

MILTON BABBITT'S DUET IS A STUDY OF SYMMETRY

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Abstract: *This article discusses the symmetry in the piano composition "Duet" of the American composer Milton Babbitt. Through an in-depth analysis of the mathematical means as revealed in the various parameters of the "Duet"'s musical structure, we are able to deduct the extraordinary symmetries of the piece's serial technique*

In 1956, when the Duet was composed, I was an undergraduate at Juilliard School of Music in New York City and it was then that I first came across the music of the American composer Milton Babbitt. He had just finished *Two Sonnets for clarinet*, baritone, viola, and cello in celebration of Juilliard's 50th anniversary. Since, time has passed for me discretely, but that first impression of Babbitt still wears well and has not changed.

The experience of Babbitt's quartet was like the math problems which I used to solve riding my bike to High School. The difference today is that I am still looking for the answers to the quartet's puzzle and indeed Babbitt's overall work. Leibnitz writes that some mathematical processes of certain quantities are "a wonderful flight of God's spirit"; in fact, there are these quantities and qualities which are explicable with numbers, yet there are those others which are in essence inexplicable and not measurable. We do not have proper explanations for certain audible and observable orders.

For Betty Ann

Duet

MILTON BABBITT

The musical score for 'Duet' by Milton Babbitt is presented in five systems. The first system includes a tempo marking of $\text{♩} = 120 \text{ MI MI}$ and a performance instruction '(una p corda)'. The score is written for piano (p) and violin (v). Dynamics include *pp*, *p*, *mf*, *f*, and *ppp*. Articulations such as accents, slurs, and fingerings (e.g., 1, 2, 3, 4, 5) are used throughout. The key signature has one sharp (F#), and the time signature is 4/4.

Example 1: Milton Babbitt, *Duet*.

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The *Duet* (dedicated to Betty Ann, Milton Babbitt's then young daughter) was composed for piano solo and it just lasts 30 seconds, yet it flows through a remarkable sound-space. Every detail of its deployment outlines an attractive architecture of freshly imagined steps and jogs. Many of its aspects are accountable; there will remain the inescapable question of how the *Duet* arrives to be. Why does it start on Ab4 and D5? Why this interval, the pitches themselves, their registration, their sound path inception; the dialogue of the two hands that this compounded initiative energizes? We might continue with our wonder and wander. Why did the composer select that very particular array of twelve-pitch sets (P0-P6, R6-R15, R11-I11, I5-R0) to design the *Duet*? (In musical set theory, P0 stands for the prime form distribution of 12 different pitches or the entire chromatic collection; R0 stands for the retrograde form of the P0; I0 stands for the inversion of the P0; and R10 stands for the retrograde of the inversion. This is based on a system of music composition first used by the Austrian composer Arnold Schoenberg in 1925. These four mirror forms may be transposed eleven times each from one chromatic step upwards to eleven, thus P6 is a prime form transposed 6 chromatic steps. Altogether there are 48 different original sets available to the composer for the composition of a piece of music using this system.) This array has very crucial set-statistics: **2 chromatic steps describe** the set's structure; it is this interval which divides the hexachord, trichords, and begins/ends the eight sets present in the entire *Duet*. The class interval missing in the set is the 6, but heard throughout the combinatoriality of the two hands in the opening measures (mm. 1-5, sets P0-P6); and in the last measures, not as incisive, between sets I5-R0. Thus, the *Duet* opens and cadences vertically with the same tritone interval, Ab4-D5 and D4-Ab4.

Let $X\{u, v, w, x, y, z\} + n$ be the first hexachord of a Prime set
 where n stands for the missing interval among the pitches and
 where $X + n = Y$ (the second hexachord of a Prime set, P0)

$X\{u, v, w, x, y, z\}$ in Babbitt's *Duet* = $A^b, G^b, D^b, E^b, B, B^b$
 where there is no interval of 6 chromatic steps among the pitches

thus $\{u\} + n = A^b + 6 = D$, $\{v\} + n = G^b + 6 = C$; etc.

thus $\{u + 6, v + 6, w + 6, x + 6, y + 6, z + 6\} = Y$ second hexacord D, C, G, A, F, E

thus $X + Y$ = Prime set or P0 of Babbitt's *Duet* $A^b, G^b, D^b, E^b, B, B^b, D, C, G, A, F, E$

The *Duet* is a study of symmetry. The sets are selected because of their group invariants: same pitches eliding the sets and the sets's characteristics mirroring each other. The registration of the 99 pitch-attacks of the eight sets (8x12 pitch-sets = 96 + three repeated pitches) are so distributed as to offer 54 pitches for register four (the middle register of the piano), half (27) of 54 for register five and one-third (18) of 54 for register three. These quantities are in harmonic mean, $(2ac)/(a+c)=b$ harmonic mean, 18:27:54.

The four pairs of twelve-pitch sets plus the three repeated pitches extend themselves into three clearly defined phrases:

- * Phrase 1 includes the pair P0-P6 and the first hexachord of the second pair R6-R15, mm. 1-7, ending on an eighth duration rest.
- * Phrase 2 completes the second pair of sets and then sounds the third pair in its entirety, R11-I11, mm. 8-14, ending again on an eighth duration rest.
- * Phrase 3 includes the pair I5-R0, mm. 14 (last quarter duration) -19, ending on a quarter rest followed by a fermata on the final measure line.

These three phrases are defined not just by the pairs of sets, but also by remarkable expressive accents:

- * Phrase 1 is rather continuous, opening with three quarters followed by an eighth triplet and consecutive eighth attacks with a *p* dynamic. It takes four measures before changing dynamics. Thus, it is quieter, more linear, and less brusque.
- * Phrase 2 begins with an upward motion from a new-low dyad of pitches (D[#]3-F[#]3), articulated staccato, reaching beyond the octave span into F4 within the same measure; from *p* dynamic to *f* on that final attack of the ascension; both hands reinforcing each other. An abrupt eighth rest at the end of the measure interrupts this gesture.
- * Phrase 3 begins again articulated with a strong stress on the left hand C[#]4. On this occasion switching to *f* followed by *p*. The total attack points of this phrase flashing back to phrase 1 as it opens the same way. The outstanding difference here is that the dynamics are constantly changing, expanding from *pp* to *ff*, and actually offering all the dynamics that the composer has chosen to color the pairs of sets.

How do the three phrases interrelate? As we have discussed above, a series of nuances touch briefly certain events of the *Duet*'s phrases. These nuances, though, closely help to clarify the phrases further:

- * Phrase 1, lasting 21 quarter-notes, opens *p* for the longest span diminishing to *pp* at the end of measure four; and for the remaining three measures of the phrase, the original *p*, twice increasing to *mf*, ending on *mf*.
- * Phrase 2, lasting 20 quarter-notes, again opens *p*; but, within its very first measure, the soaring gesture reaches a new *f* changing immediately to another new dynamic, *mp*, on the down pulsation of the next measure; ending that measure with a *p*. The next soaring gesture increasing from *mf* to *f*, again

suddenly changing to *pp* and *mp* in the only measure, m. 12 of the *Duet*, where the left hand is omitted. The phrase ends with a *crescendo* to *mf* and a *decrescendo* to *pp*.

- * Phrase 3, lasting 16 quarter-notes, opens *f* and ends *pp*. It is this phrase which introduces yet another new dynamic, *ff*, and presents eleven changes of dynamics for its nineteen total attack points, an average of one dynamic per every two attacks.

In total, the *Duet* uses six different quality dynamics. If we include the slight, short *crescendi* and *diminuendi* attached to the actual letter dynamics markings, from *pp* to *ff*, we then have 9, 14, and 11 markings for each of the three phrases. These quantities are in harmonic mean, 9:11:14.

The highest pitch of the *Duet*, A5, is heard in the first phrase as an eighth duration; the lowest pitch, D3, appears in the second phrase, also as an eighth. These two extremes are especially located in the composition. Not only are they not repeated, but the D3 is articulated staccato just like the two other attacks which anticipate this lowest point, are in the previous measure. Interruption follows after the D3 is reached from C4 through a leap of a 10 (chromatics). It is this lowest boundary which also separates the two halves of the *Duet*. The high point, on the other hand, initiates the eighth duration pulsation of the *Duet*, as it sounds immediately after the first three quarters and eighth duration triplet. It is a prominent onset projecting the highest pitch so early and almost having it merge with its environment, as it does not have a different nuance and is not articulated in any especial way. However, this A5 lies exactly at the negative Golden Section from its opposite, the D3: total quarter duration of the *Duet* is 57, if we multiply the negative Golden Section ratio of .382 times 57 we obtain 22 and it is this integer of quarter quantities which separates these poles. This is climatic! There is no markable classical climax in the *Duet*; each phrase simply becoming an invariant of the other. Climaxes exist only in the microcosm of the *Duet*'s threading of pitches as these flow in and out of steps and jogs.

The third phrase is perhaps the one accumulating the most differentiation. It narrows the space and sounds the least number of pitches (35, 40, and 24 for the third phrase). Of the 24 pitches sounded, 16 are in register four, 5 in register five, and 3 in register three. These quantities are in harmonic mean, 3:5:16. The most pitches in register five, 12, are to be heard in the first phrase. As a matter of fact, in the first three measures of the *Duet*, the right hand only plays register five. We have already mentioned the dynamics and pointed out how the third phrase opens with an accent. There are very few articulated pitches - five of them are sounding in the second phrase and the only other one opens this last phrase.

The distances between the two hands also tend to encourage differentiation among the three phrases:

- * Phrase 1 has the most number of extended intervals, thirteen, from 13 to 21 chromatics; has eight complementary intervals sounding only 8, 9 and 11; has five narrow intervals sounding only 5, 4, and 6. (Narrow intervals or class intervals from 0 to 6 in chromatic content are not reducible; complementary are the ones based on module 12 or octave from 7 to 12 complementing the narrow ones; and extended are all intervals beyond the octave.)
- * Phrase 2 has nine extended intervals stretching to 23 chromatics; has eight complementary sounding the 7; four narrow sounding 1 and 3.
- * Phrase 3 has six extended intervals narrowing to an 18; has three complementary sounding only the 10 here, the *Duet* avoids interval class 2 vertically as the horizontal lines will be rich with them; has six narrow sounding three 6, one of 1 and two of 4.

All the extended (28), complimentary (19), and narrow (15), vertical intervals are in harmonic mean, 15:19:28.

Turning to the horizontal intervals between the pitches on both separate lines, phrase 1 distinguishes itself by dwelling on narrow intervals as only six out of thirty are complementary and one an extended interval. In phrase 2, four of the wider intervals are heard at the very beginning and continuously on the right hand. The proportion is the same as in phrase 1 - nine out of thirty-two, but they are wide: five 10, one 9, and one 14. Both hands end with a 10. The right hand appears to enhance the wide intervals in the last phrase, as every other interval is wide and they equal the number of narrow (four/four). Long durations and interruptions tend to stress their soundings in this final phrase.

The *Duet* formalizes a structure of variability within its obvious symmetry and seemingly developmental drive. For instance, the first phrase is completely continuous until its end; the second phrase has two interruptions; so does the third phrase; and, of course, both the first and second phrases are interrupted at their ends. If we, however, observe the two playing hands separately, we may conclude that both hands interrupt at different places; and that the left one interrupts more and towards the end of the *Duet* is almost non-linear as it interrupts six times within the sixteen quarter duration of the third phrase. Overall, the left hand does have thirteen rests compared to the ten of the right hand. These breathing events tend to stop the temporal flow at proper moments: end of phrases, after the lowest point, to stress articulation, shift levels, at exactly the mid-point of the *Duet*, at unique dynamic levels.

The *Duet* does not really present fast activity. The slowest pulsation is the very last attack, a half duration; and the fastest is the eighth triplet. The most common is the eighth. It is the middle phrase which does not present any triplets relying wholly on the eighth pulsation. It is the last phrase, which, like with other musical aspects already

analyzed, presents all the six different pitch durations being used in the *Duet*. This being the shortest of the three phrases, it nevertheless tends to concentrate the complexity and richness that the *Duet* implies.

Similar to an architectural scheme: better, like those wondrous Gothic Cathedrals, the *Duet* shows a hierarchy of divisibility. There are not only three clearly defined phrases, but the *Duet* also falls into two exact parts in measure ten, just after the lowest pitch has been reached. At this breakthrough, both hands interrupt as they conclude their two pairs of pitch sets. The right hand hexachord cascades down into the Db4 as canonically speaking the left hand falls into that lowest D3. Yet another partition is accomplished with the Golden Section, both negative and positive. The negative occurs at the first and second phrases' division; the positive at the only measure where the left hand is quiet and the right hand, after a *pp* high point stops and restarts an articulated unique soaring line from D#4 to G#5. Finally, we have the four pairs of twelve-pitch sets, as they themselves structure yet another hierarchy.

When we draw the geometry of the pitch-sets, using the four points of start/highest/lowest/end pitches as vertexes, the resulting polygons show a vivid symmetry. Notice that just after the halfway, m. 10, and where the Golden Section (positive) occurs, m. 12, the *Duet* presents the sets mirroring each other; and that the left hand-set has no contour, as the start/lowest plus the highest/end pitches coincide. Also, how the last two figures complement their respective mirrors in the first half. In reality, each triangular shaped set has its complement in the quadrilateral polygon, except, certainly, where the Golden Section occurs, as pointed out. There is a tendency in the first half for the figures to direct themselves downwards versus the second half, where the triangle, line and polygons appear travelling onwards. It is interesting to see that the first half's figures take up the entire available space; in the second half the figures lie a distance away from the lowest limit, and there is a superimposition of the last two figures narrowing the space considerably. A variety of vertexes are at equal distances from each other, as demonstrated by the lines joining these vertexes in the example.

The *Duet* would not fit into any of the so-called modal (Medieval) or tonal (Classical) system forms; the *Duet* is simply a process of sound transformations on a canvas of symmetry. A collection of twelve different pitches in sets is conditioned to outline the process. Certain symmetrical rotations of the original collection are temporally linearized throughout a determined selected space. High and low points are constructed as to sustain dynamic symmetrical translations within the given space. There is no progression or development; rather, a series of interconnected designs juxtaposing each other, like bricks on a lateral wall. Any sequentiality is a mere result of the symmetrical offsprings. The artistic validity is on the proportion (often the mathematical harmonic means and the special geometrical means of the Golden Section), on the exact weight of the *Duet's* formation, the *Duet's* playability. The persistent, and seemingly repetitious, small partitions are a mirage. They do not repeat; they jog, they step onward and downward in a dance of joy and challenge. The *Duet* is so brief, yet it offers such a giant fortune of thinking and invention.

Inspired by and for the young hands, it could indeed be named an Etude for the Young: it separates clearly the black and white keys of the piano; it uses mostly the middle register not engaging the extremes of the piano's range; it exemplifies the two intervals which had already become so prominent in the 20th century music (class interval 1 and 6) by sounding them at important points; it is a two-part counterpoint exercise in twelve-pitch serialization; it uses an antiphonal or canonic style for the pitches spatial and temporal motion; even the metronome marking is to be easily grasped (an average of two quarter pulsations per second); it provides for dynamic differentiation, wisely offering the young performer a chance (first three measures) to learn the *p* nuance before the changes occur; uses all four mirror forms of the musical set system.

Now, at least, I know one answer in connection with Milton Babbitt's music: that it can be extraordinarily beautiful, like mathematics is beautiful.

NOTE

This essay is in honor of Prof. Ernő Lendvai. None of my work, relating mathematics and music, would have been possible without his pioneer vision and studies.

