Symmetry: Art and Science
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VISUAL MATHEMATICS

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Abstract: *Throughout history, there existed permanent links between geometry and the visual arts, painting, sculpture, and architecture, so that visual representations were often the basis for geometric investigations. We may refer to the ornamental art or to the descriptive geometry discovered by Renaissance painters. More recently, after the introduction of non-Euclidean geometry, visualization of new mathematical structures became their most comprehensive models. The visual modeling of structures belonging to the field of natural sciences (Physics, Chemistry, Biology, etc.) by means of different visual representations - diagrams, graphics, etc. - brought into being a complete visual language for the expression and representation of mathematical structures.*

1 VISUAL MATHEMATICS

In the last few decades, thanks to computers, we have a new field of art/science connections: the visual mathematics. Many of the structures known only to mathematicians, or even by them considered as "exceptions" or "monsters" (e.g., fractals), are now visually represented. For the first time, we can see real-time animated images from the hyperbolic space, rotate knots or links in the screen of our computer, or make the 3D models of different topological surfaces.

Unfortunately, most of the electronic mathematical journals that could be found on the Web are, in fact, just the classical mathematical journals in the electronic form, without using any of the new visual possibilities. We are very proud that *Visual Mathematics* (or *VM* http://members.tripod.com/vismath/ or http://www.mi.sanu.ac.yu/vismath/), the electronic journal of ISIS-Symmetry, is probably the unique on-line journal having both components: visual and mathematical. The main goal of *VM* is to show the beauty of mathematics in a broad artistic-scientific context. As a secondary aim, *VM* tries to correct the negative tendency that led to the unpopularity of mathematics in school and the lack of public understanding of this field.

2 MODULAR STRUCTURES

Modularity is the use of several basic elements (modules) for constructing a large collection of different possible (modular) structures. In science, the modularity principle is represented by the search for basic elements (e.g., elementary particles, prototiles for different geometric structures, etc.). In art, different modules (e.g., bricks in architecture or in ornamental brickwork, etc.) occur as the basis of modular structures. In various fields of (discrete) mathematics, the important problem is the recognition of some set of basic elements, construction rules, and a (exhaustive) derivation of different structures that can be generated in the given framework.

In a general sense, the modularity principle is a manifestation of the universal principle of economy in nature: the possibility for diversity and variability of structures, resulting from some finite and very restricted set of basic elements by their recombination. In all such cases, the most important step is the first choice (recognition or discovery) of the basic elements. This is shown by examples from ornamental art, where some elements originating from Paleolithic or Neolithic art are preserved till now, as a kind of "ornamental archetypes".

In many cases, the derivation of discrete modular structures is based on symmetry. Using the theory of symmetry and its generalizations (simple and multiple antisymmetry, colored symmetry, etc.) for certain structures, it is possible to define exhaustive derivation algorithms, and even to obtain some combinatorial formula for their enumeration.

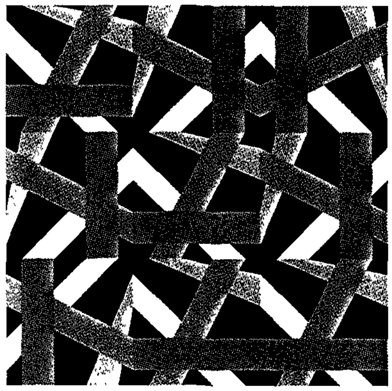
Various examples for modular structures that are lying on the border between art and mathematics will be considered:

a) antisymmetry ornaments and their derivation with a few prototiles, as well as the algorithmic approach to their generation (*OpTiles*);

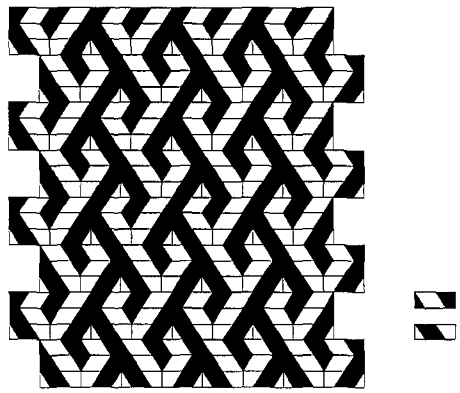
b) different knot-projections occurring in knotwork designs (Islamic, Celtic, etc.), derived from regular and uniform plane tessellations by using a few basic elements (*KnotTiles*);

c) sets of modular elements for the derivation of possible and impossible objects (tribar, etc.) and the classification of the obtained structures (*SpaceTiles*).

It is interesting that many of the modular geometrical structures originate from the prehistoric or ancient art, or from the ethnical art, and sometimes the same structures are discovered completely independently by different cultures, distant one from another in space and time. This is an important argument proving that the algorithmic way of thinking is a universal property of mankind, and not only the characteristic of the present epoch or culture, caused by the technological development.



Single prototile and knotwork tiling created from it.



Anasazi pattern and its modular reconstruction.

After showing some representative examples from *VisMath*, in the second part of the lecture several modular structures (*OrnTiles*, *SpaceTiles*, *KnotTiles*) will be presented, which were created by the author. All of them have very interesting historical backgrounds (e.g., *OpTiles* are inspired by Paleolithic and Neolithic key-patterns, *SpaceTiles* are modular elements for creating impossible figures, *KnotTiles* are the elements for creating mirror-curves). In the same time, using the principle of modularity, these structures could be used in order to explain the origins of the same or similar patterns occurring in different cultures, distant in space and time, as well as for the modular reconstruction of such patterns from archeological artifacts, where only some parts or fragments are preserved.

In a similar way, the concepts of *Mirror Curves* and *KnotTiles*, which are resulting from them, are connected with different knotwork structures occurring in ornamental art, black-and-white designs, and mathematical objects (knots and links, graphs, polyominoes, plane tilings, etc.). Such perfect singular curves, which are generated by mirror-reflections, were discovered independently by different cultures: their highlights are Celtic knotwork designs, Tamil drawings, and Tchokwe sand designs. After recognizing their mathematical basis, it is possible to consider them from the mathematical point of view and to use them very efficiently in education. This way, visual-symbolic structures of ornamental art could be used as comprehensive models of some mathematical concepts (knots, regular graphs, etc.).

References

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