

SYMMETRY IN MUSIC: A HISTORICAL PERSPECTIVE

Dalia Cohen⁽¹⁾ and Judith Cohen⁽²⁾

Address: ⁽¹⁾ The Hebrew University of Jerusalem.
⁽²⁾ Tel-Aviv University.

INTRODUCTION

Compared to other arts, symmetry phenomena in music are not immediately perceptible. They are mainly human-made, and reflect the artistic ideal in a more hidden and complex way than those in other arts. Whereas in the other arts symmetry may be part of the ready-made raw material detected in nature (e.g., the human body, colors and shapes of many objects, etc.), musical raw materials (e.g., scales and rhythmic patterns) are created by human beings. They therefore require learning – also in respect to symmetry – and differ from culture to culture.

We believe that the very existence of symmetry or asymmetry, whether conscious or unconscious, and the specific ways of their realization, together form one of the most important characteristics of style in its various manifestations – of a culture, a period, a composer or an individual composition.

Verbal references to symmetry in music appeared relatively late. Admittedly, already in classical Greece the general concept of symmetry, as embodied in the dualism of Apollo versus Dionysus, was also reflected in the classification of musical instruments and scales; and, likewise, in ancient China discussions of music merged with the broad dualism of *yin* and *yang* (Danielou 1943). But these are examples of very broad concepts and philosophies.

While in the realm of visual arts symmetry figured as a prominent aesthetic ideal of works of art in the ancient world (Weyl 1952), and was clearly expressed in theoretical writings and discussions of various periods, we do not know of any parallel written reference to symmetry in music. This is especially intriguing in the Renaissance, when despite music being undoubtedly guided by the notion of overall symmetry (explicit or implicit), this principle was not mentioned at all in theoretical musical writings.¹ Interestingly, in the Baroque era, too, where many manifestations of musical symmetry (according to our definition of the concept) may be detected, they were not accompanied by theoretical statements.²

In music, symmetry may be manifested not only in the manner in which the various parameters are distributed on the time axis, but also in their distribution on the pitch axis as well as in groups of elements serving as raw material. However, until recently symmetry in music was discussed mainly in regard to the time axis. Curt Sachs, for instance, while drawing broad comparisons between symmetrical phenomena of various arts and of different periods, related only to this tacit definition of symmetry (Sachs 1946).³

Symmetry on the time axis may be subdivided into three kinds of manifestations:

- (1) "Mirror" on various levels of the time axis (e.g., retrograde motion, also termed "cancrizans" or "crab"; "chiasmus", etc.). This kind of "mirror" may be realized by pitches, tonal centers, formal structures, textures, etc. On a more abstract level, the structure $a-b-c-...-b-a$ is also a mirror on the time axis, although not on the immediate level, since the repetitions of a and b maintain the same order of events within units.
- (2) One repetition ($a-a$) or division into two similar parts ("period", and in an extreme case a continuous division into two, according to the formula 2^n ; see Figure 1).
- (3) Various kinds of division of the whole, made in reference to a central point on the time axis (e.g., the "golden section", or other divisions in accordance with various mathematical formulae).

To these symmetries a fourth, accepted one should be added:

- (4) "Mirror" with regard to the pitch axis ("Mirror", "Inversion").

One of the detailed theoretical discussions that entails the concept of symmetry on the pitch axis (i.e., the vertical axis) is that of J. P. Rameau (1722), who presumed the

existence of undertones as a mirror of overtones in order to explain the minor-major dualism.⁴ In addition, he defined the symmetry existing between the dominant and the subdominant harmonic degrees, that are placed a fifth above and below the tonic accordingly.

As far as we know, music theorists explicitly referred to musical symmetry as late as in the eighteenth century (e.g., Mattheson 1739, Riepel 1752, Chastellux 1765, Daube 1773, La Cépède 1785, Koch 1787, and others; also summarized in Ratner 1980). However, they considered solely symmetry that results from a twofold division of phrases (i.e., No. 2 in the above list of symmetry manifestations).

In the last decades, the concept of musical symmetry has been expanded, under the influence of three main factors:

- (1) The perception of symmetry as a comprehensive phenomenon, manifested both in nature and in human activities;⁵
- (2) The new research directions, that regard musical activity as cognitive activity;
- (3) The increasing awareness of twentieth-century music (which rejected the learned schemata on which tonal music was based) of symmetrical operations functioning as basic procedures in the formation of a musical work (e.g., Oppenheim 1989).

In our research we refer to the relatively new definition of symmetry, which is common to various domains of human culture and of natural phenomena: "Symmetry [is] no more than statements as to the operations that have no effect upon the systems that we consider" (Wilkinson 1989). The adoption of this definition to the realm of music may, of course, be made only on the basis of a primary discussion of the significance of the various musical systems. In this way we hope to shed additional light on phenomena that have already been accepted as symmetrical, and also to expose latent symmetries and other similar phenomena. With the help of the new definition we shall try to examine symmetry in various styles as reflecting the change in the aesthetic ideal of different historical periods.

I BACKGROUND CONSIDERATIONS

A. Symmetry Phenomena in Music

The main concepts by which we describe symmetry in music – schema (learned and natural), transformation, and categories of operations – are frequently used in the analysis of specific musical compositions and in general analytical researches pertaining to musical theoretical principles. As yet, however, there is still no consensus as to their application, and they may often be encountered in a different terminology. In the following, we shall summarize their characteristics where relevant to our discussion (for more details see Cohen—Dubnov 1997, and Cohen 1996).

A1. Schemata

The concept of schema represents an organizational principle that may be realized in many ways, and may be seen as derived from the operation of “fusion” (see below). The schemata create links between the events and may appear on various hierarchic levels. They contribute to the creation of a system of expectations and to intelligibility, and they make complex organization possible. They originate consciously or unconsciously in our mind, and all our aural impressions are perceived through comparisons with them. Their special importance in music stems from the fact that in music all meanings are based on organizational rules rather than on semantics.

We may distinguish between learned and natural schemata. The learned schemata (intervals, scales, chords, meters, etc.), which are expressed mainly in theory, are expressed in exact magnitudes of specific parameters, and differ from culture to culture.⁶ In contrast, what we call “natural schemata” are not culture-dependent, and are not characterized by precise quantification. They are known to us from outside music, too, and they appear on various levels of abstraction (e.g., ranges of occurrences in various parameters, with reference to the normative range; curves of changes in time; types of operations; degree of definability; etc.). They contribute to basic and natural sensations of excitement.⁷

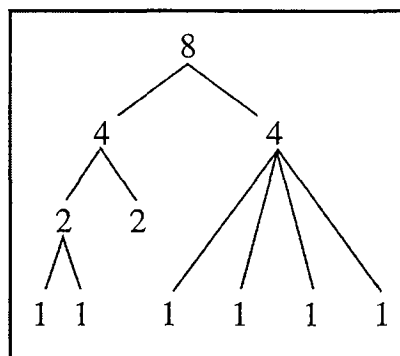


Figure 1: The Symmetrical 2^n Schema One Possibility

Let us specify two natural schemata which enable us to predict the continuation of the musical course, and whose presence or absence in different periods and cultures is very significant: (1) The 2^n formula, i.e., a continuous duple division of musical phrases, or a continuous multiplication $(1 + 1 + 2 + 4 + 8 + \dots)$.⁸ As we shall see later, this schema may be seen as one manifestation of the operation of segregation/fusion. (2) A convex curve (in all parameters: low-high-low; slow-fast-slow; soft-loud-soft, etc.). For example, most popular tunes of the world that do not aim to arouse excitement adhere to the convex curve (Nettl 1964; Huron 1996).

These two schemata function as important characteristics of style. Another natural schema which contributes to stylistic characterization is the principle of concurrence and non-concurrence between events of different parameters⁹: a full concurrence, for instance, would mean a convex curve in all the parameters simultaneously, while non-concurrence means a convex curve in one parameter and a concave curve in the other. The principle of concurrence/ non-concurrence may also apply to the relation between various natural schemata, between various learned schemata, and between both learned and natural ones. Non-concurrence may contribute to complexity and to uncertainty as to the limits of the various units and their characterization, and therefore of a sense of tension, too. This schema can be seen as one manifestation of the degree of definability.

A2. Symmetry as a Natural Schema

In this study we discuss symmetry (or a-symmetry) phenomena in their broad meaning, as manifested in three interrelated domains:

- (1) Learned schemata of human-made raw materials, in the cognitive stage, as opposed to the raw materials in the physical-acoustic stage;
- (2) Forms that may be regarded as modes of organization of musical units in terms of similarity and difference, such as *a-a* or *a-b-a*, the units being defined by learned and/or natural schemata;
- (3) Compositional rules (explicit or implicit, conscious or unconscious ones) as manifested in categories of operations, that are applied to learned or natural schemata or to various events on the immediate level.

From our point of view, the principles of organizing the variables on the basis of similarity and difference (Tversky 1977) may be considered natural schemata of high abstraction, as are the categories of operations, since they represent natural procedures of our cognitive activity. As we shall see later, formal organization is one of the manifestations of one of the operations (segregation and fusion).

Operations are, of course, the main characteristic of transformation.

A3. Transformation in Music

Transformations preserve the two essential conditions of any kind of organization: similarity (= repetition) and difference (resulting from a well-defined operation). They have frequently been discussed in musical literature.¹⁰ In order to determine their musical significance, we have examined their manifestations, while taking into account their variables. These are:

- (1) The kind of operation in which symmetry is manifested;
- (2) The parameters and the components derived from them, that are subjected to operations (e.g., interval, scale, and chord, derived from the parameter of pitch; rhythm, meter, and tempo, derived from the parameter of duration; and loudness and timbre);
- (3) The level of musical organization (a transformation of an immediate event or of a schema on a deeper level, such as a chord, a scale, or even a harmonic phrase);
- (4) The degree of change (large or small);
- (5) The contribution to clear/unclear directionality or to the sense of relaxation/tension.

A4. Categories of Operations

The operations may be grouped into five categories, that may appear in various units and dimensions, and whose principles are those of cognitive operations (Appel 1992). They may therefore be considered natural schemata that make complex and intelligible organization possible. The categories may appear in various units and dimensions.

These are the categories of all the operations: (1) contrast; (2) shift; (3) augmentation and diminution; (4) fusion and segregation; (5) equivalence. In part, these operations are analogous to mathematical ones (Weyl 1952; Leyton 1992), and may be applied to other subjects, including arts in general and especially visual arts (Avital 1996, 1998).

(1) Contrast may be manifested in many parameters and in three main kinds of operation:

- a. A simple contrast with regard to two related points on one scale with different parameters (e.g., high/low; loud/soft, fast/slow); or with regard to two directions on one scale (up/down); or in regard to the location on the time axis: *a-b-c-d-c-b-a* (for details see the discussion of form). This contrast is analogous with the mathematical operation called “reflection.”
- b. A compound complementary contrast that agrees with a few conditions not specified here (e.g., harmony/melody; a chord/a melodic second; harmony/polyphony);
- c. A binary contrast between two options only (major/minor scale; stressed/unstressed beat, minor/major second; double/triple meter).

(2) A shift within a cyclic system, such as the three inversions of the triad chords; modal scales (in the diatonic system); melodic sequence (in the diatonic system); transposition (in the chromatic system); rhythms pertaining to the metric cycle (such as the various poetic meters). This operation is analogous with “translation” or “reflection”.

(3) Augmentation and diminution, such as durational multiplication/division; addition/subtraction (including condensation/reduction of the melodic line); or expansion/contraction of a musical period in Classical style).

(4) Fusion and segregation (or grouping/splitting, which is a basic operation for any kind of organization, e.g., grouping into schemata; grouping into “variations” – as opposed to “family resemblance”), and reduction. From this operation we obtain the 2ⁿ schema and forms of repetition.

(5) Equivalence, which is one of the conditions for any living language (Powers 1976), e.g., equivalence between categories of harmonic degrees; means of emphasizing musical events; variations in Western music; and different realizations of the same melody type, mainly in non-Western music.

Let us remember that an operation is considered a symmetrical activity as long as its result does not overstep the limit of the original system, and that this system (which we consider a schema) may be of various hierarchic levels.

One of the most significant systems in Western tonal music is the diatonic system (as represented by the white keys of the piano), consisting of seven notes chosen from twelve notes. An operation may result in staying within the limits of the system, or in overstepping to the larger system of twelve, as in the dodecaphonic system. For instance, the operation of “shift” or “mirror” within the parameter of pitch may be (a) “diatonic” (i.e., not exact, but remaining within the 7-note schema, that is selected from 12 notes), or (b) “real” (i.e., exact, while the intervals are chosen from the 12-note system). For example, the real mirror of an ascending major chord is a descending minor chord, and that of a major tetrachord is a Phrygian tetrachord.

Tonal music usually has diatonic operations. Bach’s music (the duet BWV 803), however, is unusual in that we also find real operations that lead to a sense of bitonality (aside from the real answers to the subjects of fugues under certain conditions). The “real” operation indicates abandoning the 7-note system. The larger the system is – according to information theory – the less severe the restrictions are, and the less “directional” or more unexpected the composition is.

B. Interference of an Extra-Musical Factor: The Aesthetic Ideal

In different cultures and epochs we may find different manifestations of symmetry that contribute to style according to the aesthetic ideal. One may argue that the choice of the kind of symmetry – whether made consciously or unconsciously – reflects that ideal. It should be noted, that the two extreme situations – total symmetry (e.g., Pierre Boulez’s *Structures I*, 1952) and a complete lack of symmetry – are meaningless. In reality we find intermediate stages reflecting the ideal.

The following are some of the variables that characterize the aesthetic ideal and are relevant to symmetry: overall structure as opposed to concentration on the momentary

event; clear expression as opposed to a blurred one; certainty as opposed to uncertainty, and, in general various kinds of complexity and directionality (Cohen 1994). The concept of directionality relates to the importance of the sequence of events on the time axis, each event evolving from its predecessor: when there is no hierarchy in the sequence – there is no directionality. Directionality may refer to various levels of musical organization, and it represents the “logical” progression of music, enabling us to predict the succession of musical events.¹¹ Two other important variables are the connection and non-connection to the extra-musical world; and a sense of calm or excitement that represents (according to Curt Sachs [1946]), an ideal of “ethos” and “pathos,” which alternate cyclically in the West.

Despite the risk of generalization, we can draw a clear line between the ideals of Western tonal music and those of non-Western music, as well as between different periods in music history of the West. In non-Western cultures, where music has changed less in the course of history than in the West, music tends to be an integral part of life. Music in non-Western cultures does not have a closed, independent existence with overall directionality and complexity and a clear beginning and ending; instead, there is greater emphasis on the momentary event.

How does symmetry contribute to the representation of the aesthetic ideal? We shall demonstrate here merely a few specific realizations of some of the categories of operations; namely, their contribution to clear/unclear directionality and to a sensation of calm/excitement. These operations may increase the degree of clarity or obscurity, calmness or excitement of the musical expression. For example, contrast may be obtained by means of a descent following a small ascent, in which case it causes balance, or by a sudden change between extremes of pitch, intensity, tempo, or density, in which case the result is tension. As another example, the operation of contrast can be realized by the difference between a concave and a convex curve. A convex curve (i.e., a descent following a melodic ascent) contributes to clear directionality, whereas the opposite type of symmetry – a concave curve (i.e., ascent following a melodic descent) – causes excitement and a lack of directionality. The difference between these two kinds of symmetry is that the ends of the concave curve extend to infinity, and therefore we have no certainty as to the continuation of the musical events, whereas the convex curve provides for maximum predictability.

Similarly, operations may lead to nonconcurrence of various sorts between parameters, units, events or schemata – thereby increasing complexity and uncertainty, i.e., reducing directionality. As a result, we find different kinds of symmetry in different styles. For

example, we may find convex curves in styles that strive toward clear directionality, and vice versa.

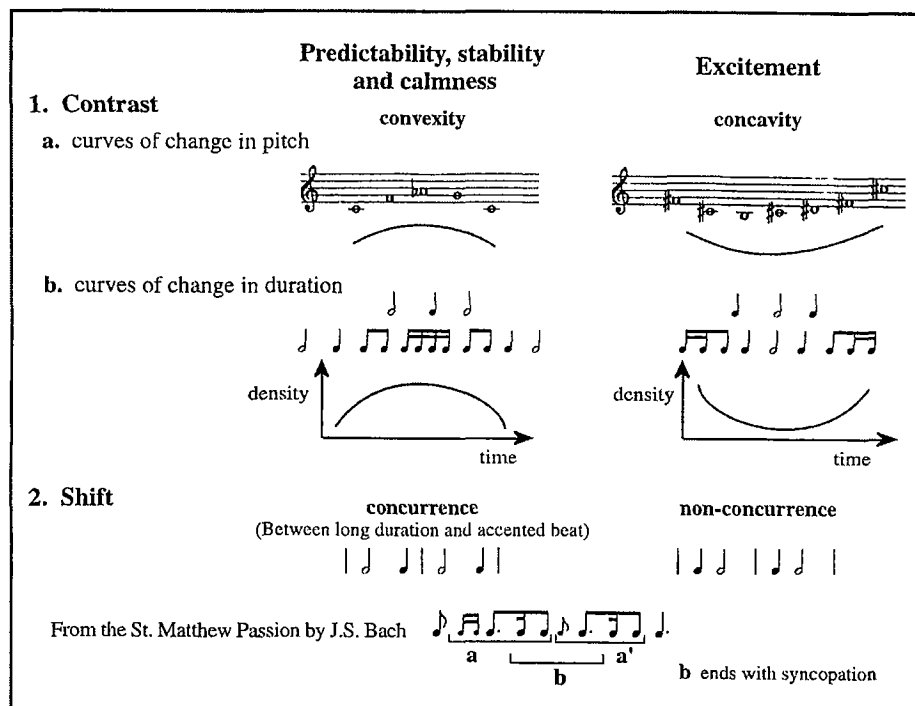


Figure 2: Symmetry causing both Calmness and Excitement: 1) Derived by the operation of contrast; 2) Derived by the operation of shift. The letter *a* at the bottom of the figure represents a characteristic rhythmic motive, repeated in variance (*a-a'*) at the opening of the aria "Erbarne dich" (mm 1-2); The letter *b* represents another characteristic motive, starting as upbeat to m. 3, that may be considered as a shift from *a*. Because of the *appoggiatura* on the beat, *b* ends with syncopation (i.e., nonconcurrency) and causes excitement, in contrast to *a*.

II KINDS OF SYMMETRY IN VARIOUS ERAS IN THE WEST

In our discussion of symmetry in various eras in Western music, we will consider the constraints of the stylistic ideal and the cognitive constraints that we assume intervene in the (unconscious) selection of specific types of symmetry.¹² The concept of an era is flexible, of course, since the chronological borders and styles of eras often overlap, and because each of them can be subdivided. Here, however, we present a bird's-eye view of the stylistic landscape in accordance with the standard division of music history. For the sake of comparison, we also take a look at non-Western musical cultures.

Our presentation follows the three domains of symmetry discussed above: the principles of (1) raw material; (2) compositional rules; (3) forms.

C. Symmetry in Musical Raw Material in the West since the Seventeenth Century

C1. Binary Contrasts

Our point of departure for comparing symmetry in different kinds of raw material is Western tonal music, in which the ideal calling for overall directionality and complexity reached its peak, and binary contrast assisted to achieve this ideal. Indeed, only in the raw material of Western tonal music, beginning in the seventeenth century, do we find binary contrast. Binary contrast is manifested in the “building blocks” in various parameters: beats (accented/unaccented; only one accented beat per cycle); meters (duple/triple); intervals of a second (major/minor); and scales (major/minor). Binarity, which means a minimum number ($=2$) of possibilities for the different building blocks in each parameter, makes it possible to get numerous complex forms of organization on high levels. Thus, for example, the paucity of scales (major-minor only) enables their multifarious realizations in different tonal centers; the system as a whole generates the schemata of the chords and of the laws of harmony that characterize Western music, and allows for an overall and complex directionality; and the limited choice of meters enables hypermetric organizations, etc.

Let us stress that the very possibility of obtaining multiple forms of organization with different styles accompanies the possibility of significant changes in the styles of different eras and composers; these changes are themselves an important element of the Western ideal.

Before the seventeenth century, there were more than two scales (modes) in Western music (with virtually no modulation); these modes were obtained through shift operations in the single diatonic scale system of seven notes (which itself constituted a specific hierarchical selection from the pool of twelve notes in an octave).

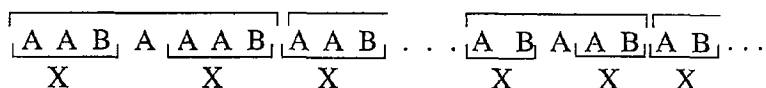
Until the seventeenth century meters, too, were not simple and did not conform to the principle of binarity: there were more than two kinds of meter and more than two kinds of beat, and in certain cases (in the rhythmic modes, for instance), meter and rhythm were not clearly differentiated. All these basic qualities are even more prominent in

non-Western music. We find here the opposite of what we see in Western tonal music: multiple relationships on the immediate level (e.g., complex metric patterns; more than two kinds of beats and more than two kinds of seconds; and numerous scales, which are derived from an abundance of scale systems). This multiplicity makes it possible to increase momentary complexity and prevents overall directionality with complexity, as well as marked changes in style.¹³ Therefore, the symmetry of the raw material, the operations, and the forms have less impact on the music of non-Western cultures than on Western music.

C2. Optimal Quantities and Minimal Asymmetry in the Scale System in the West

As stated above, the single Western system with seven named notes constitutes a specific hierarchical selection from the pool of twelve notes in the octave. This system is unique in many ways, and it allows for overarching organization with clear directionality and complexity (something that is found only in Western tonal music). In addition to being a single system,¹⁴ it also constitutes a kind of optimum regarding the number of different intervals. Certain non-Western cultures have, on the one hand, a not-well-defined multitude of intervals and scales (groups of seven), which are obtained from pools larger than twelve, and on the other hand, a small number of intervals or systems with fewer than seven notes – most often five notes (pentatonic systems), obtained from a pool of seven or fewer. In effect, in both of these extreme situations the number of intervals used for musical organization is quite limited; consequently one can hardly speak of a system of intervals. In accordance with the ideal, neither extreme allows for complex overall organization.

For our purposes, it should be stressed that the Western system, like biological systems, has minimal asymmetry, which is considered an essential condition for obtaining complex systems (Atlan 1981, 1987). Outside the West only a few systems maintain minimal asymmetry (Figure 3). Interestingly, it is found also in the rhythmic pattern that is fundamental to African polyrhythmics (Arom 1998).

General:**In scale systems:**

In the West (in units of half-tones): $\boxed{2, 2, 1} \ 2 \ \boxed{2, 2, 1} \ \boxed{2, 2, 1} \dots$

In Chinese pentatonics: $\boxed{2, 3} \ 2 \ \boxed{2, 3} \dots$

In Japanese pentatonics: $\boxed{4, 1} \ 2 \ \boxed{4, 1} \dots$

In ancient India (in *śruti* units): $\boxed{3, 4, 2} \ 4 \ \boxed{3, 4, 2} \dots$

In the *māqamāt* (rare examples)

Hejāz-qār: $\boxed{1, 3, 1} \ 2 \ \boxed{1, 3, 1} \dots$

Rast (in units of quarter tones): $\boxed{4, 3, 3} \ 4 \ \boxed{4, 3, 3} \dots$

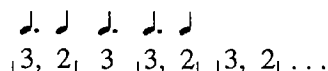
In the basic rhythmic paradigm for the African polyrhythm:

Figure 3: Minimal Asymmetry in Cyclical Systems in Various Musical Cultures

3.1 (General) - Two abstract examples of minimal deviation from the symmetrical pattern X in cyclical systems; 3 2 and 3 3 (in scales and rhythm) - Selected concrete examples of systems in regard to pitches and intervals (scales), and durations (rhythm)

In the West, the specific raw material, characterized by unique features such as minimum asymmetry, binarity, optimal quantity of elements, etc., existed from the seventeenth to the twentieth century. In the twentieth century, however, many styles discard the learned schemata in the raw material. The breakup of the system of seven, together with most of the hierarchical schemata derived from it, demands that the composer himself now determine his own schemata of the raw material, and in addition rely (in part or exclusively) on parameters other than the interval, e.g., on texture and timbre. The Western pool of 12 thus contains the maximum amount of well-defined multiplicity.

D. Symmetry in Compositional Rules

D1. The Relationship to the Raw Material

The systems of twelve and seven and the major and minor scales in the West make it possible to create learned schemata on many hierarchical levels. Let us stress once again the most important schemata inherent to the West – chords and the rules of harmony: both contribute decisively to the overall directionality and to the complexity of compositions of large dimensions. Likewise, the system enables a clear distinction between operations that do not cause any deviation from the group of seven (“diatonic operations”) and those that expand the diatonic group towards the pool of twelve (“chromatic operations”). This expansion blurs the distinction between the scales and the tonal centers and heightens the listener’s sense of uncertainty regarding the continuation of the musical progression. The system of seven and twelve thus supplies another parameter of certainty/uncertainty, which contributes to a sense of tension and relaxation; this sense is fundamental to shaping a piece of music.

D2. The operations

For the most part, the operations act on small units, from the motive to the complete musical phrase, but they may act on larger sections as well. In small segments, numerous operations may function simultaneously in various parameters, whereas in large segments there is rarely more than one operation, and it generally acts on only one parameter. Extensive use of operations in Western music began in the seventeenth century, when the major and minor scales, harmony, and large forms with complex superstructure took shape. One typical transformation that began in this era and is virtually unique to the West is modulation on various levels, obtained by shift operations that involve the tonal centers.¹⁵ As stated above, the specific selection of the type of operation and manner of realization is determined in accordance with the ideal and cognitive constraints.

In the following, a short survey of the operations in the various periods of music history is given:

Medieval music may be divided into sacred and secular genres. Even if we consider written music only (9th–15th centuries), we can classify it according to the presence or absence of religious associations, the degree of definiteness of the rhythmic

organization, and the existence and complexity of polyphony. The rules of polyphony are still flexible. Symmetry phenomena are rare; they are manifested mainly in the formal organization of secular songs according to the differences and similarities between musical and textual units. This type of melodic organization has generated several fixed schemata (*formes fixes*), most of which are based on an additive combination of two units (*a-b*), which are considered different in various respects. Symmetry is also manifested in a constant repetition of melodic and/or rhythmic patterns (motet and isorhythmic motet), and in retrograde motion and voice exchange, as in the case of Machaut's rondeau "*Ma fin est mon commencement*." In monophonic religious music (Gregorian chant), symmetry is manifested mainly in the general convex shape of the melodic curve.

In the late Renaissance, especially in the vocal music of Palestrina, whose ideal is a "calm flow" and whose directional unit is the musical phrase that corresponds to the verbal phrase, the operations are extremely limited. The symmetrical schema of the convex curve – which allows for predictability regarding the continuation of the line and meets the conditions of calm from numerous standpoints – stands out on various levels and parameters. Curves which cause tension and do not enable predictability – concave, zigzag, and flat curves – are not found in this style. Another schema absent in it is the 2ⁿ schema which, although contributing to directionality, also implies division, an undesirable stylistic feature in the religious vocal music of the Renaissance.

The convex curve in the Renaissance is manifested on various levels, from the tiny unit of a combination of harmonic intervals according to the pattern of consonance-dissonance-consonance, to the immediate level involving the succession of melodic intervals or durations, and finally to convexity on the phrase level, and even on the entire structural level (in which case the convex structure is extremely weak). In Palestrina's music the operations of shift and sudden contrast are virtually absent. Contrasts are limited, on the whole, and appear mainly on the immediate level, such as change of direction, as well as in texture, e.g., homophony as opposed to polyphony. We may also find antiphonal echo (*cori spezzati* – in a sense, a stereophonic contrast), and of course the gradual contrast of the convex curve. As to the well-known contrast consonant/dissonant, it should be noted that the main purpose of the dissonance is to highlight the consonance (Zarlino 1588),¹⁶ and its occurrence underlies certain restrictions: it may occur only gradually on an accented beat, and should be prepared by a suspension in one of the voice parts.

In the Baroque Era (the seventeenth and eighteenth centuries), many rules of composition as defined by Renaissance theorists and many balances are deliberately violated, as it were. Therefore we may find on one hand lack of change in many parameters, stretching over long durations (a feature not found in Renaissance music), and on the other hand conspicuous changes such as large skips and changes derived by the operation of contrast. Indeed, this is the era of the concerto, based on the contrast between soloist (or soloists) and orchestra, in which the concerto principle is superimposed on a great part of musical forms (Bukofzer 1947). In addition, the accented dissonance occurring simultaneously in two voice parts, the melodic sequence (i.e., the operation of shift), and contrasts between large melodic skips and stepwise motion are frequent. Another salient feature is that the operations are applied to entire or partial themes (mainly in the fugue), which are being repeated many times. These common operations are inversion (a diatonic contrast regarding the pitch parameter); expansion and contraction (with respect to the parameter of duration); and shift along the time axis (*stretto*). In Bach's works, overt and latent operations in texturally uniform musical elements are especially abundant. For example, almost the entire D-Sharp Minor Fugue from Book II of *The Well-Tempered Clavier* may be regarded as being derived through operations of parts of the first theme, in addition to operations on the theme as a whole. Moreover, Bach extends the manifestations of the operations maximally to various levels of musical organization and even to real operations.

This multiplicity of operations with concurrent or nonconcurrent relationships results in maximum complexity with respect to a minimum of different elements.

In the Classical era, the most symmetrical and directional overall form – “sonata form” – developed. Here we find many changes in texture, which help to clearly shape the parts of the form: each theme in sonata form (“first subject”, “bridge”, “second subject”, “closing theme”) may have a different texture, in contrast to the Baroque era, where a single texture often dominates an entire composition or movement. Unlike in the Baroque, in the Classical era operations generally act on parts of the theme rather than on the whole. Particularly salient is the operation of contrast, which appears distinctly, but in various forms. For example, there exist contrasts in the learned schemata (e.g., a chord as opposed to a melodic second), or a binary contrast between the two themes of sonata form in respect to their predetermined tonal centers and also their texture. A clear contrast also exists between the first and second movements of a sonata. Sudden contrast is particularly salient in the works of the so-called “Storm and Stress” composers (mainly C.P.E. Bach) and of Beethoven, where the ideal is greater excitement regarding both the quantity of contrasts and their intensity. Often, the

contrast is between two extremes, such as a high register following a very low one, *ff* after *pp*, or a large ambitus as opposed to a restricted one. (Moreover, just being in an extreme range, which is like a deviation from the norm, elicits excitement.)

The following example (Figure 4) demonstrates multiple operations, mainly contrasts, occurring within four measures.

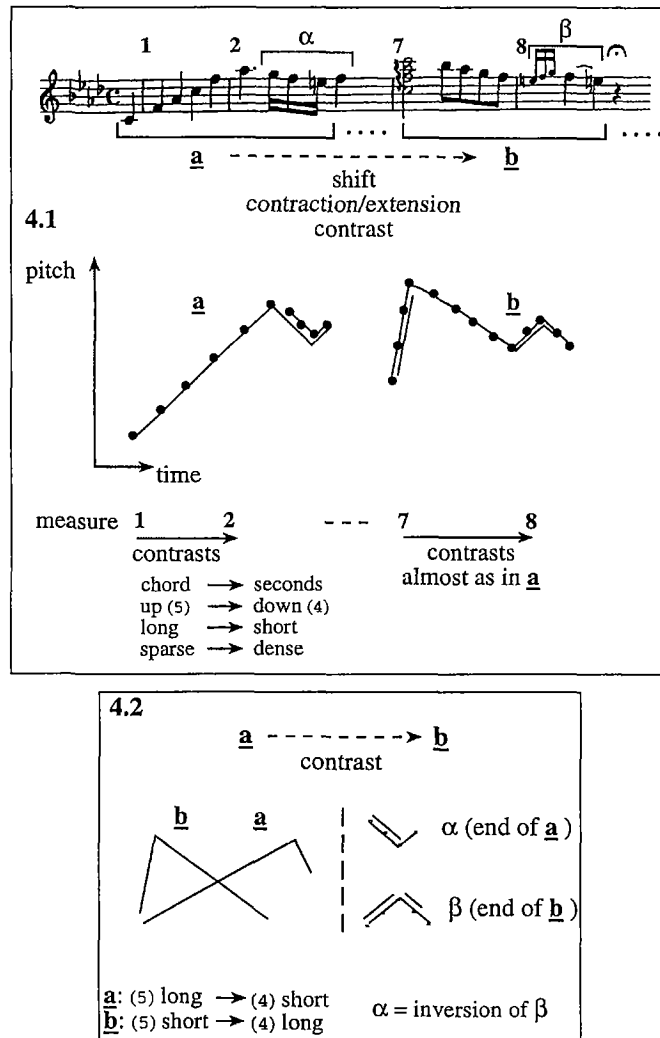


Figure 4: Symmetrical Links within and between Motives a and b in the Opening of Beethoven's First Piano Sonata (Op. 2, No. 1)

4.1 Contrasts within each of the units a and b;

4.2 Contrasts between a and b

Side by side with the operation of contrast, a salient phenomenon in the music of the Classical era is the symmetrical schema of 2^n . This schema (which can be regarded as being obtained from the operations of fusion and segregation, or augmentation by means of multiplication) is common in the so-called periodical phrases, and allows for clear directionality with maximum division into hierarchical levels. Adherence to it or deviation from it is an important characteristic of the structure. In sonata form, deviations from 2^n are common in the first theme, but the second theme is almost always based clearly on 2^n .

Deviations from 2^n are mainly of four types:

- (1) Division of a unit with 2^n measures into more than two, e.g., division of eight into $2+3+3$ (in works by Bach, Haydn, and others).
- (2) Division into two, where the units being divided are not 2^n (e.g., $5+5$; $3+3$). These deviations can be considered as the result of addition or subtraction (i.e., the operation of augmentation or diminution).
- (3) An extreme case: Division into three of a unit that is not 2^n , e.g., $3+3+3$ (found, for instance, in Scarlatti).
- (4) A lack of clarity in the division into units, because of nonconcurrency.

An interesting example of the first kind of consistent and clear deviation from 2^n is found in the first movement of Piano Sonata No. 25 in G Major, Opus 79, by Beethoven (Figure 5). The deviation is manifested in the internal division of a unit (y in the figure) of eight measures into three sub-units with $3+4+1$ measures. The idea of deviation from 2^n within a unit of eight measures is already suggested by the ambiguous division of the first theme (owing to different types of nonconcurrency), as opposed to the obvious deviation of unit y . The consistent deviation to $3+4+1$ governs most of the development section (it occurs five times), and can therefore be viewed as a specific schema (division of 2^n) that is unique to the piece.

An example of the second and third types of deviation from 2^n is found in the first movement of Mozart's Piano Sonata in C Major, K. 309 (Figure 6). The first theme consists of twenty measures which are broken down into $6+14$; the 14 is divided into $7+7$; the 7 is divided into $5+2$; the 6 is divided into $3+3$. The bridge also fits into this category, whereas the second theme definitely represents the symmetrical schema 2^n .

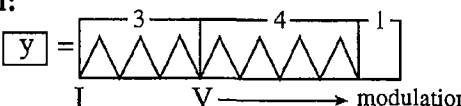
Theme	No. of measures	Division	Concurrence with 2 ⁿ
A			
First	8	Various interpretations	? (one possibility : 1 + 2+ + 2- + 3)
Bridge	16	4 x 4	#
Second	16	4 x 4	#
Closing	14	2 + 2 + 4 + 4 + 2	
Coda	4	2 + 2	#
B			
a	8	?	?
y ₁	8 (E major)	3 + 4 + 1	—
y ₂	8 (C major)	3 + 4 + 1	—
b	8	4 + 2 + 2	#
y ₃	8 (C minor)	3 + 4 + 1	—
y ₄	8 (E ^b major)	3 + 4 + 1	—
c	13 = 12 + 1 =	4 + 2 + 2 + 4 + 1	
y ₅	8 (D major)	3 + 4 + 1	—
d	4	2 + 2	#
Legend:  △ = broken chord # = full concurrence			

Figure 5: 2ⁿ in the First Movement of Beethoven's Piano Sonata No. 25 Op. 79: Adherence and Deviation. Maximal concurrence with 2ⁿ appears in the Exposition (A); consistent deviation from it appears in the Development section (B) in units y (y₁-y₅). Letters a-d represent various units between the y's; 2+ and 2- (top right in the figure) mean more and less than two measures; y consists of broken chords, and the internal division is regulated by harmony.

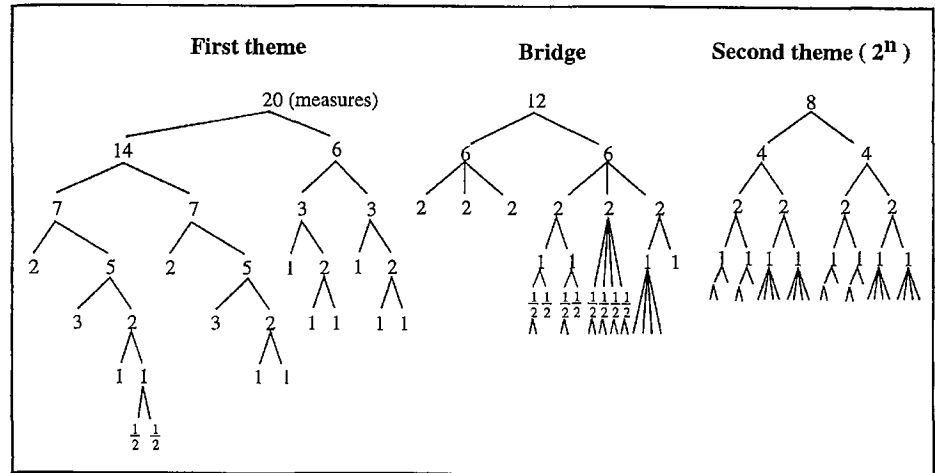


Figure 6: Division of Units into 2, but not 2^n , in the First Movement of Mozart's Piano Sonata KV 309. Only the second subject adheres to 2^n

The Romantic Era witnessed a gradual destruction of the tonal system, achieved in various ways: by the expansion of the schemata on various levels – especially the significant extension of the operations of shift and equivalence; the reduction of hierarchy within and between the schemata; the transgression of the norm, or a sudden break of a directional schema. In practice, we may find various ways of blurring the dichotomy between the similar and the different: between major and minor scales (also by the use of modes); between chords, tonal centers, or progressive and regressive harmonic steps; between sonata movements and sections within a movement (for example, between the exposition and the development section, the latter principle being superimposed also on the exposition), etc. Within these procedures, each composer chooses his own personal way of loosening the system without causing its total collapse. A few examples may serve to illustrate this point:

One of the stylistic traits of Schubert, especially in his late works, is a sudden deviation from the expected by the use of a classical harmonic schema, where the dominant degree (i.e., the most directional chord of the schema) is not led into the tonic, but is instead broken by a partial or full chromatic shift and succeeded by a completely strange chord (for instance, in “*Der Wegweiser*” from the *Winterreise*, the dominant of C – G-B-D – leads into $G^b-B^b-E^b$, i.e., into an E^b minor chord). This is a significant expansion of the false cadence, and the result is complete surprise. Here and elsewhere (e.g., in the *F* minor piano fantasy for four hands) we may also find different enharmonic interpretations of the diminished seventh chord, or of the augmented triad.¹⁷

Chopin adheres very often to the 2ⁿ schema, which helps him to keep the general notion of tonality in spite of the fact that the intensive chromatic shifts threaten to endanger it.¹⁸ A primary experiment has shown that without the 2ⁿ schema the listener tends to lose the sense of tonality. Brahms often inflects the sixth note of the major scale, and the sixth degree as a whole, thus shifting suddenly to a tonal center lying a major third beneath the tonic, and also creating a chain of equi-distant major thirds. For example, in Brahms's Rhapsody No 1, Op. 79 in B Minor, we find the major scale with a minor sixth in mm. 22-29; modulations via the inflected sixth degree appear in mm. 1-43, where the tonal centers are F# - D - B^b - G^b (=F#). Interestingly, the piece also opens with a descending line consisting of these notes. Unlike Schubert, who usually opens a work on the tonic, in Brahms's Rhapsody the tonic (B) appears for the first time only in m. 89!

In the Twentieth Century, as the learned schemata became less and less important, the planned natural schemata became more prominent. Despite the difference between tonal and atonal systems and their realizations in specific pieces, we find the same categories of operations expressing various forms of symmetry in all of them.

A well-known, almost classic, example is the "dodecaphonic system" formulated in the 1920s by Arnold Schoenberg. In dodecaphonic music, which is based exclusively on the pool of twelve notes, the learned tonal schemata are replaced by the so-called row schema, which is manifested in a specific one-time arrangement of the twelve notes in the octave. Although it does not seem possible to refer to schemata, since each piece has its own row, the theorists (Babbitt 1960) point to categories of rows in terms of the intervals and the internal organization within the row, which represent phenomena of symmetry (Perle 1972). In rows existing in Schoenberg's works, for example, the collection of the first six notes in the row is an inversion, in a sense, of the second collection of six notes; in Webern's music, the row is divided into even smaller units, which are obtained from each other through the operations of contrast and shift.¹⁹ The row schema is realized in various ways, not only in terms of parameters other than pitch, but even with respect to the pitch class. Realizations of the pitch class can thus be considered an operation of equivalence, and a division of the row into sub-units is an operation of segregation; the other operations involving the parameter of pitch, and that are discussed explicitly, are contrast ("inversion" with respect to the pitch axis and "retrograde" with respect to the time axis), and the precise shift (transposition). The method was also expanded to encompass the other parameters and the links between the rows in the parameters (total serialism).

In recent decades we find many musical styles that relate to the parameter of pitch (in the pool of twelve) without clear tonality. In some of them, the compositional rules are based on set theory (e.g., Vieru 1993; Conner 1994). From our standpoint, the concepts used in set-theory – “inclusion”, “cutting”, “difference”, and “completion” – relate to the operations of fusion and division. The tonal schemata are replaced by various sets of notes, each of which is realized in various ways by means of the different operations. The computer is gaining increasing use in all stages of composition – e.g., selection of the raw material, various parameters, the compositional rules, and the musical piece itself – and much of the work done by the computer consists of performing operations (which are still included among our five basic categories). When discussing symmetry in twentieth century music, one cannot ignore the music of Béla Bartók, in which manifold manifestations of symmetry reached a peak.

E. Symmetry in Forms

“Form” is defined in the *New Harvard Dictionary* (Randel 1986) as “The shape of a musical composition as defined by all of its pitches, rhythms, dynamics, and timbres.” This definition is shared by many authors writing on form (e.g., Leichtentritt 1951), but according to our definition form means the organization of units mainly with regard to the difference and similarity between them in various parameters, even without taking into account the nature of the unit (be it melodic, harmonic, rhythmic, etc.).²⁰ The very distinction according to similarity and difference is a basic cognitive activity (Tversky 1977); it may be perceived as a schema, and considered as derived from the extended operations of fusion and segregation on various levels.²¹

An initial categorization of form relates to (a): the clarity of the distinction between similar and different; (b) the degree of adherence to a symmetrical organization; and (c) the levels of the symmetrical phenomena.

(a) Regarding the degree of distinction between similar and different, the repetition *a-a* (referring to segments of varying size) may appear in identical manner – with precise repetitions designated by a repetition sign (*//*: *://*) – or with small or large changes, so that we would not know whether to define the section as “repetition with changes” (*a-a'*) or as a different section, professing a few points of similarity with the previous one.²² This problem gets even more involved when we consider that different cultures vary in their definition of the similar and the different. On the other hand, different sections (*a-b*) usually exhibit some similarity (even by means of the operation of

contrast, or another principle). Thus, the difference between *a* and *b* may be expressed in several parameters or in only one (as, for example, in works by Schubert in *a-b-a* form, where *a* and *b* may differ mainly in terms of the mode – major as opposed to minor). In the *a-b-a* form, the opening and closing *a*'s may profess varying degrees of similarity: they may be identical (as in the *aria da capo*), similar though not identical, with a fixed change (as in sonata form), or similar with no fixed change.

(b) As to the adherence to symmetrical organization, the most common schemata in the forms of Western tonal music are *a-a* (*a-a'*), and *a-b-a*. A single repetition (exact or not), that forms the basis of 2^n , always increases directionality (for examples of adherence to symmetry and deviations from it see Figures 5 and 6 above.). On the other hand, multiple repetitions based on the additive principle reduce the overall directionality of the collection of *a*'s. Multiple repetitions may occur in relation to units on the immediate level, or in a schema or schemata that are inherent in *a*, e.g., a harmonic pattern (common in variation form), the bass line (*passacaglia*), or the “melody type” in improvisational music in non-Western cultures. In the latter the repetition always entails some change. However, there is also a precise structure *a-a-a...* found especially in polyrhythmic African music, which lacks all directionality (in accordance with its aesthetic ideal).

Symmetries may also occur on the overall level, based on the expansion of the principle of *a-b-a*. The latter represents retrograde motion in respect to the location of entire sections, but not regarding their musical essence: the closing *a* is a repeat, and not a retrograde of the opening *a*.

Common augmentations of *a-b-a* are *//a://b://* or *//a://a-b://*, where the *b* section includes in its closure a part of the *a*. Another possibility of augmentation is *a-b-c-d ... d-c-b-a* (as in Bach's *St. Matthew Passion* with respect to the texture of the various sections; in Brahms's *German Requiem*, where there is also an affinity between the themes; or in Penderecki's *Seven Gates of Jerusalem*). From the standpoint of form, this type of organization indicates inversion (*a-b ... b-a*), but similar sections are repeated and not inverted. The question of whether the listener is able to perceive the symmetry of such large-scale compositions as Bach's *St. Matthew Passion* must remain open. An extreme case of a loose *a-b-a* form is when the opening thematic material *a* is repeated at the end of a composition, resulting in an auditory experience of “closing the circle”, a frequent procedure in multimovement compositions of the nineteenth-century.

(c) As regards the level of symmetrical phenomena, when a relatively long section in tonal music is repeated (*a-a*), the repetition – owing to the constraints of human memory – is usually without operations and relates to the entire section. The operations mainly refer to motives or themes on the immediate level (dependent on style); but also cases of repetitions with an operation on a higher, and even on the overall level may also be found.

An extreme case is the E minor Fugue in Book I of *The Well-Tempered Clavier* by Bach (Figure 7). Here the fugue falls into two equal parts (A and B in the Figure), the second one being derived from the first by the operation of contrast (a precise exchange of roles between the two hands). A and B are continuously divided into two similar units (A1, A2; B1, B2): each consists of eight measures + addition (two measures in A1, one measure in A2, and likewise in B), and each unit of eight measures is divided into two: the first four measures contain the main subject twice, and the second four – an episode. Each of the units of four measures is split into two, and the second episode – into four. Likewise, the coda is divided into two, as are the additional measures (19 and 38) that close parts A and B with parallel octaves. All in all, the division into two appears here on eight levels, including the overall level. In most cases, this division is made by operations (despite the rare case of division into two without operation, found in the above-mentioned measures with parallel octaves). A selection of various manifestations of these operations is summed-up in the following:

(1) Contrast: Between A and B; between the themes in the four measures that open the sub-units A1, A2, B1, B2; within episode 2 – the fourfold voice exchange between the upper and the lower part; between rising and falling sixths; between ascending broken chords (in most cases) and descending ones (at the beginning of the coda); between a chord and a second; between polyphony and homophony (up to unison); between melodic motives, which we shall not specify here.

(2) Shift: Between the themes; between the half episodes; between motives.

(3) Augmentation/diminution: In the motion of the triads (in eighths or sixteenth notes); the curtailment of the main theme in the coda; in the interval of the sixth – whether gapped or stepwise.

(4) Fusion/segregation: The episodes may be conceived as derived from various combinations of elements from the main theme and the countersubject (e.g., the triad, the sixth interval, etc.)

(5) Equivalence: The existence of different episodes, serving as intermediate units that cause relief from polyphonic rigor.

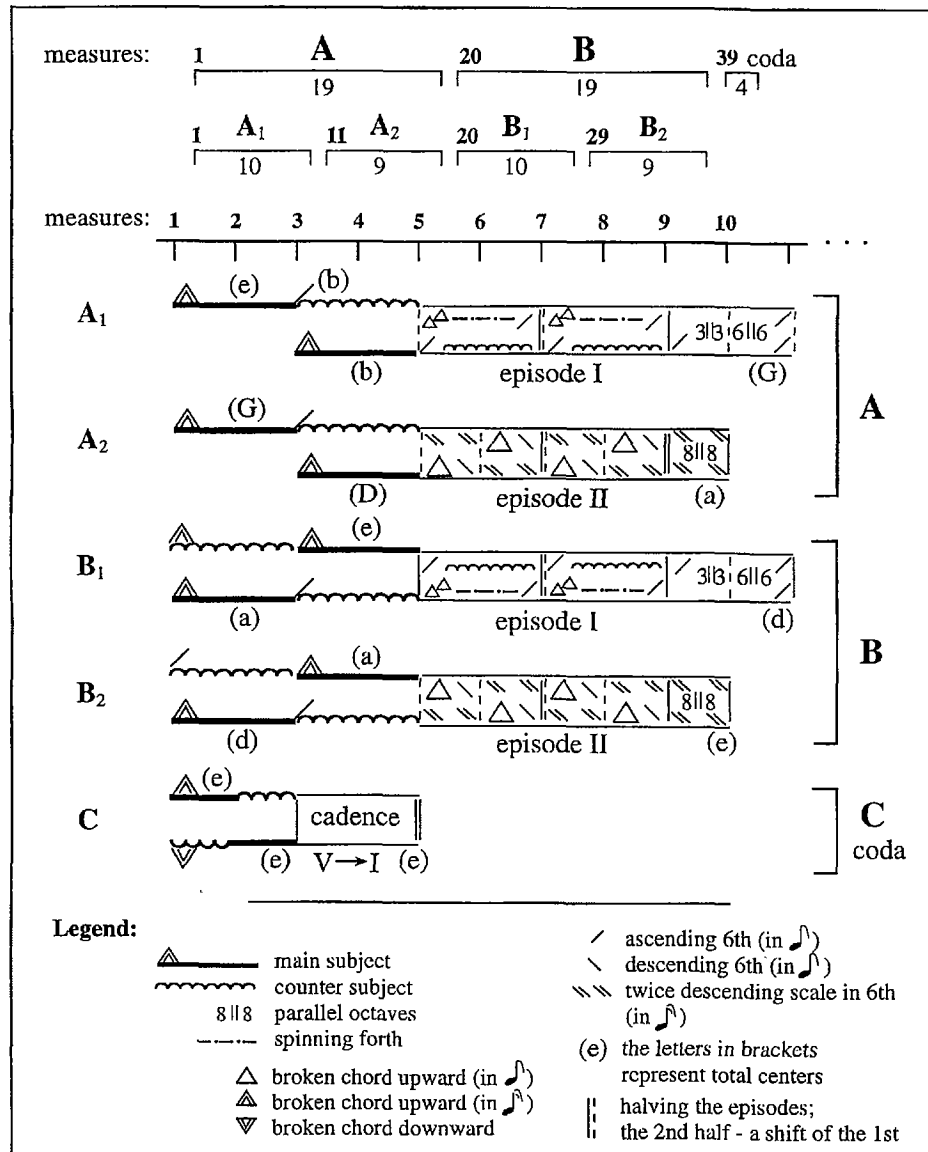


Figure 7: A Schematic Outline of the Symmetrical Two-Part Fugue in E Minor from Bach's *Well-Tempered Clavier I*: An Extreme Case of Complex Symmetrical Relations on Various Levels

It should be noted, however, that the picture gets more involved once one analyses the details of the themes and discovers many non-concurrences and uncertainty as to the boundaries of the sub-units.

Essentially, the fugue form is based on a chain of many repetitions, with operations (from all five categories) applied to entire themes. However, the overall form of the fugue is flexible, even if the general formula of $a-b-a$ is obtained with respect to the tonal centers in each fugue (for another analysis of this fugue, see Werker 1922).

Another example of overall symmetrical organization is found in Bach's "Goldberg Variations" where every third variation is a canon in a gradually ascending interval (i.e., the operation of augmentation) – in addition to various other operations, such as the mirror in Variation 15 (a canon in the fifth).

All these observations contribute to the distinction between small and large-scale forms, simple and complex, and directional and non-directional ones. These distinctions follow the stylistic ideal of the various periods of music history.

F. A Summary of the Symmetrical Phenomena in Six Eras of Western Music

We attempt to summarize here both aspects of the subject: on the one hand, the main characteristics of symmetry, and on the other hand, the characteristics of styles in the different eras (guided by different ideals) by means of the symmetry variables.

F1. The Variables of Symmetry Relevant to the Relationship with the Stylistic Ideal:

- i. The basic tonal system which serves as raw material from which the various schemata are derived: for example, the scale system of seven (with minimal asymmetry), the system of twelve (with maximum equality and without hierarchy), or both, distinguishable from each other to varying degrees.
- ii. Types of schemata in the raw material (scales, chords, rhythmic patterns, etc.): The quantity of types, manifestation of binarity, degree of distinction between schemata in the same category, and the hierarchy in each schema (the hierarchy among the notes of the scale, between chords in harmonic phrases, between beats in a meter, etc.).

- iii. The range of occurrence of the various parameters (a characteristic of texture) with regard to register, ambitus, and degree of change, and with attention to the U-function (in the optimal domain, which is a norm, as opposed to deviations in the direction of the extremes).
- iv. Concurrence/nonconcurrence regarding the simultaneous occurrence of different schemata or units in various parameters; effects the contribution of symmetry to excitement or calmness.
- v. Cases of balance or imbalance (balance may itself be considered part of the ideal), with attention to the range of occurrence: A balance between the extremes is different from that within the optimal range.
- vi. Types of curves of change (another characteristic of texture that contributes to predictability or uncertainty). The prominent representative of the first group is the convex curve; the definitive representatives of the second group are the concave, zigzag, and flat-line curves.
- vii. Reference to the directional symmetrical schema 2ⁿ.
- viii. Types of structures and forms on different levels in terms of symmetry, types of directionality and complexity, and the quantity and range of options for various realizations (in non-Western cultures we find forms that restrict the range of realizations; while sonata form, for example, may be realized in various styles).
- ix. Operations – manifestation of symmetry in the compositional rules (one of the conditions for a complex, directional super-structures): Quantity, kind and function.

F2. A Summary of Historical Eras in Western Music According to the variables of Symmetry

Early Middle Ages (9th –13th centuries): monophonic music (Gregorian chant)

Ideal: Simple, calm; maximum adherence to the text and to the religious context.

- i. Basic tonal system: The diatonic system (of seven), with minimal asymmetry.

- ii. Schemata in the raw material: Scales (of modes – more than two); the intervals are limited in size and are all melodic; rhythmically and metrically undefined.
- iii. Range of occurrence: In the optimal range of the vocal diapason; the musical instruments are technically not well developed.
- iv. Concurrence/nonconcurrence: not relevant to the simple system.
- v. Balance: Exists mainly through the melodic curve, particularly for the parameter of pitch.
- vi. Curves of change: Convex curve. A paradigmatic example of symmetry is found in psalmody, where the second half of the verse is the opposite of the first.
- vii. 2ⁿ: None.
- viii. Structures and Forms: Simple and short; dependent on the text and the liturgical structure.
- ix. Operations: Few and extremely simple. Though there are some repetitions, there are mainly contrasts between ascending/descending tones, solo/chorus, syllabic/melismatic, step/skip.

Late Renaissance: religious vocal music (16th century)

Ideal: Calm and balance; directionality chiefly regarding the unit of the verse, with full concurrence between the musical and textual verses.

- i. Basic tonal system: Mainly a single system – the diatonic system, with few chromatic notes.
- ii. Schemata in the raw material: scales (modes – more than two); chords (with no hierarchy between strong and weak progressions); melodic and harmonic intervals that are categorized meticulously and are subject to strong restrictions on separate appearances in various contexts; consonance-dissonance; homophonic or polyphonic texture; structures depend largely on the text.

- iii. Range of occurrence: Maximum adherence to the optimal range of change, ambitus, and register in all relevant parameters (durations and intervals).
- iv. Concurrence/nonconcurrence: Little nonconcurrence.
- v. Balance: Maximum in the optimal range, in a single melodic line, and between melodic lines.
- vi. Curves of change: Almost entirely convex for the parameters of pitch and duration on the immediate and phrase levels.
- vii. 2ⁿ: Hardly exists at all in vocal music, but is found in dance music.
- viii. Structures and Forms: Simple, mostly text-dependent; in a latent manner, using proportions of the durations of movements and sections, as in visual art (e.g., golden section or perfect symmetry manifested in a convex curve).
- ix. Operations: Few, mostly of the category of “soft” contrast regarding the parameters of pitch (e.g., ascending/descending, consonance/dissonance, step/skip) and duration; appear mainly on the immediate level and contribute to calm.

Baroque (17th–18th centuries)

Ideal: Multiple styles; emphasis on excitement; unclear directionality with maximum complexity (“an oddly-shaped pearl”; Pluche 1770, in Palisca 1968); related to the religious and theatrical framework.

- i. Basic tonal system consists of the two fundamental Western systems – the diatonic (with seven notes) and the full chromatic system (with twelve notes).
- ii. Schemata in the raw material: Scales (chiefly major/minor); harmony (with a noticeable preference for strong steps, but no strong hierarchy between degrees); expansion of the complexity of the polyphonic texture, homophony, and the ranges of overlap between them. A prominent example of the lack of harmonic hierarchy (equality between progressions and harmonic degrees) can be seen in the complex chains composed of sequential links (cf. David and Mendel 1946, pp. 394-398). The best known of these is the sequence of descending fifths.

iii. Range of occurrence: Numerous deviations from the optimum; notable polarity of both lack of change and salient change. For example, there may be a polar ambitus and lack of balance between instrumental groups (attested to by the development of the concerto), as well as sudden changes, as in recitative music or certain fantasies (e.g., Bach's "Chromatic Fantasy"). In contrast, we may find a lack of change from various standpoints, e.g., on the immediate level (sequence, multiple repetitions of a motive or rhythm), on the level of the movement (uniformity of texture), and on the level of the overall piece (lack of change in tonality and duration of the movements).

iv. Concurrence/nonconcurrence: Extensive nonconcurrence between various parameters (especially salient in Bach's works).

v. Balance: Numerous imbalances due to adherence to the ends of the U-function in many parameters (cf. Hargreaves 1986).

vi. Curves of change: All sorts and on all levels; in Bach's works we find – in addition to convex curves – concave curves and zigzags from various standpoints, both on the immediate level and in the relationship between movements in a complex piece, (e.g., concave and zigzag curves for the textures and lengths of movements in Bach's *St. Matthew Passion*).

vii. 2ⁿ: Appears at times, alongside a flexible schema with three links and various expansions of it.

viii. Structures and Forms: Numerous types of asymmetrical and symmetrical structures with unclear directionality. Some uncommon but interesting symmetrical structures in Bach's music: *a-b-a* in a fugue (e.g., Duet in F Major, BWV 803; in the first movement of the Violin Concerto in E Major, BWV 1042); *a-a* in a fugue (the E minor Fugue from Book I of the *Well-Tempered Clavier*, where the second *a* is obtained from the first by an operation, and the two are equal in size (see Figure 7 above); the second "Kyrie" from the *B Minor Mass*). There are also symmetrical structures that are expansions of *a-b-a* or retrograde, such as the overall structure of the *St. Matthew Passion*. Such symmetry with respect to texture is found, for example, in the Credo of the B Minor Mass and in Cantata BWV 4; with respect to tonal organization it is found in Cantata BWV 78. In Bach's compositions we may also find specific, one-time superstructures, such as in the *Goldberg Variations* and *The Art of the Fugue*.

ix. Operations: Numerous operations of all types; may act on entire sections.

Classical Era (18th century)

Ideal: Clarity; clear directionality on various levels (“polished diamond”); the peak of autonomous music, separated from the extra-musical world (with exceptions, such as opera).

- i. Basic tonal system: The two systems of 7 and 12 notes, with maximum separation between them (the diatonic and chromatic systems).
- ii. Schemata in the raw material: Clear binary separation between the major and minor scales; maximum hierarchy among the harmonic schemata with respect to the degrees and the harmonic progressions; maximum separation between meter and rhythm; mainly simple meters (duple or triple); schemata highlighted by means of textural changes.
- iii. Range of occurrence: Both adherence to the optimum and deviations from it (particularly noticeable in Beethoven's works); an optimum of well-defined categories of musical instruments to highlight the structure. For the first time meticulous attention is paid to the orchestration.
- iv. Concurrence/nonconcurrence: Multiple cases of concurrence in various parameters and on various levels, and in general between natural and learned schemata. Therefore, cases of nonconcurrence are the more salient (Beethoven).
- v. Balance: Prevalent, but may occur at the extremes, which are moving away from the optimum (Beethoven).
- vi. Curves of change: Limited in comparison with the Baroque era.
- vii. 2ⁿ: Serves as a point of departure.
- viii. Structures and Forms: Mainly sonata form, in which symmetry is salient on various levels, even in the order of movements producing *a-b-a* in terms of the tonal centers.²³
- ix. Operations: Numerous manifestations in all types of operations, in all parameters, and on all levels; particularly prominent is the operation of contrast, which occurs suddenly and not gradually. All the operations contribute to logical structure and complexity.

Romanticism (19th century)

Ideal: Excitement; unclear directionality; a close relationship with the extra-musical world

- i. Basic tonal system: Blurring of the systems of seven and twelve.
 - ii. Schemata in the raw material: As in the Classical era, but with a blurring of meters, of major/minor scales, of harmonic degrees, of strong and weak harmonic progressions, of the identity of chords, and of the close ties between harmony and melody.
 - iii. Range of occurrence: Adherence to extreme deviations from the norm (in two directions: that of “more” and that of “less”) in all the parameters and in respect to all kinds of occurrences – such as amount of change (no change/ large change) ambitus, durations of the pieces and movements, etc.; a multitude of musical instruments with no well-defined categories, which emphasize the parameter of timbre rather than of structure.
 - iv. Concurrence/nonconcurrence: Numerous cases of nonconcurrence of all sorts.
 - v. Balance: Numerous violations.
 - vi. Curves of change: Numerous manifestations. Especially salient are the concave, zigzag, and flat curves, which produce excitement on all levels.
 - vii. 2ⁿ: On the one hand, numerous exceptions; on the other hand, adherence, with deviations in the other parameters (salient in Chopin’s works).
 - viii. Structures and Forms: Numerous types of symmetrical and asymmetrical structures in small and large-scale compositions and with clear and unclear directionality; *a-b-a* form prevails.
- The functions of the components of sonata form (exposition – development – recapitulation) are blurred by various means (e.g., by adopting the development principle in the exposition).
- ix. Operations: As in the Classical era, but with an expansion of the operations of shift and of equivalence, and with manifold manifestations of operations contributing to states of excitement.

20th century

Ideal: Multiple ideals, including relationships with the extramusical world. Prominent ideals are non-directionality, frozen time (= “space”), and the use of many non-Western principles.

i. Basic system: With respect to pitch, mainly the system of twelve (dodecaphony), but also other divisions of the octave. Timbre, texture, and interim states between them are of prime importance (Cohen—Dubnov 1997); by their very nature they arouse extramusical associations and contribute to a focus on momentary events.

ii. Schemata in the raw material: Usually a group of compositions does not share any specific, predetermined learned schemata, but there are schema types, such as serial rows (in dodecaphonic music), types of pitch sets, types of modes (Messiaen), and types of textures and timbres. The composer may or may not adhere to schema types.

iii. Range of occurrence: This factor, which is the main characteristic of texture, is of the greatest importance and appears in an enormous variety of ways in all parameters.

iv. Concurrence/nonconcurrence: All sorts, in all parameters; not always relevant.

v. Balance: The balance is generally broken in various ways.

vi. Curves of change: Of all sorts.

vii. 2ⁿ: Sometimes exists, sometimes is irrelevant.

viii. Structures and Forms: Owing to the not well-defined concepts of units and forms, predetermined structures are rare. There are numerous plans for all parameters, with varying degrees of complexity and directionality, including symmetrical structures (e.g., the golden section in Bartók’s works). The mathematical organization of a musical composition, especially in computerized music, has reached the peak of complexity, but we do not as yet know how far it is significant for our capacity as listeners.

ix. Operations: The most important factor in organization, replacing the learned schemata of tonal music; especially prominent in computerized music. The operations occur with respect to all possible parameters. Interestingly, one can demarcate them by means of the same five basic cognitive categories mentioned throughout this paper (contrast, shift, expansion/contraction, fusion/division, equivalence).

CONCLUSION

As we have seen, all music has some sort of symmetry, which can be regarded as a specific case of symmetry in human life and natural phenomena. Any characterization of structural organization must relate at least in part to the presence or absence of various forms of symmetry. Moreover, as we tried to show, the various manifestations of symmetry reflect aesthetic ideals that take shape against the backdrop of numerous extramusical factors.

Clearly, the ideal is what determines (consciously or unconsciously) the selection of types of symmetry (and not vice versa). However, because the selection takes cognitive constraints into consideration, there is also an opposite relationship according to which we can draw conclusions about the ideal that guided the selection of symmetry, by observation of the laws of symmetry in a particular style.

In this study we have attempted to classify the realizations of symmetry in terms of its reciprocal relationship with the stylistic ideal. We have done so by means of theoretical considerations and analysis of examples of various styles, governed by different ideals.

We believe that the present subject deserves further study, to investigate in more depth the nature of symmetry in the music of various eras in the West, and to extend such in-depth research into non-Western musical cultures and the inter-relationship with other art forms.

ENDNOTES

¹ In some specific cases symmetrical structure, expressed mainly by the number of measures in parts of a composition, is so exact and complex that it points to a careful and conscious planning by the composer (Powel 1979; Guletsky 1995).

² We exclude here Baroque musical theoretical writings applying principles, concepts and terms from the discipline of rhetoric, especially those categorized as "Figures of melodic repetition" (Buelow 1980).

³ For a recent similar definition of symmetry see Kempf 1996.

⁴ The frequency ratios of the intervals in the major triad are 4: 5: 6, and in the minor – 1/4 : 1/5 : 1/6.

⁵ An interesting case is that of Schillinger (1948), who tried to formulate an abundance of artificial scales and rhythms that should serve as foundations of music. The very concept of *ISIS* is, of course, a notable expression of this comprehensive notion.

⁶ Schemata based on the vertical parameter (pitch) require the invention of instruments, in order to enable exact measurements. Let us stress that, although these schemata are learned ones, they are not arbitrary but derive from the aesthetic ideal and from cognitive constraints. Some of them have even been explained and formulated mathematically (see note 14 below).

⁷ Excitement may be caused, for instance, by a sudden change in contrast to a gradual one, by deviation from the norm, and other natural phenomena pertaining to the parameter of texture. Let us emphasize again, that we believe these to be universal laws, manifested even in birdcalls (Cohen 1983). Moreover, some of these principles, as well as others not specified above (e.g., rareness, ambiguity, etc.), may be applied not only to

basic parameters, but also to organizations on different levels, namely, learned and natural schemata. From the vast literature on that subject let us mention Bolinger 1972; Clynes 1982; Howel, Cross and West 1985; McAdams 1987; Sloboda 1991; Gabrielson and Juslin 1996; Cohen and Inbar 2001.

⁸ An illuminating description of 2ⁿ from the eighteenth century is given by Comte de la Cépède (1785; in le Huray and Day, 1981, p. 181):

Symmetry [...] requires that the corresponding sections of a composition shall have the same structure and the same number of components; it is essential at least that the music is composed in such a way that the ear can easily connect and divide phrases and thus discover parallel arrangements and groupings of sections. These goals have only been attained [...] when the length of phrases has been based on the number two and its multiples. It is agreed therefore that all phrases must be constructed of two, four, eight or sixteen measures and so on. The ear is easily able then to divide the pieces into equal phrases of four or two measures, or to build four bar phrases from successions of two, on the assumption that the ear wishes to be guided by the laws of song or of symmetry.

⁹ This concept (Cohen 1971; Cohen and Wagner 2000) was indirectly referred to by many scholars, who used a different terminology, such as non-congruence, ambiguity (Cooper and Meyer 1960), concinnity (LaRue 1970), conflict (Schachter 1970)

¹⁰ It should be noted, that most of the discussions on transformation (e.g., Reti 1951; Real 1970; Rosen 1980; Cone 1987; Kamien 1990; Appel 1992; Conner 1994) do not associate it with symmetry, while only few scholars refer to transformation as a manifestation of symmetry (e.g., Solomon 1973; Wilson 1986; Cohen 1996). Other scholars (e.g., Lendvai 1977) discuss symmetry, but without reference to transformation. The close tie between the two was detected only recently.

¹¹ The concept of directionality in its general significance and without sub-divisions appears in the theoretical literature under different terms, such as progression, processive forms, flow, goal, and approaching.

¹² It should be emphasized, that artificial symmetrical organization that ignores psychoacoustic and cognitive constraints loses its meaning for the listener. Such attempts have been made in modern times (e.g., by Boulez, see above), and the auditory result was complete chaos.

¹³ This idea – the von Forster Theory – was also expressed mathematically (Koppel et al., 1987; Shanon and Atlan 1990).

¹⁴ The diatonic system may be obtained from the “cyclic generator” (octave and fifth), and the interval system derived from it meets the conditions of maximum coherence and efficiency. The priority of the Western system has been proved also by its comparison with hypothetical systems (Balzano 1980; Gouldin 1983; Agmon 1989).

¹⁵ Modulations are also conspicuous in Arab music, but only on the immediate level, and especially between different maqāmāt rather than between the same maqāmāt with different tonal centers.

¹⁶ In Book III, chapter 27 of his *Instituzioni armoniche*, Zarlino states:

And although every composition, every counterpoint, and in a word every harmony is made up primarily and principally of consonances, dissonances are used secondarily and incidentally for the sake of greater beauty and elegance. (Strunk 1950, 231-232.)

¹⁷ For the “melting” of the dominant, see “Der Wegweiser”, mm. 12-13, 34-37, 47-48. Enharmonic interpretations of the diminished seventh chords, leading to an equi-distant chain of tonal centers in minor thirds appear in the same song, mm. 57-67. Both devices illustrate Schubert’s subtle textual interpretation: the road leads to a place from which nobody has ever returned, and where there is no significance of directionality of any sort.

Enharmonic interpretations of the augmented triad, creating a series of tonal centers in chains of descending major thirds may be found in the F Minor piano fantasy for four hands, mm. 65-91.

The tonal centers (all in minor) are: F - D^b - A - F.

¹⁸ Good examples are Chopin’s Prelude No. 4 in E Minor, the Mazurka in A Minor, Op. 17, No. 4, and many other mazurkas.

¹⁹ For a detailed discussion of a paradigmatic example, Webern’s Concerto for nine instruments, Op. 24, see Bailey 1996.

²⁰ Many music theorists also believe that musical form and content cannot be separated (e.g., Randel 1986). We believe that the two can be separated, although musical contents often influences the form. Thus, for

example, tonal centers are decisive in shaping sonata form, whose overall design is *a-b-a*. The degree to which content intervenes in form may be considered one of the characteristics of style.

²¹ Other variables of formal organization are conspicuous/not conspicuous, and frequent/rare, and these, together with different/similar, effect the reaction of our brain waves, as attested by ERP (Event-Related Brain Potential) research (Cohen—Granot 1995).

²² The repetition may be imprecise (*a-a'*), where *a* and *a'* differ only in their closure; examples are found in the "periodic phrase" of the Classical era. Repetition may also occur with the addition of embellishments (augmentation), or even with an operation of any kind (although this is uncommon)

²³ An interesting example of latent symmetry in harmonic progressions can be found in the first movement of Piano Sonata No. 42, Hob XVI/27 in G Major by Haydn, and in the development section of the first movement of Piano Sonata No. 5 Op. 10 in C Minor by Beethoven. The tonal centers of the modulations are arranged here in a two-way symmetrical series, where the retrograde part is extremely short:

Haydn: G - D - a - e // e - a - D - G

Beethoven: c - f - b^b - D^b // D^b - b^b - f - c

Another example is obtained when a harmonic schema composed of a chain of sequential links appears also in retrograde. Thus, for instance, the retrograde schema of the "Pachelbel Canon",

I - V - VI - V - VI - III - IV - I, is I - IV - III - VI - V - VI - V - I,

and both are found in Beethoven's *Egmont* Overture.

REFERENCES

- Agmon, E. (1989) A mathematical model of the diatonic system. *Journal of Music Theory* 33(1), 1–25.
- Appel, R. (1993) A cognitive model for the procedural aspects of the creative process in music with emphasis on transformation processes in Western music, Ph.D. dissertation, University of Haifa.
- Arom, S. (1998) Description and modelization of traditional music: the dialectic between indigenous nomenclature and structural analysis. In Braun, J. and Sharvit, U., eds., *Studies in Socio-Musical Sciences*, Ramat-Gan: Bar-Ilan University Press.
- Atlan, H. (1981) Hierarchical self-organization in living systems: Noise and meaning. In Zeleny, M., ed., *Autopoiesis: A Theory of the Living Organization*. North Holland.
- Atlan, H. (1987) Self creation of meaning, *Physica Scripta* 36, 563–576.
- Avital, T. (1996) The connectivity principle of art, *Symmetry: Culture and Science* 7(1), 27–50.
- Avital, T. (1998) Mindprints: the structural shadows of mind reality?, *Symmetry: Culture and Science* 9(1), 47–76.
- Babbitt, M. (1960) Twelve-tone invariants as compositional determinants, *The Musical Quarterly* 46, 246–259.
- Bailey, K. (1996) Symmetry as Nemesis: Webern and the first movement of the concerto, opus 24, *Journal of Music Theory* 40, 245–310.
- Balzano, G. J. (1980) The group-theoretic description of 12-fold and microtonal pitch systems, *Computer Music Journal* 4(4), 66–84.
- Bolinger, D., ed. (1972) *Intonation: Selected readings*, Penguin Education.
- Buelow, G. J. (1980) Rhetoric and music, *The New Grove Dictionary of Music and Musicians* 15, 791–803.
- Bukofzer, M. (1947) *Music in the Baroque era*, New York: Norton.
- Clynes, M., ed. (1982) *Music, mind and brain*, New York: Plenum Press.
- Cohen, D. (1971) Palestrina counterpoint – a musical expression of unexcited speech, *Journal of Music Theory* 15, 68–111.
- Cohen, D. (1983) Birdcalls and the rules of Palestrina counterpoint: towards the discovery of universal qualities in vocal expression, *Israel Studies in Musicology* 3, 96–123.
- Cohen, D. (1994) Directionality and complexity in music, *Musikometrika* 6, 27–77.

- Cohen, D. (1996) Aspects of symmetry in music, *Proceedings of the 2nd International Conference of ISIS*, Washington.
- Cohen, D. and Dubnov, S. (1997) Gestalt principles in musical texture. In Leman, M., ed., *Music, Gestalt and Computing: Studies in Systematic and Cognitive Musicology*, Berlin: Springer-Verlag.
- Cohen, D. and Granot, R. (1995) Constant and variable influences on stages of musical activities: Research based on experiments using behavioral and electrophysiological indices, *Journal of New Music Research* 24, 197–229.
- Cohen, D. and Inbar, E. (2001) Musical imagery as related to schemata of emotional expression in music and on the prosodic level of speech, in Godøy, R. and Jørgensen, H., eds., *Musical Imagery*, pp. 91–113.
- Cohen, D. and Wagner, N. (2000) Concurrence and non-concurrence between learned and natural schemata, *New Music Research* 29(1), 23–36.
- Cone, E. T. (1987) On derivation, syntax and rhetoric, *Music Analysis* 6(3).
- Conner, T. A. (1994) Techniques for manipulating the internal structure of mosaics, in *Theory Only* 12(7–8), 15–34.
- Cooper, G. and Meyer, L. B. (1960) *The rhythmic structure of music*, Chicago: University of Chicago Press.
- Danielou, A. (1943) *Introduction to the study of musical scales*, London: The Indian Society.
- David, H. T., and Mendel, A., eds. (1946) *The Bach reader*, London: J. M. Dent.
- Gabrielsson, A. and Juslin, P. (1996) Emotional expression in music performance: Between the performer's intention and the listeners' experience, *Psychology of Music* 24(1).
- Gauldin, R. (1983) The cycle-7 complex: Relations of diatonic set theory to the evolution of ancient tonal system, *Music Theory Spectrum* 5, 39–55.
- Guletsky, I. (1995) Proportion in Palestrina's masses, in *Proceedings of the Third International Conference of Study of "Palestrina e L'Europa"*, Palestrina, October 1994.
- Hargreaves, D. J. (1986) *The developmental psychology of music*, Cambridge University Press.
- Howell, P., Cross, J. and West, R., eds. (1985) *Musical structure and cognition*, London and New York: Academic Press.
- Huron, D. (1996) On the kinematics of melodic contour: deceleration, declination and arch-trajectories in vocal phrases, in *Proceedings of the 4th ICMPC*, Montreal.
- Kamien, R. (1990) The slow introduction of Mozart Symphony no. 38 in D, K. 504 ('Prague'): A possible model for the slow introduction of Beethoven's Symphony no. 2 in D, op. 36, *Israel Studies in Musicology* 5, 113–131.
- Kempf, D. (1996) What is symmetry in music?, *International Review of the Aesthetics and Sociology of Music* 27, 155–165.
- Koch, H. C. (1983, 1782) *Introductory essay on composition: The mechanical rules of melody, sections 3 and 4* (translated from German by Baker, N. K.), New Haven: Yale University Press.
- Koppel, M., Atlan, H. and Dupoy, J. P. (1987) Triviality and alienation in systems: Proof of Von Forster's conjecture, *International Journal of General Systems*.
- LaRue, J. (1973) *Guidelines for style analysis*, New York: Norton.
- Le Huray, P., and Day, J., eds. (1981) *Music and aesthetics in the eighteenth and early-nineteenth centuries*, Cambridge University Press.
- Leichtentritt, H. (1951) *Musical form*, Cambridge, Mass.: Harvard University Press.
- Lendvai, E. (1972) Duality and synthesis in the music of Béla Bartók, in Kepes, G., ed., *Modules, Symmetry, Proportion*, 174–193, London: Studio Vista.
- Leyton, M. (1992) *Symmetry, causality, mind*, Cambridge, Mass.: MIT Press.
- McAdams, S., ed. (1987) *Music and psychology: A mutual regard*, London and New York: Harwood Academic Publishers.

- Nettl, B. (1964) *Theory and method in ethnomusicology*, London and New York. Free Press of Glencoe, Collier-MacMillan.
- Oppenheim, D. U. (1989) DMix: An environment for composition, in *The Proceedings of International Computer Music Conference*, Columbus, Ohio.
- Palisca, C. V. (1968) *Baroque music*, Englewood Cliffs, New Jersey: Prentice-Hall.
- Perle, G. (1974) *Serial composition and atonality*, Berkeley: University of California Press.
- Powel, N. W. (1979) Fibonacci and the golden mean: Rabbits, rumbas, and rondeaux, *Journal of Music Theory* 23(2), 227–273.
- Powers, H. (1976) The structure of musical meaning: A view from Banaras, *Perspectives of New Music* 14(2) and 15, 308–334.
- Rameau, J. P. (1722) *Traité de l'harmonie*, Paris: Ballard, English translation (1965) New York: Broude Brothers.
- Randel, D.M., ed. (1986) *The New Harvard Dictionary of Music*, Cambridge, Mass.: Harvard University Press.
- Ratner, L. (1980) *Classic Music: Expression, form, and style*, New York: Schirmer Books.
- Reale, P. V. (1970) The process of multivalent thematic transformation, Ph.D. dissertation, University of Virginia.
- Reti, R. (1951) *The thematic process in music*, New York: MacMillan.
- Rosen, C., Influence, plagiarism and inspiration, *Nineteenth Century Music* 4(2), 87–100.
- Sachs, C. (1946) *The commonwealth of art*, New York: Norton.
- Schachter, C. (1976) Rhythm and linear analysis, *The Music Forum* 4, 281–334.
- Schillinger, J. (1948) *The mathematical basis of arts*, New York: Philosophical Library.
- Shanon, B. and Atlan, H. (1990) Von Forster's theory: Semantic application, *New Ideas in Psychology* 9, 81–90.
- Sloboda, J. (1991) Music structure and emotional response: Some empirical findings, *Psychology of Music* 19(2), 110–120.
- Solomon, L. (1973) New symmetric transformations, *Perspective of New Music* 11/12, 257–264.
- Strunk, O. (1950) *Source readings in music history*, New York: Norton.
- Tversky, A. (1977) Features of similarity, *Psychological Review* 84(4), 327–352.
- Vieru, A. (1993) *The book of modes*, Bucharest: Editura muzicola A uniunii compozitorilor ci muzicologilor din Romania.
- Werker, W. (1922) *Studien über die Symmetrie im Bau der Fugen*, pp. 268–280, Leipzig.
- Weyl, H. (1952) *Symmetry*, Princeton, N.J.: Princeton University Press.
- Wilkinson, D. (1988) Symmetry in art and nature, *Queen's Quarterly* 95(1), 107–132.
- Wilson, D. (1986) Symmetry and its love-hate role in music, *Comp. & Maths* 128(1/2), 101–112.