

SYMMETRIES ON THE SPHERE

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Publications: (see also: <http://www.cs.berkeley.edu/~sequin/BIO/pubs.html>)

C.H. Séquin: "Virtual Prototyping of Scherk-Collins Saddle Rings," *Leonardo* 30, No. 2, pp89-96, 1997.

C.H. Séquin: "Analogies from 2D to 3D, Exercises in Disciplined Creativity," *BRIDGES - Mathematical Connections in Art, Music, and Science*; Conf. Proc., Winfield KS, July 1999.

C.H. Séquin, "Turning Mathematical Models into Sculptures" *MOSAIC 2000* (Millennial Open Symposium on the Arts and Interdisciplinary Computing), Conf. Proc., U. of WA, Seattle, Aug. 22, 2000, pp 71-82.

J. Yen and C.H. Sequin, "Escher Sphere Construction Kits" *Proc. of Interactive 3D Graphics Symposium*, Research Triangle Park, NC, March 19-21, 2001, pp 95-98.

C.H. Séquin, "*Viae Globi* - Pathways on a Sphere," *M+D2001*, Geelong, Australia, July 3-5, 2001.

Exhibitions: (see also: <http://www.cs.berkeley.edu/~sequin/BIO/exhibits.html>)

"Art/Science: Six New Visions," Canessa Gallery, San Francisco, Jan. 31 - Feb. 25, 1994.

"Artists Shedding Light on Science," Art Department Gallery, SF State University, June 11 - June 24, 1994.

"Science in the Arts, Art in the Sciences," *UNESCO-ICSU World Conference on Science*, Budapest, Hungary, June 26 - July 1, 1999.

"Science, Technology, and the Artist," Danville Fine Arts Gallery, Oct. 28 - Dec. 2, 2000.

"Art & Mathematics 2000," Exhibit at The Cooper Union, Albert Nerken School of Engineering, 51 Astor Place, New York, NY 10003-7183, Nov. 7 - Dec. 15, 2000.

Abstract: *Sub-symmetries of the Platonic solids have been applied to make geometrical art objects. In one effort, Escher-like tilings are created to cover the whole surface of a sphere in a regular pattern. In another one, smoothly meandering curves are drawn on a sphere, and an appropriate cross section is swept along it to make 3D sculptures. In both cases, special purpose programs have been written to assist the designer with these constructions. The development of these constrained editors and parameterized "sculpture generators" raises an interesting issue: What mixture of symmetry and deliberate symmetry-breaking is optimal for a good abstract geometrical sculpture?*

1. ESCHER SPHERES

Inspired by Escher's numerous tiling patterns for the plane [3], we wanted to make it easy to create such patterns on a sphere, and then to create physical copies of *Escher spheres* by rapid prototyping on a Solid Free-form Fabrication machine. These goals largely defined our approach to creating a suitable CAD tool [5]. The first machine that we had access to at our school was a Fused Deposition Modeling (FDM) machine that could only make objects of a single uniform color. We wanted to use this machine to build individual tiles of different color in different runs and then assemble these tiles into hollow, lightweight, and colorful *Escher spheres*. Rather than creating a "painting tool" that would simply replicate colored pixels placed on the sphere at many different spots in accordance with the chosen symmetry group, we chose an approach based on deforming an initial simple tile, representing a fundamental domain of that symmetry group. This made it easier to create physically realizable tiles and to guarantee that these tiles would yield a perfect coverage of the sphere. So far we have also limited ourselves to monohedral tilings.

After having selected one of the given spherical tiling patterns [2], the user can then deform the boundary of one of the fundamental tile domains by clicking and dragging points on its outline. All symmetrically related tile edges on the whole sphere simultaneously reflect this change; thus the tiling pattern always remains consistent with the chosen symmetry group. Once the tile outline has been defined, the user can draw additional internal points and edges in order to decorate the tile. These points can also be used to create a bas-relief on the tile by adjusting their individual distances from the center of the sphere. The regions between the specified edges are tessellated into Delaunay meshes of triangles that linearly interpolate the heights between neighbor vertices. The finished master tile can be scaled and extruded to the desired size and thickness before it is sent as a triangulated boundary representation to a Fused Deposition Modeling (FDM) machine.

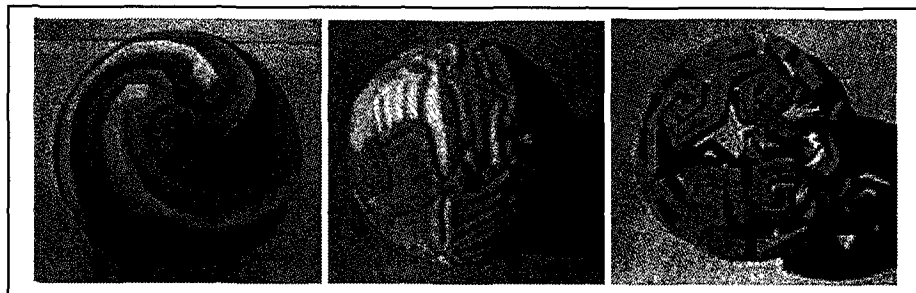


Figure 1. *Escher Spheres: a) 12 Snakes (FDM); b) 24 Birds, c) 12 Lizard Frames (3D Color Printing).*

More recently we also acquired a 3D color printer from Zcorporation on which one can fabricate multi-colored parts in a single pass. For this process, we instruct our CAD program to show all the decorated bas-relief tiles on the sphere, and we can then assign individual colors to every one of them. The colored object is transmitted to the printer in VRML format. Figure 1 shows results obtained on both of these machines.

2. Viae Globi

Another activity that prompted us to consider the symmetries on a sphere [2] quite explicitly concerned a sculpture generator for making shapes inspired by Brent Collins' *Pax Mundi* [1]. This shape reminded me of the curved edges found in sculptures by Naum Gabo. I thus use the name **Gabo curves** to describe smooth, loopy curves on a sphere that oscillate symmetrically around the equator. A generalized **n -lobe Gabo curve** would have n bulges above and below the equator and $2n$ equally spaced inflection points. These closed meandering curves divide the sphere into two identical parts. Overall, this geometry has prismatic/cylindrical symmetry of the type D_{nd} .

To create a more interesting and more dramatic sculpture, it is advantageous to reduce this symmetry from D_{nd} to D_n by allowing asymmetrically bulging lobes, or to reduce the symmetry to a lower n by not making all lobes of equal height. As an example, in *Pax Mundy*, which can be understood as a derivative of a **4-lobe Gabo curve**, every second lobe on each hemisphere is of lower amplitude. The example of *Via Globi 3*, featuring a slightly modified **3-lobe Gabo curve**, is shown in Figure 2a. The name of this series, *Viae Globi*, inspires images of curvy alpine roads winding around the globe [4].

Once I had created pleasing looking Gabo curves ranging from $n=2$ to $n=6$, I wanted to explore the possibilities in making more convoluted loopy paths on the surface of a sphere. I was looking for paths with more truly spherical symmetries as exhibited by the Platonic solids, which have higher-order symmetry axes in more than just two opposing directions in space. In one approach, I placed the control points for a closed B-spline path at the vertices or at the edge-midpoints of one of these highly regular polyhedrons and tried to connect them into a closed path of some symmetry. In this way, I was able to create several intriguing and aesthetically pleasing sculptures (Fig.2b).

There are several ways to reduce the symmetry in order to achieve more interesting aesthetic results – for instance by rotating or gradually modifying the cross section that is swept along the defined spherical path. The example in Figure 2c is based on a **2-lobe Gabo curve** enhanced with extra undulations. I started by sweeping a thin rectangular slab along it. The symmetry is broken by introducing a 180° twist in the sweep and by morphing the cross section between a thin rectangle and a shallow crescent (Fig.2c). The vertically standing ribbons are reminiscent of patterns seen in an Aurora Borealis.

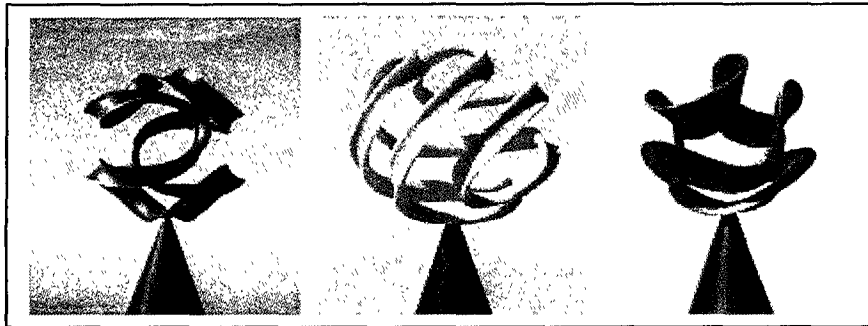


Figure 2. Sphere paths: a) *Via Globi 3* (bronze); b) *Altamont*, c) *Aurora-M* (FDM).

3. Conclusion

For the construction of *Escher spheres* as well as for the design of the *Viae Globi* sculptures, it was crucial to study the symmetries possible on a sphere. This allowed us to structure our design approach and to tailor our CAD tools to match the specific demands of our problem domain. As a result, it became easier to write the needed programs, and we also could give more effective support to the users of our programs.

In both cases, it turned out that the most symmetrical solutions are aesthetically not very interesting. The subgroups of the Platonic symmetry groups that suppress the mirror symmetries offer much more freedom for the deformation of the tile edges and thus lead to far more intriguing tile shapes for our *Escher spheres*. Similarly, the roads on the sphere become more sophisticated, if the regular waveform of a *Gabo curve* is deformed into wiggles with lower symmetry. A further reduction in symmetry and additional aesthetic gain can be obtained from varying the azimuthal orientation of the cross section that is swept along the basic spline curve on the sphere.

References

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