

SYMMETRY IN THE SELF-ORGANIZED CRITICALITY

K. NANJO, H. NAGAHAMA, AND E. YODOGAWA

Name: Kazuyoshi Nanjo, Earth Scientist, (b. Iwate, JAPAN, 1973).

Address: Earthquake Research Institute, University of Tokyo, Bunkyo-ku, Tokyo 113-0032, Japan. *E-mail:* nanjo@ism.ac.jp; nanjo@cse.ucdavis.edu.

Fields of interest: Earth sciences (also the science of complexity).

Publications and/or Exhibitions: Nanjo, K., Nagahama, H., and Yodogawa, E. (2000) Symmetry properties of spatial distribution of microfracturing in rock, *Forma*, 15, No. 1, 95-101; Nanjo, K., Nagahama, H., and Yodogawa, E. (2001) Symmetry and self-organized criticality, *Forma*, 16, No. 3, 213-224.

Abstract: *Curie symmetry principle (CSP), which states that the effects may occasionally have the same or a higher symmetry than the causes, is a powerful constraint for predicting a hypothetical physical condition with perfect symmetry. To test the validity of the CSP in the complex systems, we introduce a new concept of 'symmetry,' which measures the amount of symmetry and entropy in an object and examines its behaviour in the cellular-automaton system with self-organized criticality. During the sub-critical states, the symmetries of the causes and the results satisfy the CSP, but not always at the criticality (the edge of chaos). Our results show that symmetry breaking can occur without an anisotropic interaction of elements in cellular-automaton models and the especial tuning parameters in bifurcation process.*

1 INTRODUCTION

Curie in 1894 established a relation between symmetries of causes and symmetries of effects (Curie, 1894). This relation is called the 'Curie symmetry principle' (CSP), which is composed of three parts: 1. If certain causes yield the known effects, the

symmetry elements of the causes should be contained in the generated effects. 2. If the known effects manifest certain dissymmetry, this latter should be contained in the causes, which have generated those effects. 3. The converse to these two previous propositions is not true, at least in practical; i.e. the effects may have higher symmetry than the causes, which generate these effects. In short, the CSP states that the effects may occasionally have the same or a higher symmetry than the causes (Jaeger, 1920), which leads us to predict possible properties and to forbid impossible ones in physical process. However, previous studies refuse to invoke the CSP undoubtedly for all fields: that is, symmetry breaking is possible to be observed in some fields (Bouligand, 1985; Stewart and Golubitsky, 1992). Although previous researchers have studied symmetry breaking in the cellular-automaton model (Bouligand, 1985) and in bifurcation process (Nicolis and Prigogine, 1977; Sattinger, 1978), no attempts have been made to validate the CSP in the complex systems. In the current study we will test the validity of the CSP in each system.

The prototype model of a complex system includes a cellular-automaton model. Complexity measured by mutual information between two cells to study correlation in systems is the maximum at the phase transition between periodic and chaotic behaviour of cellular-automaton systems (Waldrop, 1992). The system at the edge of chaos is equal to that of dynamical critical state where scale-free structures emerge. The concept of self-organized criticality (SOC) was introduced by Bak and co-workers in the sand-pile cellular-automaton system (e.g., Bak et al., 1987; Bak, 1996). The SOC provides a simple mechanism with possible applications toward earthquakes, galaxy, turbulence, biological evolution, economic fluctuation, and other phenomenon. In the current study, we used the sand-pile cellular-automaton model that is regarded as the first prototype of a complex system.

To test the validity of the CSP, a tool to evaluate the symmetries of the causes and the effects is required. Mathematical group theory provides a good tool to deal with perfect symmetries (Sattinger, 1978). However, in the nature without perfect symmetry (e.g., Curie, 1894; Eigen and Winkler, 1975), it is reasonable to analyze symmetry by a continuous scale (e.g., Nagy, 1996), in spite of no general theory explaining the occurrence of continuous symmetries. Moreover, previous studies (Whyte, 1949ab; Dingle, 1949; Caillois, 1973) discuss symmetry connected to entropy. 'Symmetry' mathematically connects symmetry with entropy through information theory (Yodogawa, 1982; Nanjo et al., 2000). It measures the amount of symmetry and entropy in a given pattern. Here, we used symmetry to evaluate the symmetries of the causes and the effects.

This paper introduces a new concept of symmetry. Next, we measured the symmetries evaluating the causes and the effects of the sand-pile cellular automaton model with self-organized criticality, and the comparison was made between them in

order to test the validity of the CSP. Details of this work have been described previously (Nanjo et al., 2001).

2 SYMMETROPY

Mathematical group theory provides a good tool to deal with perfect symmetries (Sattinger, 1978). When our scope is extended to the symmetries of real objects, it is reasonable to analyze symmetry by a continuous scale rather than by a discrete feature 'symmetric or non-symmetric' (e.g., Nagy, 1996). Moreover, previous studies associated the concept of symmetry with the concept of entropy (Whyte, 1949ab; Dingle, 1949; Caillois, 1973). According to these studies, we need a tool to measure the amount of symmetry connected with entropy. 'Symmetry' meets this need (Yodogawa, 1982; Nanjo et al., 2000).

Symmetry mathematically connects symmetry with entropy through information theory. It measures the amount of symmetry and entropy in a given pattern or shape by using the two-dimensional discrete Walsh functions. A pattern becomes higher symmetric (less anisotropic) in the symmetry sense and more disorder in the entropy sense with increasing symmetry. The CSP is consistent with that the symmetry of the cause must be equal to or larger than that of the effect. For examining the validity of the CSP, we used symmetry as a continuous measure of symmetry.

3 PROCEDURE AND RESULTS

We performed the sand-pile cellular-automaton model in two dimensions (e.g., Bak et al., 1987; Bak, 1996). We started the system from an initial condition of no particles (no sands). We observed that this system evolves through sub-critical states and reaches the critical steady states. Note that in the model there is no turning parameter and the local rule at a microscopic level is the isotropic interaction between cells. The causes are particles dropped at randomly selected cells, and the effects are avalanches. We defined the symmetry of the causes as the symmetry estimated for the spatial distributions of the causal particles dropped in various time spans. The symmetry of the effects is defined as the symmetry estimated for the spatial distributions of the resultant avalanches occurred in various time spans.

During the sub-critical states, the symmetries of the spatial patterns of the causes are equal to those of the effects. This is consistent with the CSP. During the critical steady states, the symmetries of the spatial patterns of the causes are not always equal to or larger than those of the effects. This is not necessarily consistent with the CSP, and shows that symmetry breaking is possible to be observed. Therefore, we found that the CSP is not always valid in the sand-pile cellular-automaton model.

4 DISCUSSIONS

Symmetry breaking has been studied in cellular-automaton model and in bifurcation process of equilibrium statistical mechanics and mathematical group theory. In Luck's cellular-automaton model, anisotropy is built into the rules of the model at the intracellular level, and symmetry breaking is observed at a higher organization level than the single cells (Bouligand, 1985). In equilibrium statistical mechanics, symmetry breaking can be reached only by tuning a parameter to the critical values such phase transition points (Nicolis and Prigogine, 1977). In mathematical group theory, symmetry breaking is a change in the symmetry group, from a larger one to a smaller one, from the whole to the part, and its change needs that a parameter crosses a critical value (Sattinger, 1978). In the present model of sand-pile cellular-automata, local rules at a microscopic level are isotropic without fine-tuning parameter. Therefore, we found that our result regarding symmetry breaking is different from symmetry breaking occurred with an anisotropic interaction of elements in cellular-automaton models and special tuning parameters in bifurcation process.

Although there was a claim that it is natural to analyze symmetry properties in terms of a continuous scale rather than in terms of 'yes or no' (e.g., Nagy, 1996), no general theory that explains the occurrence of continuous symmetries has been made. In the present research, the estimated symmetries took various values during the critical steady states. That is, we observed various symmetries from the viewpoint of the symmetry. This indicates that symmetry is continuous rather than discrete, and meets the previous claim (e.g., Nagy, 1996). Therefore, we propose that the concept of SOC is the underlying concept for continuous symmetries.

KN thanks to JSPS Research Fellowships for Young Scientists for financial support and to Tony Y. Momma for revising the English style of the manuscript. Full list of references appeared in this paper is shown in Nanjo et al. (2001).

References

- Nagy, D. (1996) (Dis)symmetry measures, *Circular of the Society for Science on Form, Japan*, 10, No. 3, 16-17; cf., the notes by the same author in: Ogawa T., Miura, K., Masunari, Nagy D., eds., *Katachi U Symmetry*, Tokyo: Springer-Verlag, 1996, p. 45, note 2.
- Nanjo, K., Nagahama, H., and Yodogawa, E. (2000) Symmetry properties of spatial distribution of microfracturing in rock, *Forma*, 15, No. 1, 95-101.
- Nanjo, K., Nagahama, H., and Yodogawa, E. (2001) Symmetry and self-organized criticality, *Forma*, 16, No. 3, 213-224.
- Yodogawa, E. (1982) Symmetry, an entropy-like measure of visual symmetry, *Perception and Psychophysics*, 32, No. 3, 230-240.