

ITERATION INTO THE COMPLEX PLANE: SYMMETRIES AND TRANSFORMATIONS

JAVIER BARRALLO & SANTIAGO SÁNCHEZ

Name: Barrallo J. Computer Science PhD (1992), (b. Bilbao, Spain, 1964)

Address: E.T.S. Arquitectura. Plaza Onati, 2. 20018 San Sebastian. Spain. *E-mail.* mapbacaj@sa.ehu.es

Fields of interest: Dynamical Systems, Mathematical Education, Structural Analysis of Heritage Buildings.

Awards: Hobby Press Award, 1992.

Publications and/or Exhibitions: Fractal Geometry (book – 1992), The Frontier between Art & Science (Exhibition 1997 - 2001), Mathematics & Design Conference (1998), ISAMA Conference (1999), The Basque Country Gothic Churches (2000)

Abstract: *Three techniques to generate more intricate fractal symmetry patterns are described in this paper. The first one is based on the development of new fractal formulae, the second is based on mappings of classic formulae according to the 17 crystallographic groups and the third one is the use of kaleidoscopes and tessellations.*

1. INTRODUCTION

Sets obtained under iteration into the complex plane usually show simple symmetric patterns. These sets often have fractal structure, especially when using quadratic functions.

The basic symmetries shown by some elementary functions like the Mandelbrot, Julia, Koch or Sierpinski sets can be modified in order to show more intricate symmetries and visual effects. We will discuss three different techniques:

- a) The creation of new families of formulae that exhibit more sophisticated symmetries. For example a quadratic equation with simple X or Y-axis symmetry can be transformed into a new one showing more symmetric properties.

- b) The use of plane mappings that copy a certain region of the complex plane into others, according to the scheme of any of the 17 crystallographic groups of symmetry.
- c) The design of transformations that produce kaleidoscopes, self-similarity patterns, Truchet tessellations, L-Systems and Penrose and Escher like tiling.

2. FORMULA CREATION

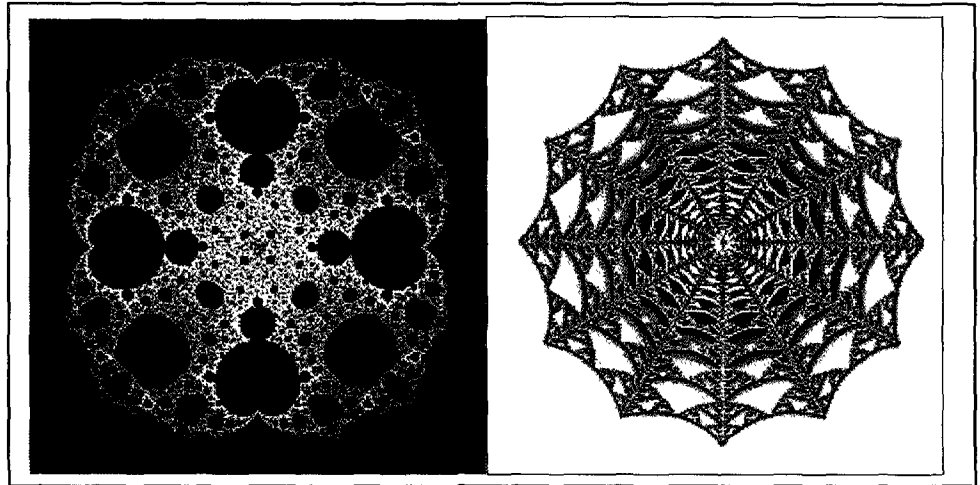
Some formulae can be altered to manifest symmetries that usually are not present on them. For example the Mandelbrot and the Sierpinski sets can be transformed as in the following example to present a new and more complex appearance.

FOUR MANDELBROTS

```
init:
  z = 1 + 0i
  c = pixel
loop:
  z = c * (z^2 + 1 / z^2)
bailout:
  |z| < 1000
```

SIERPINSKI NET

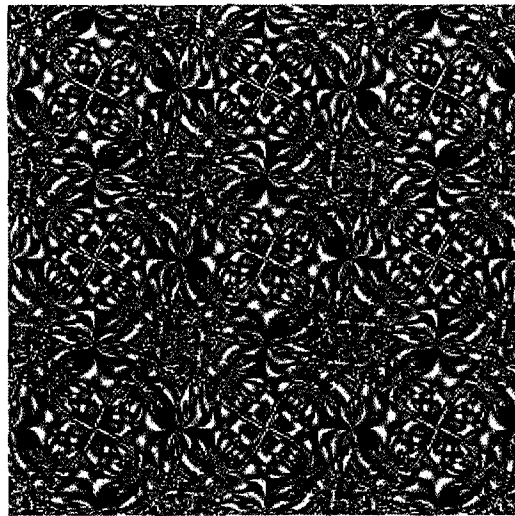
```
init:
  z = 0.2 * abs (pixel^3)
loop:
  IF (imag(z) > 0.5)
    z = 2 * real(z) + (2 * imag(z) - 1) * I
  ELSEIF (real(z) > 0.5)
    z = 2 * real(z) - 1 + 2 * imag(z) * I
  ELSE
    z = 2 * real(z) + 2 * imag(z) * I
  ENDIF
Bailout:
  |z| <= 100
```



3. PLANE MAPPINGS

Any region of the plane can be mapped into another regions with the same or different shape producing symmetric patterns. The 17 crystallographic groups of symmetry can be used as a basic catalogue to generate these transformations.

The structures obtained iterating the complex variable values into a dynamical system are mapped all along the plane creating an amazing family of graphics with several applications to Art and Design.



4. TRANSFORMATIONS

After a formula is introduced into a dynamical system, iterated enough times and mapped along the complex plane, the resulting values can be modified by external transformations, not related with the formulae.

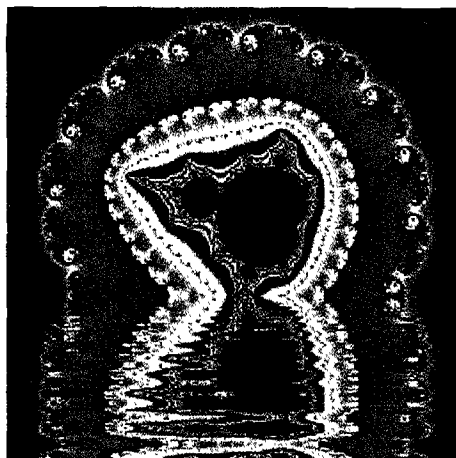
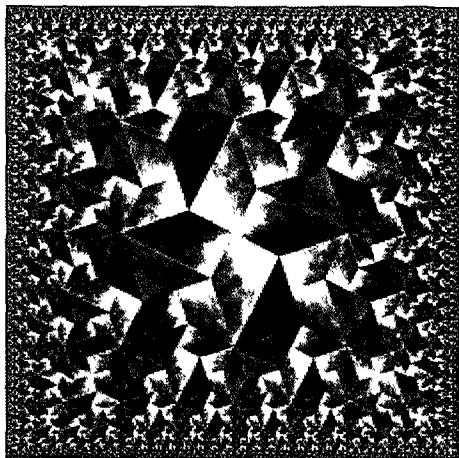
These transformations can be designed to produce kaleidoscopes, self-similar patterns, L-Systems, Truchet tessellations and Penrose and Escher like tiling.

Other transformations can be designed to create symmetry effects. For example the following code recreates the reflection of an object over the water surface.

LAKE TRANSFORMATION

```
y = imag(pixel)
if (y < imag(screen_centre))
  dy = imag(screen_centre) - y
  y = 2 * imag(screen_centre) - y
  y = y + alpha * dy * sin(beta * dy^0.2)
  pixel = real(pixel) + y I
```

All these techniques will be widely shown with several new images that have been specially created for the ISIS conference.



All images created by Javier Barrallo 2001

SNAKE PATTERNS IN EURASIA/JAPAN AND THEIR IMPLICATIONS

SZANISZLÓ BÉRCZI, OSAMU SANO and RYUJI TAKAKI

Name of coauthor: Bérczi S. Geologist (b. Szeged, Hungary, 1950)

Address: Eötvös University, Dept. G. Technology, Cosmic Materials Space Research Group,
H-1117, Budapest, Pázmány Péter sétány 1/a, Hungary, *E-mail:* berczisani@ludens.elte.hu

Fields of interest: Geometry, ethnomathematics, ornamental arts, planetary science, materials science.

Publications and/or Exhibitions:

Bérczi Sz. (1986): Escherian and Non-Escherian Developments of New Frieze Types in Hanti and Old Hungarian Communal Art. in: *M. C. Escher: Art and Science* (szerk. H. S. M. Coxeter et al.) 349-358. old. North-Holland, Amsterdam

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Name of coauthor: Osamu Sano, Physicist, (b. Ibaraki, Japan, 1949).

Address: Department of Applied Physics, Tokyo University of Agriculture and Technology,
Fuchu, Tokyo 183-0054, Japan. *E-mail:* sano@cc.tuat.ac.jp.

Fields of interest: Fluid Mechanics, Pattern Formation in Granular Material, Science on Forms

Awards: none

Publications and/or Exhibitions:

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Name of coauthor: Ryuji Takaki, Physicist, (b. Hiroshima, Japan, 1940).

Address: Graduate School of Engineering, Tokyo University of Agriculture and Technology,
Koganei, Tokyo 184-8588, Japan. *E-mail:* takaki@cc.tuat.ac.jp.

Fields of interest: Physics, Science of Forms, Physics education, (Analyses of symbolic communication).

Awards: Award for Distinguished Service of The Society for Science on Form, Japan, 1996.

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1. INTRODUCTION

People in both western and eastern worlds have been conscious about the snake (or the serpent) from the ancient ages, and also created an imaginary animal "dragon". These animals, however, seem to be considered as symbols with different implications in both worlds. For example, in the West the dragon has been an evil existence, while in East it has been a god of thunder or tempest and helped emperors to have majesty. On the other hand, there are common features of snakes on both worlds. The snake is often combined with waves or rivers owing to geometrical simulation.

The major factors of difference would be those of climates and the ways to build houses or to get foods. It is an interesting problem how these differences in cultures has produced different ideas about snake and dragon. This paper is a trial to study this kind of problems through a cooperation of scientists from both worlds. It is expected to trigger a closer cooperation in future.

2. SNAKE PATTERNS IN EURASIA AND IMPLICATIONS

Two main sources for snakes in the Western Eurasian arts are: mythology and the natural phenomena. Old arts expressed the understanding of the world in mythic compositions. Natural phenomena were related to human stories according their real and symbolic correspondences. In folk tales for example in symbolic zodiacal meaning snakes appeared as like as in the whirling sinuous-waving in the water.

Their sacral mythic appearance is in the Bible, where after Creation God places the first couple of Adam and Eve in the Paradise, the Garden of Eden. The devil - in the form of a snake - turns away the attention of the ancient couple from the right way to sin. In Christianity the snakes represented the world beneath us, and were the representatives of the bad forces of the darkness and death.

In Greek mythology the god of medicine is Asklepios, a God-pair of Apollon. The snake rolling on a rod was the symbolic attribute of Asklepios. His genera and medical traditions goes back to the island of Kos. Recently the snake around a calyx as symbol of healing on the portals of pharmacies is the heritage of Asklepios.

In the Eurasian steppe and in Eastern Eurasia the snakes are auxiliary beings and friendly spirits. Hungarian folk art preserved many old traditions from this art: pillow-ornament from Kalotaszeg, called "snaked" pattern, two small s-shaped mirror-symmetric snakes

like as in life-tree scene. On the Mishka-kancsó bottles of vine we also find snakes (with m1 symmetry).

Sinusoid surface of weave motion gave the feeling of the serpent motion deep in the water for sailors. Also the motion of whirls in streams makes people remember the "serpentine" animal motions. We can find them in rich variants in the Viking and Celtic art (with 12, 1g and mg frieze symmetries).

Planet-motions along the zodiacal belt of constellations (path of the Sun) also exhibit serpent-like motions. Such Calendars were sculptured on church gates in the 9. - 13. centuries A.D. There the serpents represented the celestial pathway of cycloid planet motions. On the zodiacal pathway serpent's knots were the animations of months, and the important celestial event points of eclipses along this line, at two draconic points, the "serpents had eaten" the Sun. (Aleppo, Lund).

3. SNAKE PATTERNS IN JAPAN AND IMPLICATIONS

In ancient times snakes seem to be worshiped as beings possessing an eternal life, because of their ability to shed an old skin and grow a new one, which was regarded as a process of death and rebirth. In addition, the spiral shape of a snake or a vine or swirl of fluid have an implication of endless motion once it started outward or inward, probably led to a concept of eternity. The earthenware from middle JOMON period (ca. 2000 BC) has top grips or sides which are ornamented with snakes. Clay figures of the same period, in which snakes are wound on her heads, are also found.

Snakes were regarded as gods of mountain. Sometimes the object of worship enshrined in a Shinto shrine is the snake-shaped mountain, and was believed to appear in the form of a snake. That explains why blue snakes living in shrines are protected as a messenger of gods. Because they appear in damp areas and were believed to control the water, which guarantee the good harvest, they were worshiped by farmers as gods of water as well as gods of agriculture. Moreover they were regarded as guardians of houses because they eat rats and other vermin. After the introduction of Chinese calendar in 6th century, snakes came into daily life as sixth animal of the twelve horary signs. People counted the days and years by using the name of these animals. This tradition is still alive today; this year (2001) is the year of snake.

On the other hand snakes were considered to be regeneration of dead persons, which originate in the idea of never-ending cycle of reincarnation in Buddhism. They were believed to reflect spirits of a dead person, and were objects of fear and adoration. There are many folk tales in which a snake, that had an appearance of a man, made a woman pregnant and gave birth to a new family. Sometimes a revengeful ghost was believed to appear as a shape of snake.

There are variety of symbols called KAMON (family crests, or coats of arms), which characterizes family line or history. One class of Kamon is called "TOMOE" (a kind of a

protector in archery). The shape, which resembles a comma, developed into a spiral, or a coil of snake, or a swirl. Basic shape of TOMOE has bigger round head and curved tail. This figure resembles a comma-shaped bead or an embryo, which shows a yearn for the prosperity of descendant. There are varieties on the number and size of the head as well as the thickness of the tail, with clockwise or anti-clockwise sense of rotation (reminding a vortex which is a charm against fire). In feudal period, social classes were fixed. But as far as the KAMON was concerned, no severe restriction seemed to be imposed, which gave good chance for creating a new design keeping basic part in common. These contradicting requirements naturally result in limited number of patterns. In this respect KAMON is not the same as a trademark nor a brand, the latter being simply a design. Today the significance of family, and hence the meaning of "KAMON", seems to disappear together with the old system. These beautiful and unique symbols, however, will probably be succeeded from parents to children as one of the identity of a new family, just as the family name does.

4. CHINESE CHARACTERS FOR SNAKE / DRAGON

There are several Chinese characters for snake, or more generally creeping animals. They were established about 3000 years ago by simplifying pictographs of these animals. Their meanings later changed and were confirmed by combination with other elements. In fact, the character for "snake" is a combination of "worm" and "snake", the latter being used for another purpose.

The character for dragon originates in a snake with a crown (king snake). The lower part of the character is similar to another element with meaning "extend" or "thunder light". Therefore, the character for dragon is considered to have a relation to the thunder and the storm. In eastern Asia storms bring water from the ocean and contribute much to agriculture. Therefore, the dragon has been accompanied with the ideas of both "strength" and "benefit". Hence, the emperors in Chinese dynasties used a design of dragon as their symbol.

After all, this paper has elucidated that the snake patterns have variety of implications, which are partly common and partly different according to respective cultures. However, this kind of research will not be complete until information from other important regions is included, such as India and middle-eastern countries.

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