

PAPER FOLDING AND PROPORTIONS IN POLYGONS

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Fields of interest: Geometry, proportion, symmetry (Art and Architecture, Mathematics with Paper Folding, Phyllotaxis, and Mathematics in Daily Life).

Publications and/or Exhibitions:

- Proporciones y Arquitectura. ICME, (Congreso Internacional de Educación Matemática) Sevilla 1996.
- Estudio y desarrollo geométrico del Modulor de Le Corbusier, VIII JAEM, (Jornadas Nacionales para la Enseñanza y Aprendizaje de las Matemáticas.) Salamanca 1997.
- Proportions mathématiques et leurs applications a l' architecture. Bulletin IREM (Institut de recherche pour l'enseignement des mathématiques de Toulouse) Toulouse. France, 1998, 39 pp.
- Papiroflexia y proporciones dinámicas en rectángulos. JAEM (Jornadas Nacionales para la Enseñanza y Aprendizaje de las Matemáticas.) 1999
- Mathematics: a determinant strategic for the development of an architectonic project. Symposium on Symmetry. Alhambra 2000, Granada (Spain)

Abstract: *In this exposition I will focus mainly on the concept of Polygonal Proportion, (PP). At the same time I present some constructions of polygons using paper folding. It is very important the justification of these constructions with mathematical operations. Teaching mathematics through paper folding is becoming a methodological way understanding mathematics. Today many average citizens usually only remember traumatic classroom experiences related with the process of learning mathematics. One of the purposes of this paper is defining a common message to improve the image of mathematics in contemporary society. Sharing properly designed activities and classical proofs can help a long way towards fruitful and productive mathematics education Teaching mathematics has been enriched with using paper folding environments of exploratory type where students can experiment with geometrical objects, investigate their properties, formulate conjectures and verify them.*

The concept of proportion in segments and rectangles is well known, but the polygonal proportion (PP) is a concept not so readily studied.

We define the proportion of polygon P_n , written $p(P_n)$, as the quotient between the area of the polygon and the minimum side square. In case of a square this formula give us 1 and if R is a rectangle whose sides are a and b , $a > b$, we obtain $p(R) = (a \cdot b)/b^2 = a/b$.

This definition applied to the regular octagon O_8 provides :

$$p(O_8) = 2(\sqrt{2} + 1) = \theta, \text{ where } \theta \text{ denotes the silver number.}$$

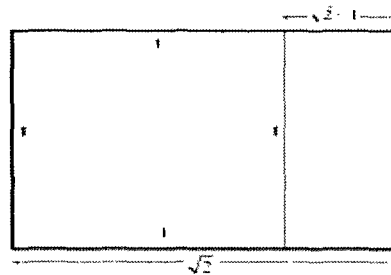
After establishing the definitions and their main properties, we can pay attention to geometric relations in some of convex polygons. The ratio between the radius of the circumscribed circumference of a polygon and the side of this corresponding polygon, is very important. In the case of the decagon, this relationship gives us the golden section and in the case of the octagon, the "cordobesa proportion".

Every case of polygonal proportion can be illustrated with many examples in: 1.- Art: classical and modern paintings as well as ultramodern painters. 2.- Architecture: with several plans and elevations of buildings. 3.- Phyllotaxis: with many examples where the PP is important. 4.- Zoology: with examples of animals where the polygonal shapes appear. 5.- And finally other aspects in daily life such as: mechanics, needlework, barred windows, lattices, as well as pavement patterns. The wide range of examples in Art and Nature will produce several ideas which can be useful in improving this learning-teaching mathematics process.

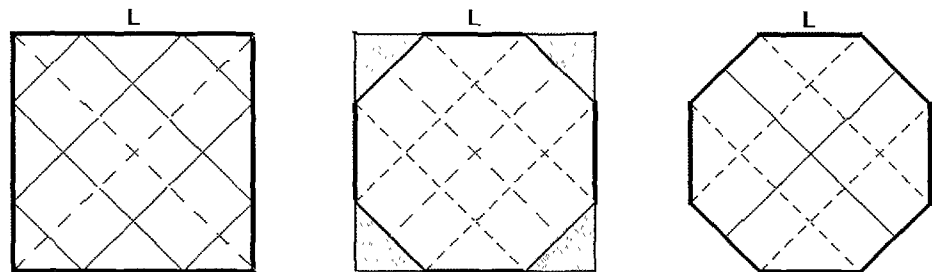
Related with educational aspects I think the paper folding is an important tool when it is incorporated in classes in all levels. My interest focuses on research the underlying mathematics involved in paper folding process. I will construct some of the polygons by paper folding, justifying their constructions with mathematical operations. I will prove that it is possible to obtain an octagon starting from an A4 sheet of paper.

It is well known that the DIN A 4 format has the proportion $\sqrt{2}$. So we can consider $1 \times \sqrt{2}$ the dimensions of the paper.

If we cut the gnomon of a DIN A4, e. g. one square, we have a little strip of dimensions $1 \times (\sqrt{2} - 1)$.

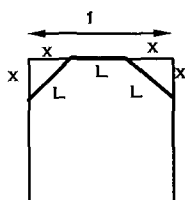


Therefore the strip of paper is placed in parallel with the diagonal of the square and tangent to its sides. Repeat same operation with the other diagonal of the square.



Folding the corners of the square, we obtain the regular octagon. The proof of this result is the following:

Denoting by L the side of an octagon obtained from a square whose side is 1, we have:



$$2x+L=1; \quad L^2=2x^2; \quad x^2=\frac{L^2}{2}; \quad x=\frac{L}{\sqrt{2}}; \quad 2x+L=1 \Leftrightarrow 2\frac{L}{\sqrt{2}}+L=1;$$

$$(\sqrt{2}+1)L=1; \quad L=\frac{1}{1+\sqrt{2}}=\frac{\sqrt{2}-1}{1}. \quad \text{Then } L=\sqrt{2}-1.$$

What implies that the paper folding process is correct because the side algebraically obtained of the octagon has exactly the same value as the side of the octagon obtained from the smallest side of the strip. Because the proportion in a pentagon is related with the golden number ϕ , the format of DIN A4, in proportion $\sqrt{2}$ is not suitable for the pentagon. We'll use the paper size's proportion combined with the Polygonal Proportion to make constructions of some polygons and justify them algebraic and geometrically.

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