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R. N. ELLIOTT ON SOCIAL MOOD AND SOCIAL CHANGE

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Abstract: *Nineteenth century physicists called into question the mechanistic approach of Newtonian physics, which was based on atomism, reductionism, the unity of the sciences, and the primacy of external causality, by developing various field theories. Developments in thermodynamics also called into question the reductive approach of mechanism. R. N. Elliott, an American social thinker who lived from 1871 to 1948, applied the principles of the new physics, i.e., holism, waves, pluralism, and internal self-organization, to society in order to develop an alternative to social mechanism. He though that mass psychology, impelled by emotion and not reason, was the driving force of human society and that these emotional waves swept through society in a five-wave sequence. According to Elliott, the most direct and detailed measure of these moods is provided by aggregate stock market prices. His work is consistent with the work of Mary Douglas and her followers in cultural theory.*

R. N. Elliott was an American social thinker who lived from 1871 to 1948. Just as nineteenth century physics called into question the mechanistic views of Newtonian physics, so Elliott called into question the application of such mechanistic models to society.

According to mechanism, an entity consists of fundamental particles or atoms. Any whole within a given discipline is merely the sum of its parts. Thus, the proper procedure is to analyze a whole into its fundamental parts and then sum those parts to constitute the whole. Further, all disciplines can ultimately be resolved into a single discipline, physics, so that the doctrine of the unity of the sciences is also fundamental to mechanism. Finally, according to mechanism, the universe is basically inert so that atoms change only as a result of outside forces acting upon them. Newton formulated these laws of external causality in his three laws of motion. Thus, atomism, reductionism, the unity of the sciences, and the primacy of external causality are the four pillars of mechanism.

A key to Elliot's alternative intellectual orientation can be found in his praise of the work of Marconi, the inventor of the radio. Marconi's work was the result of a series of century-long challenges to the mechanistic conception of the universe and seemed to vindicate a field theory over an atomistic one. The first challenge to mechanism came when Young proposed the wave theory of light, which, by the middle of the nineteenth century, was widely accepted. The second challenge came from developments in electromagnetic theory. The work of Ohm, Faraday, and Maxwell further developed a field theory as an alternative to mechanism. Faraday rejected the atomic theory of matter in favor of a field theory. Maxwell developed the electromagnetic theory of light, and Hertz provided experimental verification of Maxwell's theories as well as the fundamental discoveries upon which the development of radio broadcasting and radar were based. Hertz produced and detected waves over short distances. Marconi's achievement was to produce and detect such waves over long distances. This laid the foundations for radio. His work seemed to vindicate the concept of a field over that of particles. Elliott's work extended the field concepts of this revolutionary new philosophy, as Einstein and Infeld (1938, p. 125) called it, to society.

The rise of thermodynamics in the nineteenth century also called into question some of the tenants of mechanism. As a result of the work of Carnot, Clausius, and Kelvin, the first and second laws of thermodynamics were formulated. These describe the behavior of complex systems in more holistic terms. As a result of this work, it was shown that macrostates could be used to describe a complex system and that these could be accounted for in terms of macrovariables that did not make reference to the atomistic constituents of the complex system. Indeed, any given macrostate is compatible with many different microstates and thus is not uniquely determined by a single microstate. Thus, the properties of wholes could be studied in and of themselves since they have properties of their own that were not merely the sum of their constituents. This left it open to study the properties of fields and systems independently of their parts.

Applying the above innovations from physics to social theory, Elliott developed a conception of society based upon systems as wholes and upon field concepts. His significant innovation at this point was to develop a method for identifying and studying the properties of wholes that cannot be reduced to the properties of their constituents. Philosophically, Elliott's conception of a system is similar to Spinoza's approach to organic systems. Like Elliott, Spinoza thought that a whole should be viewed as a whole and not as a mechanical interplay of constituting parts. As Jonas (1973, p. 269) states in his explication of Spinoza's theory, a whole is "the sustained sequence of states of a unified plurality, with only the form of its union enduring while the parts come and go". Thus, a whole maintains its identity by maintaining a certain form or pattern. As was the case with electromagnetic phenomena, this form or pattern was, for Elliott, given by a wave structure. By developing a procedure for identifying the properties of wholes in terms of wave structure, Elliott rejected the reductionism of mechanism.

Elliott identified a five-wave sequence that he thought was a fundamental pattern of nature. In a growth wave, wave 1 consists of an initial expansion. Wave 2 is a partial retracement of wave 1. This is followed by wave 3, which is often a strong growth spurt. Wave 4 consists of a partial retracement of wave 3. Finally, wave 5 constitutes the last and final growth spurt of the system. At this point, the growth has reached its maximum point. Then a decline begins, which partially retraces the whole progress of the previous five-wave expansion. The first wave, wave A, of the retracement is a downward thrust. This is followed by wave B, which is an upward thrust that corrects the first wave down. Finally, there is the third wave down, wave C, which completes the correction of the five-wave sequence. The process is then ready to begin another five-wave expansion period (Elliott, Jan. 16, 1940, pp. 156-158).

Elliott applied the principles of holism, waves, pluralism and internal self-organization independently of external causes to develop an understanding of human society. Elliott thought that mass psychology, impelled by emotion and not reason, was the driving force of human society. Thus, the waves that passed through society were emotional moods (Elliott, Apr. 20, 1943, p. 182; See, also Dec. 15, 1939, p. 146; Oct. 1, 1940, p. 162; Dec. 15, 1942, p. 109; and 1942, p. 171). Such waves of emotion express themselves in all sorts of social phenomena. However, according to Elliott, the most direct and detailed measure of these moods is provided by aggregate stock market prices. Elliott (Apr. 20, 1943, p. 182) states that stocks are "an emotional human activity", and the New York Stock Exchange because of "its marvelous machinery and organization reflect psychology immediately and to perfection" (Aug. 6, 1945, p. 137). Investment decisions are typically not made by a rational assessment of risks and rewards but by relying upon others and what they are doing. As Prechter (1999, p. 153) states, investors "are driven to follow the herd because they do not have firsthand knowledge adequate to form an independent conviction, which makes them seek wisdom in numbers". Because such movements are emotionally based, they can give rise to what people perceive (usually after the fact) as "abnormal markets" in which there is extreme irrational exuberance (Elliott, Apr. 20, 1943, p. 182). Under such conditions, individuals are overly optimistic and take on excessive risks, which leads to an economic crisis of over-investment. This is what happened in the late 1920s. After being financially burned in the economic collapse following a period of over-optimism, the crowd then goes to the opposite extreme and becomes extremely pessimistic and risk-averse. As a result, economic conditions stagnate, as they did in the 1930s. Elliott's biggest challenge to modern economics, though, may be that the one-dimensional conception of human being that underlies traditional economic theory is flawed because it ignores the emotional side of human existence. Elliott's work is consistent with the work in cultural theory by Mary Douglas and her students. Human beings are social animals who find meaning through identification with others. Identification with others requires acceptance by others, and this in turn requires that an individual adopt the preferences of his/her target group and that he/she behave accordingly. Social acceptance and identification with a group brings with it a set of preferences and a set of social relations consistent with those preferences. Preferences and social relations act in a self-reinforcing manner so as to form a consistent whole among groups of self-organizing individuals. This is true of people's perception and assumption of risk, which is also socially constructed. Thompson *et al.* (1990, pp. 25ff) identify four socially constructed perceptions of risk. According to cultural theory, social changes are due to the movement of people from one of these orientations to another. What the work of Elliott suggests is that there is a pattern to such movement and that the movement toward these ways of life can reach extremes.

Though it is under attack, the primary hypothesis today purporting to explain the behavior of financial markets is the Efficient Market Hypothesis. This holds that the decisions of all market participants are always informed and rational so that investment values are equilibrated immediately. This theory finds it difficult to account for the irrational exuberance that leads to stock market bubbles, such as the one in 1720, the one in 1929, and the current one that is unraveling before our very eyes. During such times when assets are overpriced, investors should be selling instead of buying. Nor can it account for times of extreme pessimism in which investors withdraw wholesale from the markets at the very time that assets are underpriced and they should be buying instead of selling. Proponents of "Behavioral Finance" have pointed out that since the market is supposed to reach "equilibrium" efficiently, this theory cannot account for what some would call "far-from-equilibrium states". Elliott would say that the very idea of equilibrium is erroneous in the social setting and that every level of financial valuation is no more than a point along the continuum of mood change. For Elliott, both of these purported states, and those in between, are part of the natural ebb and flow of social mood.

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THE HINGED DODECAHEDRON

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Abstract: *The trisectioning of the cube into congruent parts is one of several three-dimensional assignment of my formative design studio that has been intermittently investigated with my students for nearly 30 years. It grew out of a wider pedagogic challenge: the sectioning of the cube into any number of congruent parts and, even, the sectioning of other regular and semi-regular solids in such wise.*

1. Trisectioning of the cube and the Rhombic Dodecahedron

A remarkable design from the unpredictable pool of student creativity (in this case, from John Lape) came out of the early phase of this studio task: a rhombic dodecahedron, sectioned into 24 congruent, non-regular, superposable tetrahedra. The art of this artifice—which, arranged into 12 square-based, split pyramids, can lie flat on a plane (Fig. 1)—is derived from the fashion in which the 24 parts are hinged together; for it is most remarkable that there are two strikingly different ways to assemble this hinged wizardry into a rhombic dodecahedral body. One may be called the *wrapped cube*; the other, the *radiant pentahedra*. A telling sign of this dualism lies in the recognition that, in one assemblage, a joint cuts across the short diagonal of each rhombic face and, in the other, a joint cuts across the long diagonal of the rhombic face.

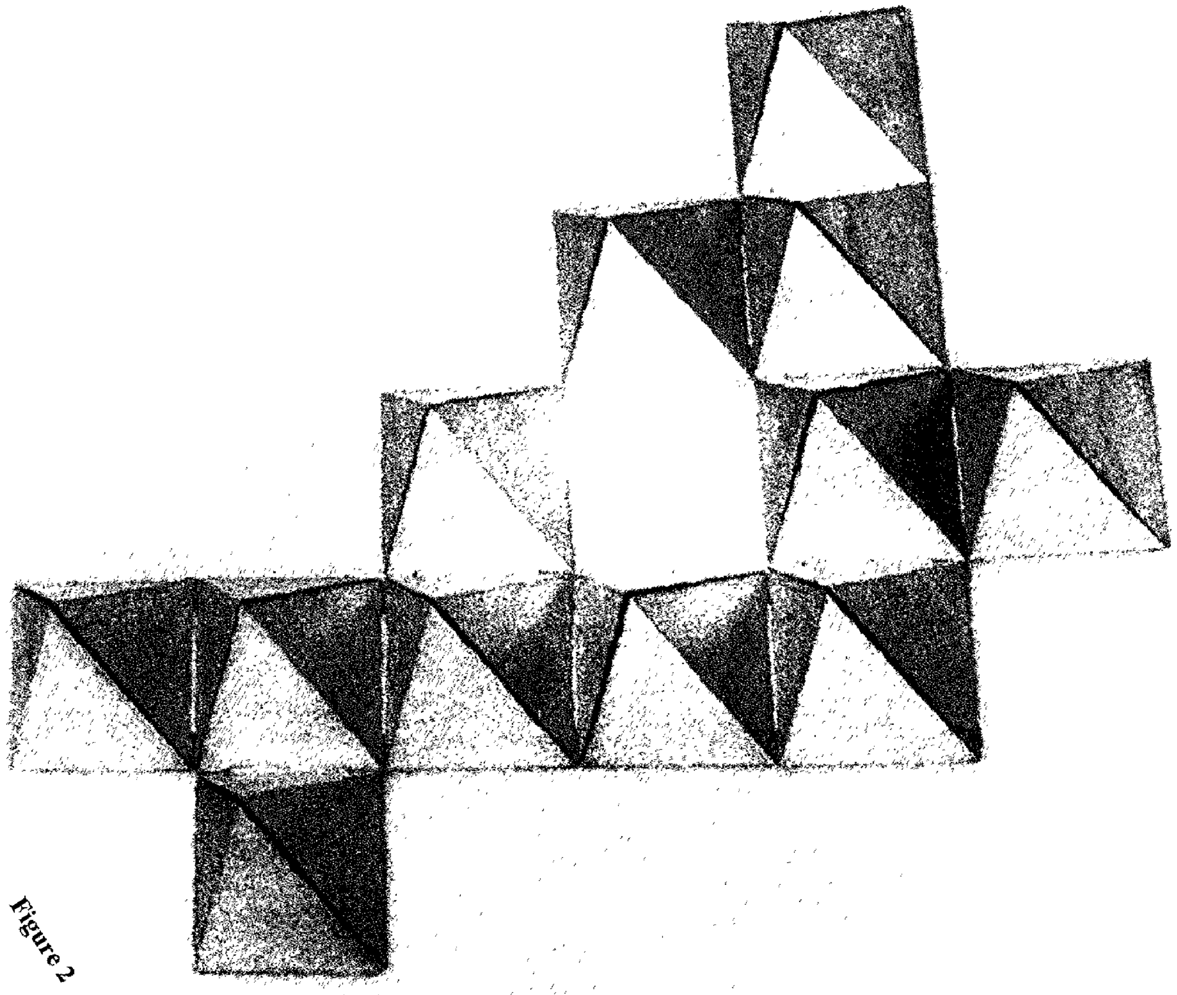

Figure 2

2. The wrapped cube

At the midpoint in the assemblage of the *wrapped cube* (Fig. 2), with one cube formed, the remainder is easily plied into a conjoined twin cube. The assembler can, then, dissemble either of the cubes and wrap the free dangling pieces around the still assembled cube (somewhat like a hull fits around a nut) and complete the rhombic dodecahedron—the one with unconnected butt joints running across the short diagonals of the rhombic faces. (This vividly suggests how the rhombic dodecahedron belongs to the cubic system of the crystallographer.)

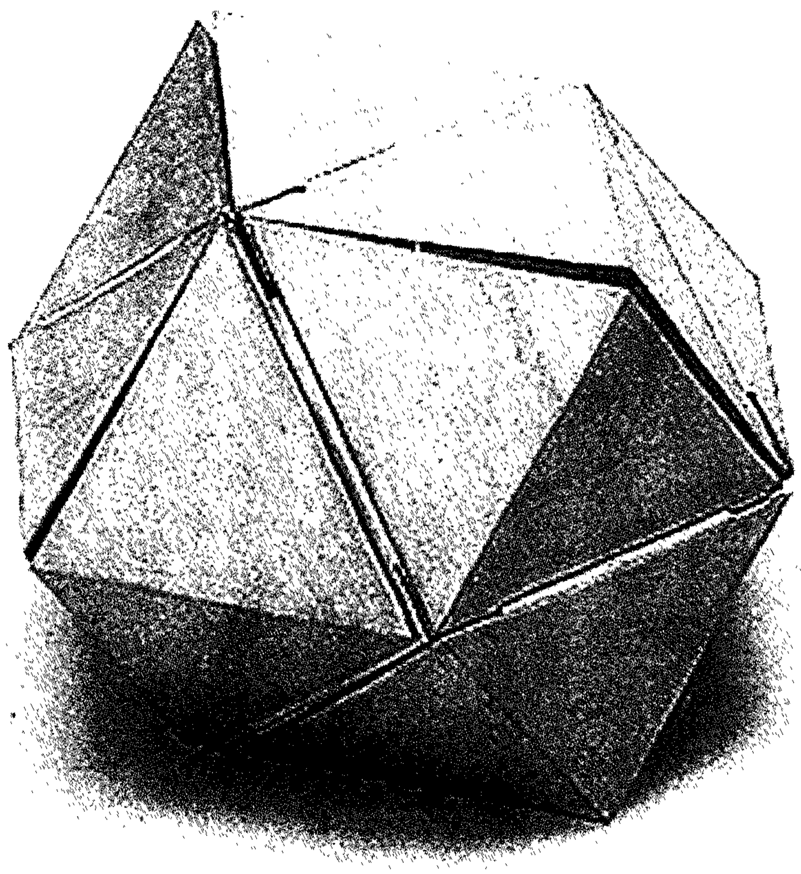

Figure 2

3. The radiant pentahedra

Before attempting the more difficult feat that is met in the assemblage of the *radiant pentahedra* (Fig. 3), it does well to return the hinged configuration to the planar layout. By pivoting two elemental units on the hinge that pairs them into a square-based, split pyramid, a rhombic-based, split pyramid is formed; and as one after another of these are in the like manner paired, the whole configuration automatically begins to pull together (the apices of all 12 pyramids meeting at the body's center) into the alternatively assembled rhombic dodecahedron, which displays hinged joints running across the long diagonals of the rhombic faces. It is seen in the planar layout that four of the pyramids are connected at their basal edges to three other pyramids, two are connected to two, and six are connected to one. The six singly connected, square-based pyramids, being re-paired into rhombic-based pyramids, will flip-flop into alternate positions, as this version of the assembled dodecahedron is taking shape; and an incorrect positioning of any one of them will throw the assemblage out of kilter,

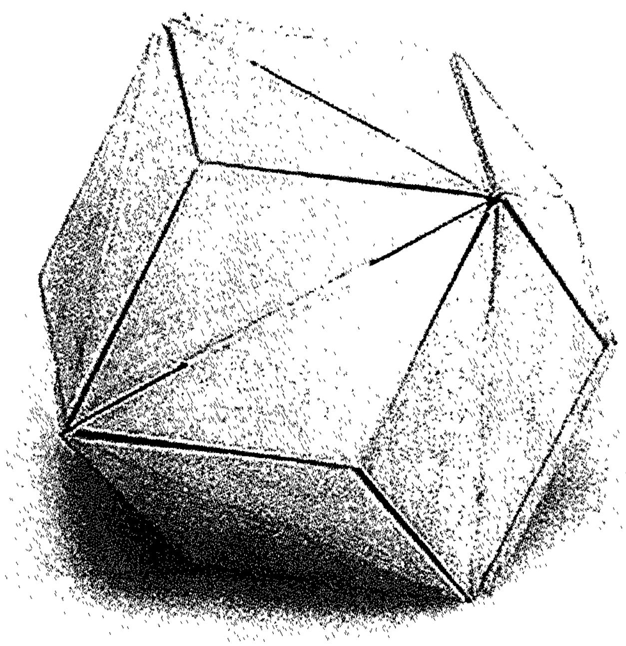

Figure 3

THE GOLDEN SECTION AS AN OPTIMAL SOLUTION

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- Organiser of a forthcoming ISIS-related seminar. See: http://www.matomium.org.

Abstract: *The golden number f arises when a line segment of length x (>1) is divided into pieces of lengths 1 and x-1, such that x = 1/(x-1), of which f = 1,6180... is the positive solution. The golden rectangle of width 1 and length 1.618... is said to be the most elegant, but as G. Markowsky pointed out, the statistical proofs for this popular assertion are not reliable. Nevertheless, the present paper will try to provide an answer to the question that the golden section corresponds to an optimal solution. It could reboot the mathematical career of the golden section.*

1. THE GOLDEN NUMBER

The golden number ϕ (or golden section, mean, or ratio or divine proportion etc.) arises when a line segment of length x (>1) is divided into two pieces of lengths 1 and x-1. This division must be such that the whole length stands to 1, as 1 is to the remaining piece, x-1. Thus, the ratio x/1 must equal the ratio 1/(x-1). This produces the equation $x^2-x-1 = 0$, of which $(1+\sqrt{5})/2 = 1,6180... = \phi$ is the positive solution; note that $\phi - 1 = \phi^{-}$

[1] = 0.618... More generally, the positive roots of x^2-nx-1 = 0 yield the family of *"metallic means"*, for n = 1, 2, ... For n = 2, it is the silver mean $\sigma_{Ag} = 1+\sqrt{2}$, etc.

The properties of these numbers have been described extensively (for a comprehensive survey, see *De Spinadel*). Of course, the diagonal of a rectangle with length 2 and width 1 corresponds to the "irrational part" $\sqrt{5}/2$ of the golden section, but this is so obvious that it is hardly a 'mathematical application'. Still, the Fibonacci series 1, 1, 2=1+1, 3=2+1, 5=3+2, 8=5+3, 13=8+5, ... is linked to ϕ, since the quotient series, formed by of each term divided by the previous one, tends to it. The numbers 2, 3, 5, 8, 13, etc. yield many beautiful applications, when mixed with a portion of fantasy. Furthermore, the geometric properties of the equilateral triangle with a top *"golden"* angle of 36° yield another large category of interpretations. Since a pentagon has related angles, there are various often-surprising forms. Together with P. Labarque, the author recently introduced "golden grey", containing 61.8% pure light (the remaining being "black"). It lead to one of the very few devices to determine the golden number experimentally.

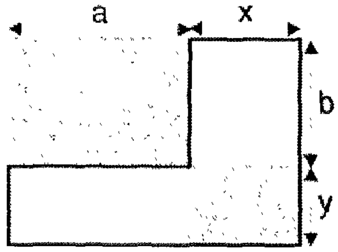
2. THE MOST ELEGANT (GOLDEN) RECTANGLE

To some authors, an explanation for the exceptionality of the metallic means is found in their representations as continued fractions, while some point to its emergence in proportions in the human body. Yet, these justifications do not use the approved mathematical language, to state it gently. Others turn to the golden rectangle of width 1 and length 1,618... It can be subdivided in smaller and smaller squares, and a spiral neatly fits into it. However, this is not so exceptional as often presented in introductory textbooks. Another justification of the importance of the golden rectangle would be its use in various designs. The Parthenon for instance would fit "exactly" in such a golden rectangle. Yet, a comparison of the various illustrations given in literature show this is a rather loose statement. Of course, painters and sculptors often subdivide their (rectangular) canvas or stone into proportions, and 3 – 5 or 5 - 8 arrangements are frequent. This provides a connection to the Fibonacci numbers. Yet, 2 – 4 – 8 schemes are popular too, and even used by great masters such as Seurat, Monet or Cězanne. It would be hard to prove that the Fibonacci subdivisions lead to the "most" beautiful artworks.

The very few valid statistical experiments about the golden rectangle do not seem to confirm a preference for the golden rectangle. As G. Markowsky pointed out, it is not indisputable that one can pick out the "most elegant" (golden) rectangle from an arbitrary list. In spite of these vigorous considerations in disfavour of the golden section, many mathematicians continue to spread the myth, while others encounter it. Mathematicians may be somewhat biased about the subject. A moderate conclusion comes from a psychologist, Christopher Green. *"Although many researchers have concluded that the effect is illusory"*, says Ch. D. Green, *"more carefully conducted studies have fairly consistently shown, that there is, in fact, a set of phenomena that require explanation, though no one has yet produced an explanation, both adequate and plausible, that has been able to stand the test of time."*

3. OPTIMAL SOLUTIONS

Here is an attempt to describe that (in)famous golden rectangle. It uses the common mathematical vocabulary, starting with a function to describe a geometric construction. Then, its derivative is set equal to zero, in order to find exceptional solutions. It appears that they are the golden number, its inverses or opposites. For a rectangle of arbitrary length a and arbitrary width b, we increase a by x and b by y (see fig. 1). This creates two rectangles on the main diagonal (in grey), and two rectangles on the border (in white). The extended area, made up by the rectangles on the border and the added grey rectangle, is $x \times b + y \times a + x \times y$ (surrounded be a tick black line). We compare this area $a \times b + x \times y$ on the main diagonal (shaded).



[Fig. 1: extending a rectangle.]

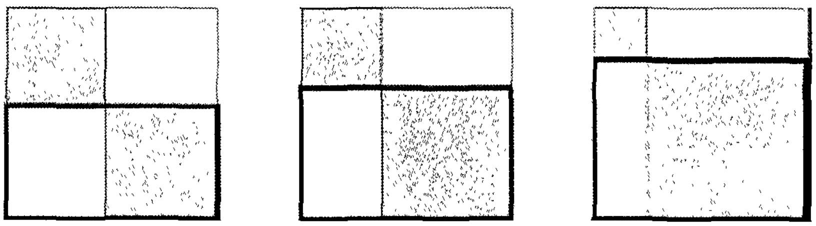
It produces a function $f(x, y) = \dfrac{x.b + y.a + a.b}{a.b + x.y}$. Its extreme value is computed through

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \end{cases}, \text{ or } \quad \begin{cases} y^2 - by - b^2 = 0 \\ x^2 - ax - a^2 = 0 \end{cases}.$$

The solution yields the possibilities: $x = a\dfrac{1 \pm \sqrt{5}}{2}$ and $y = b\dfrac{1 \pm \sqrt{5}}{2}$, of which only the positive signs yield positive lengths x and y (an interpretation for the negative values could be given though).

Now $\left(a\dfrac{1+\sqrt{5}}{2}, b\dfrac{1+\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2} \right)$ corresponds to a saddle point, while its z-value is the golden number, independently of the value of a and b. This could have been expected, as an initial division by a and by b would not have influenced the result.

The fact that we get a saddle point is remarkable enough to point out the particularity of the golden section. Since it does not correspond to a maximum, another approach may be of interest. We can simply start with a square (a=b=1). The rectangle on either sides should not be too long nor too high; thus, for reason of symmetry and because of the previous experience, we could propose to take x=y. The extended area is compared to the area of the squares on the diagonal (in grey), and this is given by a function $g(x)=\frac{2x+x^2}{1+x^2}$. Its derivative is $-2\frac{x^2-x-1}{(1+x^2)^2}$, annihilating for x = 1.618... and this corresponds to a maximum. Thus, if we want to extend a square by another square to form a larger square, the added area is maximal, with respect to area of the two squares, if the golden section is used (see fig. 2).



[Fig. 2: the rectangle in a thick line is too thin on the left and to thick on the right. It is a golden rectangle in the middle.]

The silver section can be obtained through such an optimisation procedure. If the given rectangle is a unit square, the area cut off at one side is now compared to the total area of the two squares on the diagonal. The silver section is the positive solution of s'(x)=0, where $s(x)=\frac{x+x^2}{1+x^2}$.The expressions for the s(x), g(x) and f(x,y) functions were inspired by completely different mathematical problems (see *Huylebrouck 2001*). Yet, the given formulation seems to correspond to what was probably intended by the expression of "optimal solution", as given by so many authors, and refuted by others. It seems but a rephrasing exercise, using mathematical vocabulary, but it could help to reboot the mathematical career of the golden section.

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