proportions and also headed a lodge of the Theosophical Society. (Indeed the title of his paper on proportion is "Verhouding" in the journal *Architectura*, Vol. 5, pp. 175-176 and 178-180, 1987.) The case of Lauweriks is interesting to us, because he linked many ideas that we discussed earlier: started with Art Nouveau (*Nieuwe kunst*) decorations, in 1904 moved to Germany, following the invitation of Peter Behrens, and became an important figure of applied art and also designed various houses, in 1916 returned to the Netherlands and worked as a professor. With great probability, he was the designer of the villa at Bremen, which inspired Le Corbusier's interest in mathematical proportions. (Corbu's data on the villa are fuzzy in his book *Modulor*, but Reyner Banham later pointed out that it must have been Lauweriks's building, cf., *Theory and Design in the First Machine Age*, London, 1960, p. 142). Hendrikus Peter Berlage, another influential figure of Dutch architecture, was also associated with the 19th century "rational" movement, as well as the Dutch school of proportions. He visited Frank Lloyd Wright in 1897 and again in 1911, and, following his reports, the American architect became an influential figure in Europe, too. The young generation of architects in the Netherlands, Belgium, and some other countries were confused with the sharp criticism against Art Nouveau and against the use organic decorations, even Horta switched to a more academic style. The helping hand came from Berlage who distributed information on Wright's works that offered "organic simplicity". It was new, but also gave a feeling that the earlier "organic" approach had a meaning. Berlage, in his textbook in German, pointed out that the role of the architect is "the creation of space, not the sketching of façades" (*Grundlagen und Entwicklung der Architektur*, Berlin, 1908).

The importance of geometry and creation of space, while requiring simplicity, provided the intellectual basis for the painter Theo van Doesburg and some young artists and architects who founded the journal *De Stijl* (The Style, 1917-1928 and 1932). Even the title was associated with the views of de Groot and Berlage who often referred to Gottfried Semper's book *The Style in Applied Arts* (*Der Stil in den technischen Künsten*, 2 vols., 1860-63) and his approach to practical aesthetics. The "Dutch phase" and the "international phase" of *De Stijl*, with a special emphasis of architecture, are discussed by Reyner Banham's referred to book on theory of design in detail (pp. 138-200). I would say, however, that the first phase was also international in some sense. Their group included, beyond Theo van Doesburg who edited the journal, not only the Dutch painters Piet Mondrian, Bart van der Leck and the Dutch architects J. J. P. Oud, Gerrit Rietveld, Rob van t'Hoff, but also the Hungarian painter Vilmos Huszár, the Belgian sculptor and painter Georges Vantongerloo (van Tongerloo), and the Italian ex-futurist Gino Severini. Around 1922, at the beginning of the "international phase" the group included just two non-Dutch members, the German Hans Richter and the Russian El Lissitzy, while Theo van Doesburg and two architects, Gerrit Rietveld and Cornelis van Eesteren, were the Dutch participants. Mondrian and some other members of the original group advocated total abstraction, without any subjectivity, and called their approach Neo-Plasticism (*Nieuwe beelding*). Mondrian was influenced not only by cubism, but also by the spiritual-mathematical works of the Thesophist M. H. J. Schoenmaekers, who wrote a book about the *Principles of Plastic Mathematics* in Dutch (*Beginselen der beeldende wiskunde*, 1915). Although Mondrian's interest in Theosophy is often mentioned, Robert P. Welsh presented some important new details more recently (see, e.g., his paper "Mondrian and Theosophy" in the book *The Spiritual Image in Modern Art*, Wheaton, Illinois, 1987). While a large part of Europe was "bleeding" during World War I, the peace in the Netherlands led to remarkable studies in *beelding*, plasticism. The main meaning of the noun *beeld* is "image", but it is also associated with "beauty"; its derivative is used in *beeldende kunsten* "plastic arts". (In German, both van Doesburg and Mondrian rendered the Dutch expression as *Gestaltung*.) What is Neo-Plasticism? Mondrian reduced everything to horizontal and vertical lines and three primary colors, red, blue, and yellow. Later van Doesburg also allowed diagonal lines in his "counter-compositions" and called his new approach as Elementarism, which move led to a split with Mondrian. As a counterpoint to the extensive decorations and the use of curved lines in Art Nouveau, strong geometrical restrictions took command. Note that modern music made a similar step in Arnold Schonberg's serial music. The scientific interest of van Doesburg and his colleagues is well represented by the fact that the subtitle of *De Stijl* was changed from "Monthly Magazine for Plastic Arts and Crafts" (*Maandblad voor de beeldende vakken*), the last word means not only arts and crafts, but also professions, trades, subjects of study, and even, as a nice play on words, squares and panels, to "Monthly Magazine for New Art, Science and Culture" (*Maandblad voor nieuwe kunst, wetenschap en kultuur*). This reference to science was not an empty word: the journal published, for example, excerpts from a paper by Henri Poincaré, a leading figure of mathematics ("Why does the space have three dimensions?", *De Stijl*, Vol. 6, No 5, pp. 66-70, 1923). The movement associated with the journal *De Stijl* had an important impact on architecture where geometric simplicity had an advantage. Although modern technology does not exclude curved surfaces, but economical reasons may limit the structure. The architects demonstrated how to enrich the world of form, how to make new harmonies in the restricted "universe" of right angles, straight lines, and interlocking planes

These new ideas from Cubism to Neo-Plasticism, together with the needs of rebuilding many parts of Europe after World Word I (1914-1918), contributed to the popularization of modern design and the establishment of two movements with educational institutions:

- the Constructivists (1918-1922 and, in a restricted form, 1922-1934) in Russia, soon after the socialist revolution in 1917, and

- the Bauhaus (1919-1933) in Germany, soon after the establishment of the Weimar Republic in 1919.

In both countries, the dramatic political changes contributed first to the birth of these movements and later to their suppression. However, their influence remained essential via their achievements and via the activities of some of their members who continued their works in other countries, as well as via their failure. (It is a very important that we should also consider the failures and learn from these; the method of trial-and-error has two sides and both are important.). Let us see these two brothers (or sisters), the Constructivists and the Bauhaus in the context of design and mathematics.

4 Constructivism: art → industrial design → architecture (→ art) (an architect turned professor of geometry)

The Russian Constructivists interest in scientific and technical questions were immediately emphasized in the names of some of their institutions that were quickly established after the Socialist Revolution in October 1917. In Moscow, they united two schools of art and architecture as the Free State Art Workshops or shortly Svomas in 1918, with the leadership of Vladimir Tatlin, but soon renamed as Higher State Art-Technical Workshops or Vkhutemas (the abbreviation of Vysshie gosudarstvennye khudozhestvenno-tekhnicheskie masterskie, 1920-1926), which became the Higher State Art-Technical Institute or Vkhutein (1926-1930). In Petrograd, later Leningrad, now St. Petersburg, they organized similar institutions and Tatlin himself moved there for a longer period. Some constructivists and avant-garde artists also participated at the foundation of the Russian Academy of Artistic Sciences (Rosiiskaya akademiya khudozhestvennykh nauk or shortly RAKhN) in 1921. Since the expression "artistic sciences" is puzzling, we may illustrate this diverse field with two examples. The painter Wassily Kandinsky (Kandinskii) became the head of the Physico-Mathematical and Physico-Psychological Department of this new Academy, and he initiated various research topics from the basic elements of art to the psychology of aesthetics. At the same department, Leonid Sabaneyev, who earlier studied both music and mathematics, conducted his research on the possible importance of the golden section, the proportion $a/b = b/(a + b)$, in the temporal organization of musical works and analyzed 1,770 works by 42 composers. Characteristically, some of the constructivists had training at engineering schools. Thus, El Lissitzky (Lazar Lisitskii) was educated at the Technische Hochschule Darmstadt (1909-14), while Naum Gabo, one of the Pevsner brothers, studied medicine (1910), science (1911), and engineering (1912) in Munich. Constructivism originally benefited from the various geometrical and abstract tendencies that were popular among Russian artists (Larionov, Goncharova, Malevich, Gabo, and others) and even from the combination of abstract art with a psychological and spiritual interest (Kandinsky). However, these tendencies of pure art were soon overshadowed by the emphasis on industrial design and production, including such diverse fields as textile design and typography. "Down with guarding the traditions of art. Long live the Constructivist technician", claimed Aleksandr Rodchenko and Varvara Stepanova in their manifesto of the "Productivist Group" in 1920. These "constructivists" or "artist-engineers" believed that they are able to build a new industrial culture and participating in the social changes. (Perhaps we may find some similarities between the views of the Russian "Artistic Left" in the 1920s and the British "Scientific Left" in Cambridge in the 1930s; indeed, John Desmond ("Des") Bernal's representative book *The Social Function of Science*, London, 1939, frequently refers to science and industry in the Soviet Union, although not to the artists.) Constructivism was started as a new form of art (or an "anti-art"), became associated with industrial production, tried to serve a socio-political system, but its main stream was politically destroyed from 1922 by the "socialist-realist" artists. The "Declaration of the Association of Artist of the Revolutionary Russia" claimed that their real goal is to depict "the life of the Red Army, the workers, the peasants, the revolutionaries, and the heroes of labor". (Obviously, there were enough "volunteers" to pose for such pictures and sculptures!) On the other hand, the constructivists also contributed to this attack against them by their mistakes. They did not try to defend the autonomy of art, but became involved in "agitating propaganda" with poster design and street decorations, then in "production", without even checking that the politicians like or dislike their works. In fact, the leading political figures, with the exception of Lunacharskii, the People's Commissar for Enlightenment (1917-1929), had little sympathy with their art or even opposed it. They also provoked other artists by claming that they represent the only form of artistic expression. The first sign of troubles were well demonstrated by the fact that even Kandinsky and Gabo left Russia in 1921 and 1922.

However, constructivism did not die immediately, but survived in various fields of applied art and even advanced in architecture where the goals of depicting the revolutionaries had no meaning. The architects partly responded for the great demand for building new industrial complexes, houses, and other objects, partly tried to gain support for some advanced building. For example, the Vesnin Borthers proposed a transparent glass-iron-concrete building for the Leningrad Office of the newspaper *Pravda* with the elevators outside (1923), El Lissitzky initiated skyscrapers ("Cloud Hengers") where the offices or the dwelling units were on the top of elevator-towers (1924), and I. I. Leonidov designed a spherical auditorium with 4,000 seats for the Lenin Library (1927-28). Most of these futuristic ideas had the same fate as the earlier French plans: they were not built, but influenced later generations. This period also produced many important theoretical works where architecture and mathematics were linked, including

- Nikolai Karsil'nikov's quantitative method for testing all the possible arrangements of given elements in architectural design (in the journal *Sovremannaya Arkhitektura*, i.e., Modern Architecture, 1927),

- Yakov Chernikov's book *Construction of Architectural and Machine Forms* (*Konstruktsiya arkhitekturnykh i mashinnykh form*, Leningrad, 1931), which was written in Russian, but also advertised in a brochure in English.

Perhaps the swansong of constructivism was another book by Chernikov: *Architectural Fantasies* (*Arkhitekturnye fantazii*, Leningrad, 1933). Ironically, this title immediately hints that the goal was not anymore "real" architecture, but play with geometrical forms. John E. Bowlt, in his book *Russian Art of the Avant-Garde* (London, 1988, p. 156), concluded that constructivism both began and ended as art. We may add that this final form of constructivism is such an art that has a special openness toward mathematics. Using compass, ruler, and even mathematical formulas was not unusual, as it is demonstrated by El Lissitzky's picture *Tatlin at Work on the Third International* (pencil, gouache, photomontage, ca. 1920). In Tatlin' s hand there is a ruler and at his eye a compass. It is true that the mathematical formulas written next to Tatlin's body are meaningless, but these were obviously "workable" for the general public as a metaphorical reference to higher mathematics. However, Chernikov went far beyond basic geometry and even anticipated the field of shape grammar that we should discuss later. Characteristically, Chernikov earned a degree in Architectural Sciences in 1935 and left Leningrad to take up the position of the head of the Department of Descriptive Geometry and Graphics at the Institute of Engineering Economy. Indeed, the constructivists lost their last hopes to continue their work as artists. In 1930, the movement had a second death by "renaming" its main educational institution, the Vukhtemas/Vukhtein (1920-1930), which pioneered a strong cooperation between art and crafts and also initiated the links to industry. The new name, Moscow Institute of Art, had no reference to technique (*-te-* in the earlier names) anymore. Finally, in 1934 the third and final death of constructivism came with the political declaration that all forms of art and literature should be based on Socialist Realism, which ideology soon occupied the field of architecture and design by rejecting the luxury "formalist" ideas. However, constructivism retained an influence via some of its members who moved to other countries and via the help of its "younger brother" in Germany.

5 Bauhaus: art → industrial design → architecture (→ basic design) (*Emaille* by email order, or else the roots of computer-aided manufacture)

The Bauhaus (1919-1933), "building" (*Bau*) a "house" (*Haus*) where architects, sculptors, and painters work together and there will be no barrier between artists and craftsmen, is perhaps the best known movement and educational institution in the field of modern architecture and design. However, its history was not so straightforward as it is widely believed and there were many struggles in order to find its direction. The Bauhaus was established, similar to the Vukhtemas in Moscow, by uniting two existing art colleges in the German city Weimar. It is interesting to note that the director of these institutions was the same Henry van de Velde who contributed to the birth of Art Nouveau in Belgium and had a strong interest in industrial design. The founding director of the Bauhaus was the architect Walter Gropius, who was nominated by van de Welde some years earlier, but no move was made until the end of World War I and the birth of the republic system in Germany. Gropius, in his manifesto of 1919, immediately announced the establishment of a new department of architecture, the reunification of sculpture, painting, handicrafts, and the crafts as "inseparable component of a new architecture", and promised "constant contact with the leaders of the crafts and industry" (Programme of the Staatliches Bauhaus in Weimar, April 1919). However, no real emphasis was given to industrial design and functionalism until the arrival of the Hungarian self-taught artist László Moholy-Nagy in 1923 and no department of architecture was established until 1927, which was also the last year of Gropius at the school. These steps show a striking similarity with the Russian constructivists' initial interest in pure art, then a shift to industrial design and later to architecture and applied arts Incidentally, some authors call the very first period of the Bauhaus, until 1923, as expressionist, which was followed by a constructivist phase. What factors did delay an immediate interest in industrial design at the Bauhaus? What was the reason of interrupting those links between art and industry that were available in Germany via van de Velde and, more intensively, via the German Werkbund? Obviously, the horrors of Word War I and the importance of standardization and mechanization for the military industry gave some negative feelings and did not encourage the artists to rush to the industry. The second reason could have been the personal attitude of the first instructors. Gropius hired creative artist, painters and sculptors, of great ability (Lyonel Feininger, Johannes Itten, Gerhard Marcks, later Paul Klee, Oskar Schlemmer, Vasily Kandinsky, and others), but they had less interest to move towards industry. Still, the early period was very important to work out a progressive structure of education with a half-year, later one-year, Preliminary Course (*Vorkurs*) and then a three-year dual system of instruction in form and in crafts problems (*Formlehre* and *Werklehre*) in various workshops organized according to the used materials, specifically stone, wood, metal, clay, glass, colors for wall-painting, and textiles. The idea was that two instructors should lead each workshop: an artist and a craftsman whose title is "form-master" (*Formmeister*) and "craft-master" (*Werkmeister*), respectively. The Swiss painter Johannes Itten was the most influential instructor of the first period. He organized the Preliminary Course, with an intention "to liberate the student's creative power", and also headed some of the workshops. The students created individual pieces of art-works; these were carefully crafted, but without too much consideration of the possibility of industrial reproduction. Since Itten also favored a kind of Oriental mysticism and introduced breathing exercises, diets, and prayers, while Gropius had a growing interest in social questions and the reestablishment of the links to industry, a growing tension appeared between them. The move towards the constructivist phase of the Bauhaus was also encouraged by the private seminars of Theo van Doesburg, the founder of the journal *De Stijl*, who spent a longer period in Weimar in 1921-22. Finally, Itten left the school in 1923 and Gropius' view became the dominating force: "Art and technology: a new unity".

The appointment of László Moholy-Nagy, who was associated with the Hungarian Activism, a Constructivism-related group, became a very successful step into this direction. In 1922, shortly before his arrival at the Bauhaus, Moholy-Nagy introduced the idea of "telephone-pictures" (*Telefonbilder*), which were made using vitreous enamel (or, in German *Emaille* or *Email*) on steal, and the description of the figures were such exact that, theoretically, a factory was able to produce it via a telephone order, without seeing its sketch. This method included, in preliminary form, those ideas that later led to the algorithmic description of figures, their presentation by computer graphics, and producing by computer-aided manufacture. (*Nomen est omen*: we are able to order *Emaille* pictures not only by telephone, but also by email.) Moholy-Nagy was in charge of the new one-year Preliminary Course, with the participation of Albers, Klee, and Kandinsky, and he also headed the metal workshop where many prototypes were developed for industrial production. Perhaps the best-known "objects" of the Bauhaus were those chairs that Marcel Breuer, a Hungarian student and later a leading architect, designed in the furniture workshop. On the other hand, Moholy-Nagy did not follow those Russian constructivists who favored "production art" and rejected "pure art", but suggested a balanced view. He intensively dealt with the use of new media, from photo and film experiments to works with kinetic light and mobiles. Moholy-Nagy also had a special interest in graphic design, which lead to his "asymmetric" typography for the Bauhaus books. In this series, edited by Gropius and Moholy-Nagy, 14 books were published between 1925 and 1929, including not only the works of the Bauhaus members (Kandisnky, Klee, Schlemmer, and the editors own books), but also representative works of the French cubism (Gleizes), the Russian suprematism (Malevich), and the Dutch *De Stijl* (van Doesburg, Mondrian, J. J. P. Oud).

The Bauhaus "brotherhood" with the Russian constructivists and other avant-garde movement was also reinforced with the appointment of Kandinsky in 1922 and the short visits by El Lissitzky and Malevich. Although the Bauhaus did not have such a member as the Russian Chernikov, who continued his career as a professor of geometry, the importance of mathematics was directly emphasized by Gropius in his second manifesto "The theory and organization of the Bauhaus" in 1923. Specifically, he pointed out that the dual system of instruction in form and crafts problem needs a general coordination (*Harmonisierungslehre*), and he included here, among others, mathematics, physics, and mechanics, as well as the synthetic study of space. The program of the instruction in form (*Formlehre*) also had a section of descriptive geometry. In 1926, the Bauhaus moved to Dessau into a new building designed by Gropius. This glass and reinforced concrete building, with an asymmetric shape, symbolized the Bauhaus interest in functionalism and demonstrated the relationship between design and modern technology. Finally, in 1927, the Bauhaus established its Department of Architecture with the chairmanship of the Swiss architect Hannes Meyer, who also become the new Director in 1928 when Gropius, together with Moholy Nagy and some other masters, left the school. Meyer represented a Marxist ideology and was strongly production-oriented, while reduced the teaching commitments. In 1930, he was removed from office and replaced by the German architect Mies van der Rohe. The Bauhaus, similar to the Russian constructivists' workshops Svomas/Vukhtemas/Vukhtein (1918-1930), had three different full names. Staatliches Bauhaus in Weimar (1919-1925), Bauhaus Dessau: Hochschule für Gestaltung (1925-1932), and Bauhaus Berlin (1932-1933). The lack of founding in Weimar did not cause too much problem and the Bauhaus moved to another place. However, the closure in Dessau and later in Berlin were politically motivated by the Nazis.

6 Beyond the Bauhaus (Briefly about the International Style, two schools, and two personalities)

The Bauhaus-idea survived in various forms, including some remarkable works of the "classical period" (the Bauhaus chairs are still available commercially) and the later teaching activity of its former members (for example, Gropius at Harvard University, Mies van der Rohe at the Illinois Institute of Technology in Chicago, Albers at the Black Mountain College, North Carolina). The Bauhaus' impact on education of design is significant, although not without controversy. Peter Collins remarked, with some irony, that at Harvard University, where Gropius was teaching from 1937, virtually all elements of the Bauhaus curriculum, with the exception of the Preliminary Course, were abandoned. Moreover, the Bauhaus's only graduate who furthered the ideal of "architectonic art" in the case of buildings was Marcel Brauer, who studied only furniture design there (*Changing Ideals in Modern Architecture*, London, 1965, p. 269). Perhaps the best legacy of the Bauhaus in the field of education is those Basic Design Courses or Foundation Courses that became popular almost elsewhere at schools of architecture and design in order to develop the students' creative skills. Here we should pay tribute to Johannes Itten who "designed" the first Basic Design courses before his "forced" departure. The fact that such type of courses may also contribute to mathematics is demonstrated by the Rubik's Cube, which was originally a simple tool for design students in Hungary, later became an international obsession, and ended up as a research topic for mathematicians. Interestingly, the Bauhaus had a very strong impact on Japanese design education: many schools introduced similar foundation courses and all Bauhaus books were translated into Japanese. In the history of architecture, the expression International Style was introduced for those achievements that the European architects represented from Gropius to Mies van der Rohe, from Oud to Le Corbusier, although they never formed a coherent group in such a broad scale. The name was obviously inspired by Gropius's book entitled *International Architecture* (*Internationale Architektur*, 1925), the first volume in the series "Bauhaus Books", but he himself never referred to a new "style". In 1932, the title of the first architecture-related exhibition in the Museum of Modern Art in New York, just three years after its opening, and the book published for this occasion, "canonized" the name "International Style".

There were two institutions that emphasized the Bauhaus-traditions in their names, although none of them tried simply to "copy" it. The first institution had three names (this number became "magic" in this field): the New Bauhaus in Chicago (1937-1938), which became School of Design (1939-1944), then Institute of Design (1944-), which was later incorporated by the Illinois Institute of Technology in Chicago.

Moholy-Nagy was the first head of this institution and his commercial works was marked, among others, the designed of the fountain pen *Parker 51* in 1941. (Note the conflicting interest of Hungarians in the world: Moholy-Nagy designed the best-known fountain pen, but Biro invented, with a little help of some friends in the Budapest cafés, the *Biro pen*, the original form of the ball pen.) Sadly, Moholy-Nagy's wide-ranging activities were stopped by his early death of leukemia in 1946. Between 1946 and 1951, Serge Chermayeff was the new director. His name is known as both a successful industrial designer, including various chairs, and the author of theoretical works. Gyorgy Kepes, a close coworker and compatriot of Moholy-Nagy, became a leading figure of building bridges between art and the newest results of modern science and technology, first in Chicago and later, as the founding director of the Center for Advanced Visual Studies at MIT, in Cambridge, Massachusetts. Kepes's books, with contributions by very many artists and scientists, attracted a special attention (*Language of Vision*, Chicago, 1944; *The New Landscape in Art and Science*, Chicago, 1956; Seven volumes of the series "Vision + Value", New York, 1965-72). The other institution, the Hochschule für Gestaltung in Ulm (1953-1968), was initiated soon after World War II, by a foundation for the memory of Hans and Sophie Scholl, who were members of the "White Rose" resistance group and executed by the Nazis. Similar to the Bauhaus and the Constructivist Movement, the founders were eager to contribute to the solution of social and intellectual problems after the war. Note that the German name of the institution is identical with the secondary name of the Bauhaus in Dessau, while it is usually referred to in English as Ulm School of Design. As an emphasis on the traditions, Henry van de Velde, Walter Gropius, and Mies van der Rohe were among the early visitors. We should discuss this institution in more details because of an interesting controversy in connection with mathematics. The Ulm School of Design was first headed by the Swiss artist Max Bill, a former student of the Bauhaus, and later by a collective leadership including the Argentine designer Tomás Maldonado, the German artist Otl Aicher, and others. During the first years Johannes Itten and Josef Albers, former "masters" of the Bauhaus' Preliminary Course, participated in the teaching of the new school's Foundation Course (*Grundkurs*), and later, among others, the mathematician Hermann von Baravalle and the architect William Huff (who is a Honorary Member of ISIS-Symmetry) continued this work. In the meantime, the curriculum was changed from a Bauhaus-type teaching to a new one with two types of courses: (1) design practice, and (2) complementary sciences, which included both natural and social sciences. The school had four departments: visual communications, information, industrial design, and building departments. While the Bauhaus attracted some professors with great ability in intuitive mathematics, the Ulm School of Design made a further step and regularly invited mathematicians, crystallographers, and engineers to give courses or lectures (Z. S. Makowski, Frei Otto, Paul Schatz, K. L. Wolf, and others). The school was eager to introduce the newest results of communication and information theory, cybernetics, and semiotics into the curriculum and attracted a remarkable group of people from these fields as instructors (Max Bense, R. Gunzenhäuser, Abraham A. Moles, H. Stachowiak, and even a visit by Norbert Wiener). This scientific component of the curriculum was never challenged, however, the question how much "mathematization" led to a conflict. Horst Rittel, originally a mathematician who was in charge of design methods and also a member of a three-person team that headed the school between 1959 and 1961, preferred the use of more mathematics in all fields of industrial design, for example modular systems and complicated grids, while the pragmatists rejected the "worshipping" of some mathematical methods. They suggested concentrating on the links to industry. The battle was "won" by the latter group.

We should emphasize that not just the Russian Constructivists and the Bauhaus made contributions to linking architecture and mathematics in the "machine age", the first decades of the 20th century. For example, Le Corbusier and Ozenfant's aesthetic program Purism (ca. 1918-1926) in Paris and later Buckminster Fuller's Dymaxion house in America (from the 1920s) also initiated such links. In the case of both "Corbu" and "Bucky", this interest remained long-lasting, which is well-illustrated by the golden-section-related proportional system *Modulor* and the theoretical works on *Synergetics*, the cooperative forces ("energies") in tensile structures, respectively. (We should not confuse, however, Bucky's *Synergetics* with the later usage of the same expression by the physicist Hermann Haken and his interdisciplinary group.) We should stop at Bucky for a while. He was trained as an engineer, not as an architect. Thus, his Dymaxion house and geodesic dome, which became popular first at expos and later at many places, reiterate the 19th century story with Eiffel's contribution to the development of architectural design. Playing on words, Bucky used not the analogy of the handcrafts, but immediately the aircraft. (Much later Frank O. Gehry used a computer program for shaping the surfaces of the Guggenheim Museum in Bilbao, which was originally developed for designing the French Mirage fighters; indeed, the shape immediately hints some relationship with aircrafts and the design encouraged some "fight" around the building, fortunately without fighter jets, just verbally.) Bucky himself was a "fighter" for his own ideas, which were often not fully his, but "adopted" from others and popularized with a remarkable efficiency. One may say that the hexagonal shape of the Dymaxion house was also an adoption, but it was legal because the bees did not patent it. However, Bucky went far beyond the economy of nature and introduced prefabricated high-tech components made by aircraft construction methods. The structural components were prewired, preplumbed, and equipped with various devices. Thus, the house became fully operational immediately after assembling. Fuller's design took advantage of the tensile strengths of materials and later used the same idea in the case of his geodesic domes, which were shaped with grids based on the great circles (geodesic lines) on spherical surfaces In the same time, Bucky made a sharp criticism against the Bauhaus and the architects of the International Style, claiming that they "never went back of the wall-surface to look at the plumbing". He accused the Bauhaus with "design-blindness" and the ignorance towards modern technology. Reyner Banham, in the conclusion of his book *Theory and Design in the First Machine Age* (London, 1960), agreed with Bucky's view and declared that the architects of the First Machine Age did not understand the contemporary science and technology and, in this sense, they were wrong. Fortunately, we have here just a smaller problem: architecture and modern mathematics. Do architect utilize modern mathematics? Do mathematician care those problems that are important for architecture and urban design? The answer is: usually not, but there are exceptions ..

7 Mathematical architecture (?): the systematic usage of modern mathematics in architecture and urban design (a Cambridge center with some encouragement from constructivism)

I am not quite sure that the expression "mathematical architecture" is the best one, but, using the analogy of mathematical crystallography, mathematical physics, mathematical biology, and even mathematical aesthetics, why not to try this combination. Of course, this is not about the "architecture of mathematics" or the "foundations of mathematics", which usually refers to the system of axioms, definitions, theorems in mathematics and to the related questions of logic, but a field of architecture and design or, using the idea of the Ulm School, a "complementary science" to architecture. From the 1950s structural engineering made important advances. Beyond the geodesic domes (Buckminster Fuller reinvented in the U.S.A.), the thin shell structures (Félix Candela in Mexico and Pier Luigi Nervi in Italy), and its "inverse", the suspended roofs (Frei Otto in Germany) became very popular. Of course, these structures provided further mathematical questions We may consider "mathematical architecture" as a topic associated with "building science" or "architectural science", which became a well-established field with strong links to applied physics and engineering. Let us see the partnership of mathematics and architecture from the mid 20th century. As we have seen, various groups of architects and designers turned to mathematical methods. Most of these people, however, used elementary mathematics, for example proportional systems, modules, basic geometric transformations. A few others applied higher mathematics, but just occasionally in a few concrete cases. The complex problems in architecture and urban design, however, led to the reappearance of "mathematicians-architects". (We should not forget that Sir Christopher, as we have seen in Chapter 1, was both a mathematician and an architect.) Some architects decided to learn the necessary mathematics or, in some fortunate cases, they were trained in both fields. Since we "spent" a longer period in continental Europe, it is time to visit the United Kingdom and then the U.S.A. again. Earlier we have seen the birth of *Art Nouveau, Jugendstil, Sezessionstil* in continental Europe, but we should add that the Glasgow style in Scotland and the Tiffany style in the U.S.A. represented similar developments and their representatives had regular interactions with other movements. Remember that the Belgian Horta was interested in the Arts and Crafts Movement in England (Chapter 2) and the German Muthesius published a book about the English house (Berlin, 1904-05), before he founded the *Werkbund* (Chapter 3). About a decade later, in 1915, the British architect W. R. Lethaby wrote a paper entitled "Modern German architecture and what we may learn from it", which is available in his book *Form in Civilization* (London, 1922, pp. 96-105). In another paper, he wrote about the need of systematic research on the geometry of architectural structures, as Lord Kelvin investigated the geometry of crystalline structures. The first help came, however, from the side of Constructive art. The architect Leslie Martin, later Sir Leslie, met the Constructive artist Naum Gabo, later Sir Naum, who left Russia in 1922, and these two and the English painter Ben Nicholson, edited a collection of papers with almost 200 plates:

- *Circle: International Survey on Constructive Art* (London, 1937).

Among the contributors, we may find, among others, Gropius, Le Corbusier, Moholy-Nagy, Mondrian, and, as Lethaby "dreamed" earlier, a leading British crystallographer, J. D. Bernal (earlier we mentioned his name as a representative of the "Scientific left" in Britain). Fortunately, J. D. Bernal remained active in "building" bridges between crystallography and architecture and he wrote two more papers, with almost symmetric titles, "Architecture and science" and "Science in architecture" for the *Journal of the Royal Institute of British Architects* (Vol. 44, No. 16, 1937 and Vol. 53, No. 5, 1946). The latter was based on a lecture given at an informal meeting of the same organization. All of the mentioned three papers were reprinted in Bernal's book *The Freedom of Necessity* (London, 1949, pp. 185-213). However, these contributions were brief survey papers, without going into technical details. Their importance was to identify two mathematical topics that are useful for architects: symmetry and topology.

If we are looking for the systematic usage of mathematical methods in architecture and urban design and the establishment of an educational institution where such problems became prominent, we should remain with the intellectual circles of Sir Leslie Martin. After World Word II, he faced a situation that was similar to Sir Christopher Wren's circumstances after the Great Fire of London in 1666: he helped the post-war reconstruction of London as the Chief Architect. From 1956, he taught at Cambridge University, where he became the first professor of architecture, then, in 1967, the Director of the new Centre for Land Use and Built Form Studies at the same university, which was later renamed after him as the Martin Centre for Architectural and Urban Studies. Martin declared at the very beginning of his professorship that it is necessary to establish a bridge between faculties, between the arts and the sciences. His institution became, beyond dealing with social and economical problems of architecture, a center for the application of mathematical methods to architecture. Later some of the members of his circles founded the Centre for Configurational Studies at the Open University in

England and started a cooperation with the School of Architecture and Urban Planning at the University of California Los Angeles (UCLA).

The desire to synthesize new ideas in Europe and America, architecture and modern mathematics led the Austrian-born Christopher Alexander to graduate in both architecture and mathematics at University of the English Cambridge, then earning a doctorate in architecture at Harvard of the American Cambridge. His thesis was later published as a book, while he moved to University of California, Berkeley:

- Christopher Alexander *Notes on the Synthesis of Form* (Cambridge, Mass., 1964).

From graphs theory to statistics, he applied various fields of modern mathematics, together with some results from social sciences. His next work attracted a special attention: "A city is not a tree", claimed Alexander in the title of his paper, which refers not a green tree with leaves, but to a type of semi-lattice with mathematical inspirations (*Architectural Forum*, April and May 1965, pp 58-62 and 68-71). He concluded that if we develop our cities like trees, they will "cut" our life into pieces. This paper became a widely discussed document in the field of city planning and was reprinted several times (*Design*, February 1966, pp. 46-55; Jeffrey Cook, editor, *Architectural Anthology*, Tempe, Arizona, 1969, pp. 580-590; etc.). Alexander also worked with Serge Chermayeff (*Community and Privacy: Towards a New Architecture of Humanism*, New York, 1963) and contributed a paper to the series of books edited by Gyorgy Kepes ("From a set of forces to a form", in: *The Man-Made Object*, New York, 1966, pp. 96-107). Note that earlier we referred to both Chermayeff and Kepes in the case of the history of Moholy-Nagy's New Bauhaus in Chicago (Chapter 5). In 1967, Alexander became the president of the Center for Environmental Structure in Berkeley, California, and worked on a pattern language that considers verbal criteria for generating architectural designs. Interestingly, Alexander decided not to use mathematical weighting or valuation of the criteria, the often conflicting requirements by the client, the local council, and so on, but to consider these as verbal statements and the achieve the best possible solution with a concrete pattern. (Some problems of this methodology, and a sympathy with Bruce Archer's alternative approach based on operations research with weighted criteria, were discussed in Lionel March's paper "The logic of design and the question of value" in the book *The Architecture of Form*, Cambridge, 1976, pp. 1-40). Alexander and his coworkers continued refining their methodology and published a large number of works on this topic, partly in the book series of the center, partly at other publishers, for example:

- Christopher Alexander at al. *A Pattern Language: Towns, Buildings, Construction* (New York, 1977).

The interested reader may find the list of more recent publications of Alexander and his colleagues at the home page: patternlanguage.com. Note that Alexander united the theoretical works and the practice of the architect: he designed more than 200 buildings.

Returning to the circles of Leslie Martin and the activities in the triangle of Cambridge University, Open University, and UCLA, a group of scholars published many works on mathematical architecture (note again that it is not their term, but my trial the name an important movement), where they applied a variety of mathematical methods, from symmetry groups to matrices, from graphs to networks. Their activity is marked with monographs and collections of papers, for example:

- Lionel March and Philip Steadman *The Geometry of Environment: An Introduction to Spatial Organization in Design* (London, 1971),

- Leslie Martin and Lionel March, editors, *Urban Space and Structures* (Cambridge, 1972),

- George Stiny *Pictoral and Formal Aspects of Shape and Shape Grammars: On Computer Generation of Aesthetic Objects* (Basel, 1975),

- Lionel March, editor, *The Architecture of Form* (Cambridge, 1976),

- Philip Steadman *Architectural Morphology: An Introduction to the Geometry of Building Plans* (London, 1983),

- as well as, by papers in the journal *Environment and Planning, B* [series] (1974-).

Note that the first item in this list, March and Steadman's book, also has an American edition (Cambridge, Massachusetts, 1974), while its Hungarian translation has a modified title *Geometry in Architecture* (*Geometria az építészetben*, Budapest, 1975). Independently of this, William Blackwell used the same English title for his book (New York, 1984). The second item of the list, which was edited by Martin and March, also inaugurated a new series of books "Cambridge Urban and Architectural Studies", which is, of course, not limited to mathematical architecture. Perhaps it is necessary to discuss the idea of "shape grammar", which features in the title of George Stiny's book. The word "grammar" was frequently used and even overused in architecture and design, starting with Owen Jones's monumental work *The Grammar of Ornament*. (London, 1856) and Charles Blanc's *Grammar of the Arts of Drawing* in French (*Grammaire des arts de dessin*, Paris, 1867); both books were frequently used by architects. In most cases, including these two examples, "grammar" is just a metaphor: we have a set of geometrical patterns or designs, these are somehow arranged, and we should "feel" that the arrangement is based on a grammar. In Stiny's case, however, "grammar" is not a metaphor, but it actually manipulates shapes, which are defined by sets of lines. Shape grammars are very useful to generate all the possible designs in the framework of a given grammar. We may compare it with a corner stone of mathematical crystallography: finding all the possible types of periodic patterns (crystallographic symmetry groups). Incidentally, the 17 two-dimensional or wallpaper patterns are useful for analyzing ornamental art and urban patterns: March and Steadman's book devotes a chapter to this topic. Although some of the mathematical methods in the listed books were worked out prior to the micro-electronic revolution, the research in the field of mathematical architecture, old and new, contributed to the "computerization" and also befitted from it. Even more methodologies were worked out for using computers in the design process. Here we mention just one monograph:

- W. J. Mitchell *Computer-Aided Architectural Design* (New York, 1977).

We separated this book from the others since it represented an emerging new field. Incidentally, Mitchell is a frequent collaborator of Stiny, who initiated the application of shape grammar in the field of computer-aided design. On the other hand, the same methodology can be used not only for generation of shapes, but also for formal analysis of objects of art, as Terry W. Knight demonstrated in her book *Transformations in Design: A Formal Approach to Stylistic Change and Innovation in the Visual Arts* (Cambridge, 1994). It is a special benefit that modern methods can be applied for dealing with historic topics. For example, Stiny and Mitchell coauthored papers on such diverse topics as "The Palladian grammar", which is continued by "Counting Palladian plans", and "The grammar of paradise: on the generation of Moghul gardens" (*Environment and Planning, B*, Vol. 5, pp. 5-18 and pp. 189-198, 1978; Vol. 7, pp. 209-226, 1980). In the first two papers, they dealt with the layouts of the villas of Andrea Palladio, the 16th century Italian architect, and enumerated all the possible plans of fixed sizes. Returning to the early inspiration from the Russian constructivism, the names of Gabo and Martin appeared together again in 1957, exactly twenty years after the publication of the book *Circle: International Survey on Constructive Art* (London, 1937). Specifically, Gabo's album was published by Martin's preface (London, 1957). Moreover, the publisher of *Circle* came out with another interdisciplinary and international collection *Data: Directions in Art, Theory and Aesthetics* (London, 1968), which was edited by the artist Anthony Hill. Although "data" here is an acronym of the subtitle (it is better to write it in all caps), we may find a nice metaphor here for the changing time: from the to utopian discussions in a circle to modern data processing, from the perfect geometric figure to the information age. The theoretical works of Russian architects (Chapter 4) were also rediscovered, March started his 1976 book with a reference to Karisl'nikov, and Stiny applied Chernikhov's drawings for illustrating a shape grammar. The growing in interest in Chernikhov and Krasil'nikov is represented by many publications, including Arthur Sprague's paper "Chernikov and constructivism" (*Survey*, No. 39, pp. 69-77, 1961), Catherine Cooke's book *Chernikhov: Phantasy and Construction* (London, 1984), and her paper "Nikolai

Krasil'nikov's quantitative approach to architectural design" (*Environment and Planning, B*, Vol. 2, pp. 3-20, 1975).

We do not refer to more recent works in the field of mathematical architecture, because the home pages of the referred to institutions and some related groups provide detailed information. We also confess that our survey is very far from being comprehensive, we missed many important developments. Our main goal was to demonstrate the beginnings of a new field and its early inspirations.

8 A symmetric link (and two notes on the golden section)

In modern times, no architect and designer can graduate without completing courses in mathematics. It is true, that this statement is not "symmetric": most mathematicians graduate without taking any course in architecture and design, but it would be not a bad idea to offer such courses in the framework of general studies programs. More recently, computer aided architectural design (CAAD) is a field where mathematicians, computer scientists, and architects should cooperate. Obviously, there are many problems that require discussions in broader circles of architects and mathematicians. For example, modern architecture may need new transformations and new algorithms that are available in mathematics. In the same time, some questions in architecture and design may inspire mathematicians. The historic reconstruction of building is another problem where we may have fruitful cooperation among representatives from various fields. Even history of mathematics may provide some help in special cases (I will give an example later).

In this paper, we mentioned the topic of symmetry as a link between mathematics and architecture several times. Thus, the crystallographer J. D. Bernal recommended it for architects, March and Steadman's referred to book *The Geometry of Environment:* (London, 1971) has a chapter on the basic symmetry operations and another one symmetry groups in the plane, Steadman's book *Architectural Morphology* (London, 1983) deals, among others, with the symmetries of rectangular plans. We may give further examples, including a mathematical contribution

- J. A. Baglivo and J. E. Graver *Incidence and Symmetry in Design and Architecture* (Cambridge, 1983),

and a video program made by the Open University in Great Britain,

- P. Steadman *Symmetry* for the course "Design: Principles and Practice" (1992).

If we are interested how the topic of symmetry became a part of the curriculum in design education in a systematic form, we should go back to the complementary science courses at the Ulm School of Design (Chapter 6). The leaders of the school invited the chemist K. L. Wolf to give lectures on symmetry. He coauthored two interdisciplinary books on symmetry in German:

- K. L. Wolf and D. Kuhn, D. (1952) *Gestalt and Symmetry: A Systematic Presentation of Symmetric Bodies* (*Gestalt und Symmetrie: Eine Systematik der symmetrischen Körper*, Tübingen, 1952),

- K. L. Wolf and R. Wolff *Symmetry: An Attempt towards an Instruction in Seeing Gestalt and Meaningfully Creating Gestalt, Systematically Described and with Numerous Examples Explained* (*Symmetrie: Versuch einer Anweisung zu gestalthaftem Sehen und sinnvollem Gestalten, systematisch dargestellt und an zahlreichen Beispielen erläutert,* Münster, 1956).

Note the presence of the term *Gestalt* in both tiles, which also featured in the name of the Ulm School of Design (*Hochschule für Gestaltung*). The second book has not only an extraordinary long subtitle, but also an interesting structure: the first volume gives the main text, while the second one is an exciting selection of illustrations. The American architect William S. Huff, who was the guest instructor of the same school's Foundation Course between 1963 and 1968, made the next step, a "symmetric" response. While the chemists wrote a book on symmetry that is useful for designers, too, the architect published a series of booklets as a visual introduction into the subject, which is also interesting for people with scientific background. The series includes not only many rarely seen illustrations, but also presents interesting documents from the history of the related scientific fields:

- William S. Huff *Symmetry: An Appreciation of its Presence in Man's Consciousness*, Parts 2-6, Designed by Tomás Gonda (Pittsburgh, 1967-1977).

This publication was distributed in Northern America for those universities that had design programs in that time, while the *Oppositios: A Journal for Ideas and Criticism in Architecture* reprinted three parts of the series (Nos. 3, 6, 10, 1974-1977). Another step was due to a duo, a designer and a mathematician, who wrote a textbook jointly:

- Sadao Kumagai and Yasuaki Sawada *Ornamental Patterns and Symmetry* (*Moyou to shinmetorii*, Kanazawa, 1983).

This book presents a visual approach to symmetry groups in the plane (wallpaper groups), including black-and-white and colored ones. Although the book is in Japanese, it is dominated by illustrations and tables that are given in English. A similar interdisciplinary cooperation between an anthropologist and a mathematician led to a comprehensive book with the symmetry-analysis of very many ornaments:

- Dorothy K. Washburn and Donald W. Crowe *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis* (Seattle, Washington, 1988.

Polyhedra with various symmetry properties are frequently discussed in both mathematics and design; these are very important tools for the development of 3-dimensional thinking and also can be applied in practice. Beyond a large number of specialized mathematical books on polyhedra, many authors wrote non-technical, or less technical, books that favor visual approach and consider the needs of designers. The modern pioneers of this approach, beyond Leonardo, Dürer, and some Renaissance artists, were H. S. M. Coxeter, who added a chapter on polyhedra to W. W. Rouse Ball's book *Mathematical Recreations and Essays* (London, 1939), and H. M. Cundy and A. P. Rollett, who presented many useful illustrations and data on polyhedra in their book *Mathematical Models* (Oxford, 1951). Later, both designers and mathematicians contributed to the growing number of such works, including Ugo Adriano Graziotti's *Polyhedra: The Realm of Geometric Beauty* (San Francisco, 1962), Magnus Wenninger's *Polyhedron Models* (New York, 1971), Anthony Pugh's *Polyhedra: A Visual Approach* (Berkeley, 1976), Peter and Susan Pearce's *Polyhedra Primer* (New York, 1978). We should also refer to three more complex books where the study of polyhedra play a central role: Keith Critchlow's *Order in Space: A Design Source Book* (London, 1969), Alan Holden's *Space, Shapes, and Symmetry* (New York, 1971), and Robert Williams's *The Geometrical Foundation of Natural Structures: A Source Book of Design* (New York, 1979). As a clearly interesting development, some scholars with backgrounds in architecture and design achieved not only interesting structures, but also produced new mathematical results:

- A. Wachman, M. Burt, and M. Kleinman *Infinite Polyhedra* (Haifa, 1974),

- Haresh Lalvani *Transpolyhedra: Dual Transformations by Explosion-Implosion* (New York, 1977),

- Koji Miyazaki *An Adventure in Multidimensional Space: The Art and Geometry of Polygons, Polyhedra, and Polytopes* (New York, 1986),

- Michael Burt *The Periodic Table of the Polyhedral Universe* (Haifa, 1996),

as well as, some papers in the Canadian journal *Structural Topology* (1979-).

Of course, the more recent works are available at home pages. We suggest starting with the mathematician-artists George Hart's web site.

Last, but not least the concept symmetry is also associated with proportions, which attracted, from Vitruvius to Le Corbusier, many architects, including serious theoretical works and non-sense proportion "hunts". Here I would like to discuss just one "special" proportion, the golden section, $a/b = b/(a + b)$, which attracted, unfortunately, very few serious works and very many non-sense trials for "gold digging", although these people always claimed that they found some gold. We referred to some of the more serious type of works: the discovery of the spiral phyllotaxis in the 1830s and the survey in D'Arcy Thompson's book in 1917 (Chapter 2), the Russian musicologist Sabayev's study in the 1920s (Chapter 3), and Le Corbusier's *modulor* (Chapter 6). We may add that Klee and Moholy-Nagy, two masters of the Bauhaus (Chapter 5), also used the golden section in their teaching practice. It is widely believed that the golden section (or, in Latin, *sectio aurea*) was often used in ancient and medieval art and architecture. Some scholars even used the argument that the preference of the golden section was associated with an interest in gold, which was used in decorations or occupied the minds of medieval alchemists, and so on. However, there is a serious problem with this argument: the golden section was not related to "gold" until the 19th century, but was called the extreme and mean ratio (since Euclid's time), the divine proportion (Luca Pacioli), the continuous proportion (German mathematicians), and the medial section (British mathematicians). Incidentally, the mathematical-historical works on the subject give credit to Max Ohm, the brother of the physicist Georg Simon Ohm, for the first known printed usage of the expression "golden section" (*der goldene Schnitt.*) in 1835. This information is repeated in many etymological dictionaries in English, French, German, and Italian. Some years ago, I reported an earlier example of 1833 and now, for the first time, I present a further one of 1830, which is available in the first edition of Ferdinand Wolff's textbook of geometry (*Lehrbuch der Geometrie*, Berlin, 1830). Interestingly, Wolff was a professor of the *Gewerbe-Institut* (Trade Institute), which played an important role of popularizing industrial design about 70 years later (Chapter 3). His textbook, however, deals with pure mathematics and there is no evidence that artists and craftsmen contributed to the new terminology. Interestingly, the *sectio aurea* is not an ancient or medieval expression, but a 19th century Latinization of the German term; it demonstrated that the mathematical concept, not the expression, is ancient. I still maintain my earlier view that the expression "golden section" was coined by educators of mathematics. Although we moved "backwards" just by five years, from 1835 to 1833, then to 1830, this result is still significant from the point of view of an interesting question. Did the discovery of the botanical importance of the golden section in the case of the spiral phyllotaxis inspire the new term "golden section"? The answer is negative: the recently discovered example of 1830 shows that it was available shortly before the discovery of the phyllotaxis by German and French scholars, Braun, Schimper, and the Bravais brothers, in the early 1830s.

I should refer to a new finding in connection with Leonardo, too. Many authors claim that Leonardo was interested in the golden section: he illustrated the polyhedra in Luca Pacioli's book *Divina proportione* (Venice, 1509), which title refers to the golden section, and introduced it into his proportional system of the human body. The latter is best represented in Leonardo's "Vitruvian man", which was associated with a proportional system that Vitruvius (1st c. A.D.) described in an unclear paragraph. The first part of this statement that Leonardo worked with Pacioli is correct, but the referred to book is not about the application of the golden section in art. This proportion is discussed in the mathematical parts of the book, while Pacioli's separate essay on proportion in art suggests using simple ratios of integers. Even more problematic is the statement that the "Vitruvian man" is based on the golden section. The usual view is that the navel divides the height of the man according to the golden section. Let us disregard the possible chronological problem that the "Vitruvian man" was drawn around 1490, many years before Leonardo met Pacioli. Indeed, one may argue that the dating of Leonardo's manuscripts is also problematic or Leonardo's interest in the golden section could have started earlier. The "Vitruvian man" with outstretched arms and legs is inscribed into both a square (*homo ad quadratum*) and a circle (*homo at circulum*), which are not concentric. This was Leonardo's important contribution, all earlier trials to illustrate Vitruvius's text considered concentric figures. In Leonardo's case, the center of the square is the point where the penis begins, while the center of the circle is the navel. On the other hand, the distance between these two points is not given, and this missing particular prompted various studies to reconstruct Leonardo's canon. These trials usually introduce new regulating lines and other figures, and many of these are associated with the golden section directly or indirectly. Here I offer an alternative approach, which is based on the accompanying text and does not require the extensive use of new regulating lines. Let us start with the square. It is drawn on the basis of the following observation: "The span of a man's outstretched arms is equal to his height" (I quote Edward MacCurdy's translation with some corrections). Of course, a side of the square is equal to the man's height. Leonardo described various lengths of the body by simple ratios of integers: 1/2, 1/4, 1/6, 1/7, 1/8, 1/10 lengths of the man's height. I projected the marked "maximum width of the shoulders", which is "a fourth part of the man", to the top horizontal side of the square, and calculated the distance between an end-point of this projected line-segment to the point where the circle intersects this horizontal line. The point of intersection is marked by the tip of the middle finger of an outstretched arm. I calculated the described horizontal distance by considering the right-triangle where the other two sides are: (1) the vertical distance "from the top of the breast to the crown of the head", which is "the sixth of the man", and (2) the length of the arm "from the elbow to the tip of the middle finger is the fourth part; from this elbow to the end of the shoulder is the eighth part", that is, all together, $1/4 + 1/8 = 3/8$ height of the man. Using the Pythagorean theorem, the horizontal distance in question is $\sqrt{65}/24 = 0.335$...times the man's height. If we know the exact position of this point, it is easy to calculate the radius of the circle as the function of the height: $(325 + 3\sqrt{65})/576 = 0.606$... times the height. Since the center of the circle is at the navel, this number also gives the ratio of the height of the navel to the full height of the body. It is significantly less than the golden number, $(\sqrt{5} - 1)/2 = 0.618...$ (Incidentally, the 19th century German scholar Adolf Zeising was the "pioneer" of considering the navel as the golden-section-point of the height; he tried to explain almost everything with his new theory of human proportions.) I also noticed that Leonardo's ratios included a redundant one: the distance from the top of the breast to the crown of the head can be calculated from earlier data and it is 47/280 ($1/8 + 1/7 - 1/10 = 47/280$), but later he gave it as 1/6, which is a good approximation of the other one. This means that Leonardo did not hesitate to round the complicated ratios in order to have simple ones. Consequently, we may believe that our "key ratio" of 0.335... should be 1/3. I am less enthusiastic to round 0.606... as 3/5 = 0.6. Moreover, by fixing 1/3, we determine the entire system. Thus, the ratio 1/3 is either a well kept "secret" of Leonardo or, at least, the key to a method to approximate his system. If we mark this point on the top horizontal side of a given square, it is very easy to construct the center of the circle on the vertical midline of this square, which also marks the man's height at the middle. First, we connect our point with the midpoint on the bottom horizontal side of the square (where the circle and the square should have a tangential point), and then the perpendicular bisector of this new line-segment will intersect the midline of the square at the necessary point. I do not go into further details, but will publish this approach elsewhere. This result is a further piece of evidence that the books on basic design should not rush to explain Leonardo's "Vitruvian man" with the golden section or other sophisticated methods. The importance of the golden section in the history of art is strongly overemphasized in the literature, as we discussed in the paper "Golden section(ism): From mathematics to the theory of art and musicology" (*Symmetry: Culture and Science*, Vol. 7, 413-441 and Vol. 8, 74-112, 1996-97). Although computer-aided design programs are able the handle proportional systems based on the irrational value of the golden section (earlier it was not so easy), I do not recommend "introducing" this proportion for the analysis or reconstruction of historic monuments in an uncritical way. Since the golden section was much less popular among artists and architects of the past than it is believed, we should use this proportion only in those cases where we have some evidence that it was really used. We may end up with less gold, but a few relevant cases may shine better...

9 The Symmetry Society and Brussels

Although the cooperation between architects and mathematicians may have various advantages, the meetings between them are relatively rare. Most symposia deal with specialized questions and there are very few broadly interdisciplinary meetings where representatives of science and of art may actually meet. The International Society for the Interdisciplinary Study of Symmetry (ISIS-Symmetry), or shortly Symmetry Society, pioneered such a forum with a triennial congress and exhibition: (1) Budapest, 1989; (2) Hiroshima, 1992; (3) Washington, D.C., 1995; (4) Haifa, 1998; (5) Sydney, 2001; (6) planned in Brussels, 2004. The concept "symmetry" is very useful to symbolize the interdisciplinary co-operations since it is widely known that it has both scientific and aesthetical meanings since ancient times. Interestingly, the very term "symmetry" was strongly associated in particular with mathematics and architecture in a complex way (cf., D. Nagy "The 2500-year old term symmetry in science and art and its 'missing link' between the antiquity and the modern age", *Symmetry: Culture and Science*, 6, No. 1, 18-28, 1995):

- the original Greek expression was first a mathematical term in the sense of commensurability, but later it was also used in aesthetics as good proportion (the term was not associated with "mirror symmetry"!)

- it was adopted into Latin as "symmetria", but was rarely used since the mathematical meaning of the Greek term was rendered in Latin as *commensura* or *commensuratio*, while the aesthetical one as *proportio*; the expression still survived because Vitruvius frequently used it by making a small distinction between the theoretical vs. practical aspects of proportions as *symmetria* vs. *proportio*,

- the expression "symmetry" became an international word in architectural texts and was used in many languages following the Renaissance period, when many artists-scholars translated the Vitruvian text and they needed not only the equivalent of *proportio*, but also *symmetria* (the term was not yet associated with balance in architecture)

- it was readopted by mathematicians as "mirror symmetry" since the original Greek meaning can be associated with the "common measure" of two equal parts; this modern understanding of symmetry became the everyday meaning of the expressions and it was also utilized by architects;

- architects utilized the new meaning of symmetry and used it to describe arrangement with a mirror-reflection; however, many of them emphasized that we need a more sophisticated balance, not just geometric symmetry

- mathematicians, crystallographers, and biologists generalized the concept by considering not only mirror reflection, but also other operations that transform a geometric figure or figure-system into itself (isomorphism); thus we may consider, among others, rotational symmetry, translational symmetry, crystallographic symmetries, color symmetries, similarity-symmetry, biosymmetries, etc. (later physicists continued this process and considered symmetry as invariance of a property, not definitely a geometric one)

- it became clear that the idea of crystallographic symmetries and biosymmetries is also useful in the theory of architecture, partly as a composition principles in the case of individual buildings, partly as an organizing principle in town planning.

Although the main interest of the Society is symmetry and the related concepts (proportion, balance, equilibrium, invariance, etc.), as well as the relationship between symmetry and asymmetry, it is not limited to these. We may understand the concept also metaphorically as the search for "symmetry" or "bridges" between concrete fields of art and science. Last, but not least, we should remark that the some of the major figures of the historic developments discussed earlier are associated with the Symmetry Society directly or indirectly. The mathematician Heinrich Heesch, an Honorary Member of the Society, discovered various tilings in the 1930s and 1940s, which were later commercially produced. He established connections with both Peter Behrens, a pioneer of industrial design (Chapter 3), and Johannes Itten, the initiator of the first Preliminary Course of the Bauhaus (Chapter 5). Earlier we referred to the architect William S. Huff (Chapters 6 and 8), another Honorary Member of the Society, who gave the Foundation Course of the Ulm School of Design and published a series of booklets on symmetry. He is a regular contributor of the Society and attended all of the five congresses and exhibitions in four continents. Indeed, we are very happy to have him and his fresh ideas at our events, which links architecture and mathematics.

There is a symbolic meaning that Brussels became a European center of the Symmetry Society in the symmetric year of 2002 (we should wait until 2112 for the next one). Almost fifty years ago, Brussels hosted the World Exhibition of 1958. Interestingly, but not surprisingly, some people who featured in our survey were also there. The architect Le Corbusier and the composer Edgar Varése presented their joint multimedia show entitled *Electronic Poem* in the Philips Pavilion, which was designed by the composer Iannis Xenakis who worked as an architect in the office of Le Corbusier in that time. The designer Tomás Maldonado presented the new curriculum of the Ulm School of Design during a lecture at the Brussels Exhibition. The text of this lecture was printed in the journal of the school; it was the first declaration of some distance from the Bauhaus. The Exhibition was also special for the engineer Frei Otto. At the beginning of this paper (Chapter 2), we discussed how the "expo architecture", the Crystal Palace and the Eiffel Tower, contributed to the development of modern architecture and design. Now the suspension structures, which were suggested by Frei Otto in his book of 1954, became dominant. The main symbol of the exhibition, however, was the Atomium, a building representing an iron molecule enlarged 150 billion times (designed by Polak and Waterkeyn). Eight spherical rooms ("atoms") were arranged, according to the body-centered cubic crystal-system, around a central one. The cubic structure stands on one of its vertex (the bottom spherical room) in a position where one of the diagonals is vertical (the elevator shaft is there). Earlier we discussed various utopian buildings that were never built, including Chernikov's designs and Leonidov's "spherical library" (Chapter 4). As a further coincidence, Ulrich Conrads and Hans G. Sperlich's book *The Architecture of Fantasy* (New York, 1962, pp. 90-91) put the Atomium, the sketch of Chernikov's Observation Station, and Leonidov's referred to design after each other after. The Atomium, however, is not only fantasy, but became reality...

The first Matomium conference of 2002 was opened in the Atomium building, in the presence of the Brussels Television, and concluded in a Horta Building...

Appendix: MM = Manifesto on Mat⊕mium (in the third millennium)

The Atomium Building symbolizes modern architecture, the beauty of mathematical structures, the importance of chemistry and crystallography, and, directly or indirectly, many fields of art, science, and technology. Since it was built for the Brussels Expo and preserved some of the original exhibitions, it also gives an insight into history and its meaning for the future. Last, but not least the Atomium is a symbol of Brussels, the capital of the united Europe, which is just before a historic expansion by reuniting Western and Eastern, Northern and Southern Europe, including such regions that were divided by political walls. This historic time is useful to express our wishes to reunite some efforts in different disciplines, symmetrically from all directions. Of course, the goal is not the unification of these disciplines, but to provide an informal forum where some representatives of different fields of art, science, and technology may come together and discuss mutually interesting topics. Another important goal is to contribute to the modernization of education.

The discussions in design about a century ago focused on the question: "standardization or individuality". The U-turns in the history of design show that we need both. The too much standardization, for example, contributed to the military machine before World War I, while the too much individualism led to very many –isms and movements in art and design, which disappeared very soon. The Constructivists and the Bauhaus tried to unite the positive aspects of all of these, with a strong social commitment and, indeed, produced some remarkable results. They tried to unite various forms of arts and crafts, as well as to educate artists-craftsman in a unified system, but some of their idea remained naïve utopia, while they failed to make a real unity between modern art and modern science and technology. There are many examples in our history that too much unification leads to failures. Thus, our goal is not to find some artists-craftsmen, as the Bauhaus tried, or the few people who can be considered as an architect-mathematician, a painter-biologist, a sculpture-chemist, a musician-physicist, and so on. Our actual goal is to find artists *and* scientists, including architects *and* mathematicians, painters *and* biologists, and others, who are interested in the dialogues and have a social commitment. Instead of a unified global view, we should consider the different opinions and contributions from all regions. We need *unity in plurality*, using a phrase that was used from ancient philosophy to modern architecture by various groups of people.

In the age of globalization, we should make a special effort to listen to the views in different countries and regions, including the smaller ones. Both design and mathematics provide good examples where something totally new came from small countries. For example, the initiator Art Nouveau was Victor Horta in Belgium, while one of the co-discoverers of the new non-Euclidean geometry was János Bolyai in Hungary. We are facing complex problems and the search for solutions should be also a complex interdisciplinary and international effort. For the preparation of this, however, we need regular meetings for *dialogues*, *trialogues*, *quadrologues*, ... and a "parliament" of the disciplines of art and science...

D & D in B & B (and A & A):

Dénes in Budapest (and Australia)

Dirk in Brussels (and Africa)