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ASPECTS OF SYMMETRY IN MUSIC

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1. INTRODUCTION

Despite the tremendous importance of its presence or absence in the various stages of musical organization, symmetry did not garner explicit theoretical attention in the past, even when it was quite clearly used (as in Renaissance masses and some of Bach's works); this situation formed a sharp contrast to the visual arts, where much has been said about symmetry. Today, however, unlike in the past, many researchers are dealing with various aspects of symmetry in music for various purposes: for analyzing contemporary music (e.g., Baroni *et al.*, 1922; Lendvai, 1972; Cope, 1992; Scolnic, 1993; Conner, 1994) and formulation of systems of rules to be used by composers today, as a substitute, in a sense, for the tonal system (Oppenheim, 1989; Ames and Domino, 1992); for discovering symmetrical relations that exist on various levels of organization in tonal music from various periods (e.g., Lendvai, 1972; Josephson, 1991; Wolff, 1991; Vieru, 1993); and for understanding the very concept of symmetry in music (e.g., Solomon, 1973; Wilson, 1986; Vieru, 1993).

Here we shall attempt to examine the manifestations of symmetry, both in the rules governing the raw material and composition, and in the rules of the musical works, so as to shed light on the creative process in music, which represents cognitive activity, and to introduce a new tool for musical analysis, taking the stylistic ideal into account.

The stylistic ideal

Our assumption is that the various manifestations of symmetry reflect the stylistic ideal, which is a 'message' that informs the rules of the style and serves as a starting point for both the composer and the listener (Cohen, 1995b). We can describe the ideal by means of three main interrelated variables: (1) the degree to which the musical piece is related to or cut off from its environment; (2) types of directionality and complexity on various levels; (3) the extent of excitement or calm. In other words, the presence or absence of a particular kind of symmetry, the extent of asymmetry, and various breakdowns of symmetry in the course of the piece may contribute to possibilities of musical organization in keeping with the ideal of the era, culture, or composer. The main possibilities are as follows: simple or complex organization, short or long term on the immediate or = overall level, and certainty or uncertainty regarding the continuation of the progression.

2. REFLECTIONS OF SYMMETRY IN SCHEMES AND TRANSFORMATIONS

Symmetry in music is manifested primarily in two concepts that are extremely fundamental to music and to cognitive activity in general: the scheme and the transformation, which are interrelated to some extent. Transformations can act on the schemes and on realizations of the schemes up to the most immediate level, and some transformations can be thought of as schemes of principles of composition that can be realized in various ways. It should be stressed that the concept of transformation is closely bound to the two variables that are essential to any type of organization – difference and similarity – because transformation means repetition with a change defined by a particular operation.

The general definition of symmetry phenomena, which "in fact are no more than statements as to the operations that have no effect upon the systems that we consider" (Wilkinson, 1988, p. 108), includes, as we shall see, the two best known, standard forms of symmetry: mirror-image symmetry and symmetry obtained through a single repetition of any unit to produce 'balance'.

2.1 Symmetry in schemes

We can divide the schemes into natural and learned schemes (which are not necessarily arbitrary) or into schemes of compositional rules and schemes of the raw material on a more preliminary level.

2.1.1 Natural schemes of compositional rules: The convex curve and the division into 2^n units

2.1.1.a. The convex curve

This concept is also known as the 'arched form' (e.g., Green, 1965). Sometimes, however, by stressing the symmetrical structure that revolves around a central axis (e.g., *a*, *b*, *c*, *b*, *a*), one fails to distinguish between convex and concave curves. Here the term refers to the change along the time axis – a gradual increase to a single peak followed by a gradual decline. This variation can apply to any parameter that can be arranged on a scale: pitch (low-high-low), loudness (*p-f-p*), tempo (slow-fast-slow), density, and so on. A convex curve creates maximum predictability regarding the continuation of the progression (in such a curve the rate of change is constant), unlike a zigzag, flat line (i.e., no change at all), or concave curve. (Although the concave curve, too, is symmetrical, it is not bounded, and therefore we cannot predict where the musical progression begins and ends.)

A convex curve can occur on many levels. For example, for the parameter of pitch on the most immediate level, the large interval precedes the small while ascending and the small precedes the large while descending; for the parameter of duration, the rhythmic pattern starts and ends with a large value (Fig. 1).

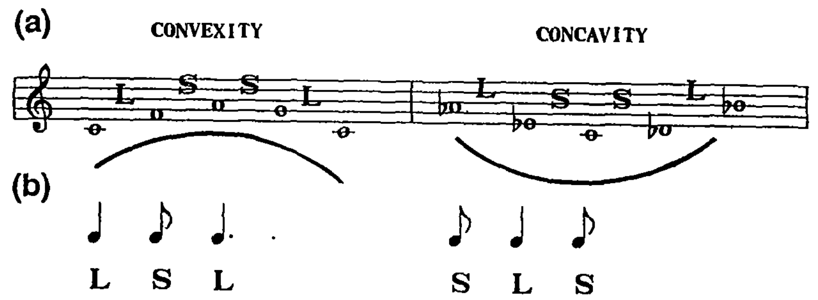


Figure 1: Convexity and concavity on the immediate level
(a) for intervals (b) for durations; L large S small

On the level of the musical phrase, there is at least one peak, neither at the beginning nor at the end, and precise repetitions and numerous large changes are avoided. Interestingly, the compositional rules of Renaissance music, in which the ideal is tranquillity and calm, require this convexity for the parameters of pitch and duration (Fig. 2; Cohen, 1971).

Figure 2: Convexity on the phrase level with respect to the parameters of pitch and duration, from the upper voice at the beginning of the motet in four voices by Palestrina.

On the other hand, some of the rules of Baroque music, in which the explicit ideal is excitement, in a sense deliberately break the rules of convexity (Buelow, 1980); thus we find multiple repetitions of a small unit: substantial, frequent changes, etc. In the Romantic period, when the ideal of excitement was particularly prominent, convexity was violated even more noticeably.

Convexity can also be manifested in extremely general superstructures; the climax of this phenomenon is reached in Palestrina's masses (Fig. 3; this was only recently discovered by Gurletsky, 1995). For the sake of comparison, let us note that Bach's huge piece *St. Matthew's Passion*, which is also symmetrical, approximately follows a concave curve and contains numerous zigzags.

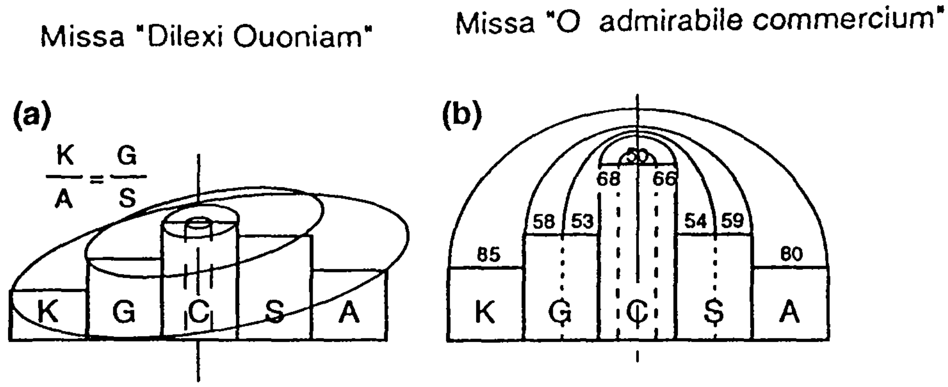


Figure 3: Convexity (here, perfect symmetry) and a "golden section" in the superstructure that uses the numbers of the measures in masses by Palestrina (Gurletsky 1995).
K = Kyrie; *G* = Gloria; *C* = Cardo; *A* = Agnus Dei.

In the overall structure of non-Western art music, the gradually rising curve is common and well known for all parameters (e.g., "Alap-Jor-Jahala" in Indian music and "Jo-ha-kue" in Japanese music). We also find the concave curve on the most immediate level (as in the *Rig Veda* hymns, considered to be the oldest music tradition – cca. 3500 years old (see Cohen, 1983). The performance of these hymns is full of tension, because it is believed that if they are not performed exactly, the harmony of the whole world will collapse.). There can, of course, be a combination of different curves on different levels; a meticulously planned combination can be found in Bartók's works (Stern, 1991). Among the evidence for the fundamentality of the convex curve is its frequent occurrence in folk music around the world (Nettl, 1965) – but not in songs, such as lamentations, that are meant to arouse excitement – and its occurrence in the calm 'syllables' of bird calls (Cohen, 1983). In states of calm the parameters of pitch and duration have concurrent convex curves, whereas in states of tension the curves are non-concurrent and deviate from convexity (Fig. 4).

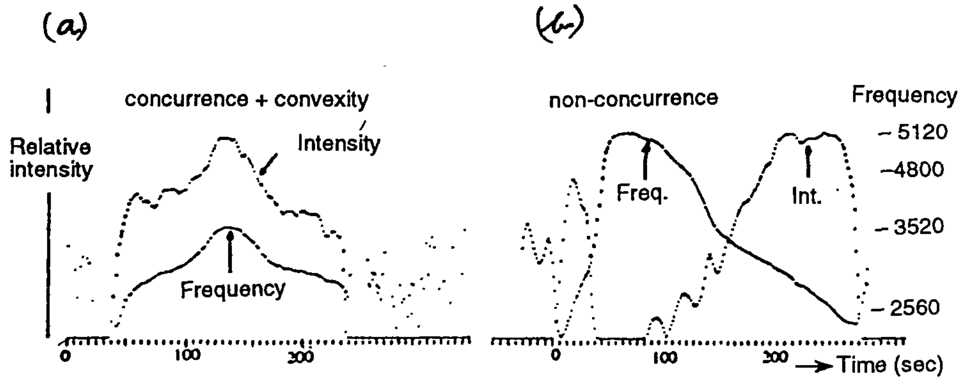


Figure 4: Two types of "syllables" in bird calls (by babblers), corresponding to two emotional states of the bird: *a* = relaxed; *b* = tense.

2.1.1.b The 2^n scheme

This scheme – which can mean either a protracted division of units into two or a single, long repetition (1 + 1 + 2 + 4 + 8 . . .), with varying degrees of precision, with or without transformation – is a natural process that, as with the convex curve, makes it possible to make predictions about the continuation of the progression. It comes as no surprise that this scheme appears in the classical period, the ideal of which calls for maximum clarity. The idea that the manifestation of this process in four-measure phrases is natural was, to my knowledge, first mentioned – but not explained – by the theoretician Koch (1782; 1983, p. 84); Lerdahl and Jackendoff (1983) also took up the issue in their theory of the rules of grouping, and most theoreticians of Western tonal music base their analyses on this kind of symmetry (e.g., Rosen, 1972; Leichtentritt, 1951).

This scheme was common in Far Eastern (especially ancient Chinese) music, and currently appears mainly in *gamelan* ('orchestral') music in Indonesia, which is structured on layers governed by a cycle of 2^n beats (*gongan*) that recurs throughout the piece, such that the range of directionality is set mainly by the range of the cycle (which is generally 32 beats but can reach 256). The melodic events on the various levels blur the 2^n structure, but this may be what makes it possible to have complex structure (within the range of a *gongan*), the tradition of which is transmitted orally.

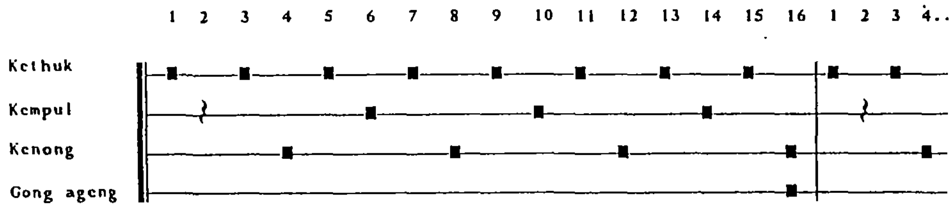


Figure 5: *Gongan* – Rhythmic pattern in *gamelan* music: 2^n beats (here, $n=4$), performed by four percussion instruments with different timbres that form an integral part of the pattern.

The 2^n structure is rare in monophonic music (e.g., Arabic, Persian, and Indian), in which the ideal calls for focusing on the moment and avoiding complex, directional, overall organization (note that the overall organization obtained by the principle of intensification in the various parameters, as noted above, is not complex); neither does it occur in African polyrhythmic music, which, on a certain level, can be viewed as consisting of $A, A, A \ldots$.

The relationship between these two processes – the convex curve and the 2^n – is interesting. Both are efficient, extreme methods of achieving predictability, but one (convexity) represents a minimum of division and levels and the other permits a maximum of levels in musical organization (Fig. 6, taken from Cohen and Wagner).

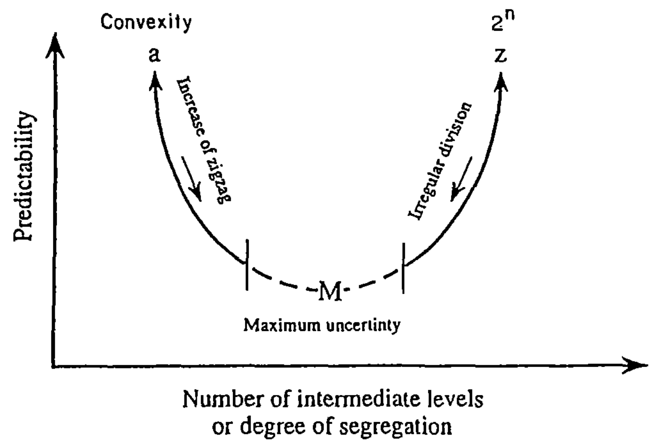


Figure 6: "Convexity" and 2^n: Two extreme solutions to maximal predictability.

Of course, maximum predictability causes boredom; and no predictability at all means no structure and no intelligibility. Music spans the entire area between these two poles, treating convexity and 2^n differently in accordance with the stylistic ideal. Moreover, adherence to 2^n resembles a ball-shaped organism, in which the cells are perfectly symmetrical and can be divided only into two identical parts. In both biology and art, to produce variety one must break up the symmetry. Thus, for example, Mozart often breaks the 2^n structure in the first theme of his sonatas, but never entirely lets go of this scheme, which usually appears in its pure form in the second theme. Wagner, on the other hand, rarely uses this scheme, so that discussion of deviations from it are irrelevant.

2.1.2 Minimal asymmetry in learned schemes in the raw material

These schemes (e.g., scales, chords, and rhythmic patterns) can usually be represented by means of hierarchical levels; each level is, in a sense, a selection from a scheme on a previous level or one realization of the scheme. For example, for the parameter of pitch in Western music, on the first level there are 12 notes in an octave; on the next level up, the diatonic system contains seven notes in the cycle; and then come the mode scale, major/minor scales, chords, harmonic phrases, and so on. The various musical cultures differ from each other as early as the first level of the raw material. In Arabic music, for example, the octave is divided into 24 parts.

Here we will only mention the scheme of the periodic scale system, the cyclicity from which the various scales are derived. In Western music, as we know, the diatonic system (only one) consists of two types of seconds, one of them double the other. They are organized as two identical tetrachords (each composed of a series of seconds: major, major, minor; or in units of semitones: 2, 2, 1). Between the tetrachords is a major second (two semitones).

2 2 1 2 2 2 1

Thus, the only system in the West contains 'minimal asymmetry' and makes possible complex organization on various levels (in accordance with the Western stylistic ideal), like the minimal asymmetry in biological systems (Atlan, 1981 and 1987). Non-Western music, on the other hand, usually has multiple systems of scales that generally include more than two sizes of seconds; some of the systems have minimal asymmetry and others have no symmetry at all. In Arabic music, for example, the *rast* system has minimum asymmetry (here in units of quarter-tones): 4, 3, 3, 4, 4, 3, 3 . . . and the *hejaz* system has no symmetry; 2, 6, 2, 4, 3, 4, 4 . . . These systems increase the possibility of complexity on the most immediate level and reduce the possibility of overall, complex, directional organization (in accordance with the stylistic ideal). Interestingly, minimum asymmetry also occurs with respect to duration, as in the skeleton of the cyclical metric pattern in African music (Arom 1991), which consists of a series of durations with the ratios

3 2 3 3 2...

This skeleton makes extremely complex rhythmic polyphony possible. Minimal asymmetry is only one of several aspects of the raw-material schemes through which the various schemes in the different styles can be examined and compared.

2.2 Symmetry in transformations

Transformations constitute a large portion of the rules of musical composition, and on various occasions some transformations have even been given their own names; the best known are inversion and retrograde. On the one hand, they can teach us about the structure and style of a specific piece, and, on the other hand, they can tell something about the creative process. Researchers have looked at them from various perspectives: for musical analysis (e.g., Reti, 1951; Reale, 1970; Cone, 1987; Conner, 1994), as a tool for musical composition, and even for illustrating symmetry (Solomon, 1973; Wilson, 1986). Note that studies that examine symmetry, a subject prominent in Lendvai's work (1972), do not always discuss transformations explicitly. Here we would like to mention briefly the nature of transformations.

2.2.1 The variables

A comparison of many analyses of Western tonal music indicated that there are about 40 different common transformations (Appel, 1992). By breaking the transformations down into their factors, we obtain five main variables: (1) the operations; (2) the parameters upon which they operate; (3) the schemes and their realizations on the various levels on which the operations act; (4) the magnitude of the change produced by the transformations; (5) the direction of change, toward more or less stability, clarity, and directionality.

2.2.2 The operations

We grouped the operations into five categories based on cognitive principles:

1. The *contrast* operation, which has numerous manifestations and is the most basic (Berlyne, 1974) and best known operation ('inversion', 'retrograde', and many other transitions between members of pairs of simple and complementary contrasts, which are so numerous in music).
2. *Shift* operations that relate to cyclical patterns (e.g., 'transposition', or obtaining mode scales from the diatonic system, or the sequence that

relates to the scale degrees, and rhythmic patterns with respect to the metric cycle and even for the beat cycle; and, in a more sophisticated manner, preserving a fixed event or scheme and shifting the whole system such that the same event means something different each time).

3. *Expansion/contraction* (or *augmentation/diminution*), which is accomplished either by inflating or contracting a complex event or scheme with respect to its various parameters or by making additions and deletions on the time or pitch axis.
4. *Unification/division* (or *fusion/segregation*) into learned and natural units (*inter alia*, the 2^n structure and the convex curve). This operation is essential for any perception and understanding (Bregman, 1990). Unification of events to form units may occur with concurrence or non-concurrence between learned schemes and natural schemes (ways of grouping). Accordingly, we obtain units of varying magnitude and with varying degrees of definability. The degree of certainty or uncertainty of the units is an important characteristic of the style and of the stylistic ideal.
5. *Substitutability* or *equivalence*, which is a prerequisite for every living language (Powers, 1976); for example, there may be equivalence between ways of accentuating a certain event in various parameters; between harmonic degrees; between 'types' of tunes in a single mode; or between various realizations of a single scheme.

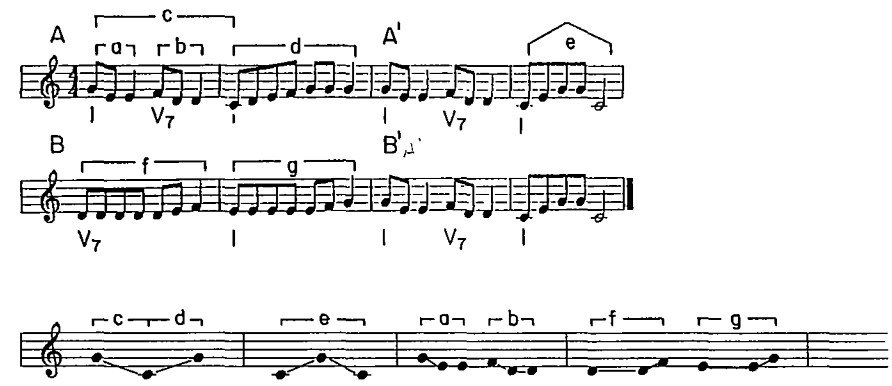
These operations are the ones that represent symmetry in music. Using them may leave the units operated on in the 'original' systems (schemes) or take them out into broader schemes. The more contracted they are, the more directional the process caused by the operation (e.g., a diatonic, rather than a real, 'mirror' or 'shift').

2.2.3 Some examples

As an example, let us look at the operations that prevail in a simple Western folk song (*Lightly Row*, Fig. 7). The most obvious structure of the song is also related to the harmonic scheme I V I (see Fig. 7d). It is obtained by combining two relatively large units, *A* and *B*, each of them two measures long (*A A' B A'*). The operations are emphasized with respect to small units (*a-g*) which, with the exception of unit *c*, end on a relatively long note (Fig. 7a). These operations apply both on the most immediate level (level 0) and on one level more abstract (level 1); the latter is represented mainly by notes located at points at which the direction of the melodic curve changes (Fig. 7b). The unity of the song is also underscored by the single repetition of a unit that increases in size according to the principle of 2^n (Fig. 7e): $a + b + \ d\ + A' \ + \ B + A'$

(a)

(b)



(c) $a \xrightarrow{(Sh_0)} b$ $c[\searrow] \xrightarrow{(I_1)} d[\nearrow]$ 3 notes $\xrightarrow{(U_0)} e$ $c+d \xrightarrow{(I_1+D)} e$ $d \xrightarrow{(U)} e$

$f \xrightarrow{(Sh_0)} g$ $a \xrightarrow{(R_1+A)} g$ $b \xrightarrow{(R_1+A)} f$ $a+b \xrightarrow{(R_1+A)} f+g$

(d) Unification into harmonic scheme: I V I

measures 1,2 (A) : I V I I] I
measures 3,4 (A'): I V I I]
measures 5,6 (B) : V V I I] V
measures 7,8 (A'): I V I I] I

(e) Unification into 2^n: 2 + 2 + 4 + 8 + 16

(in units of number of beats)

Figure 7: Various operations on units in a Western folk song:

(a) The entire song (on the immediate level 0), with the large units denoted by capital letters (A, B), the smaller units by lower-case letters (a-g), and the harmonic degrees on the most immediate level;

(b) The units on one level more abstract (level 1);

(c) The operations, marked in parentheses above the operation arrow, in capital letters and with the number of the level (0 or 1): I_1 = Inversion on level 1 (the category of contrast); Sh_0 = Shift on the immediate level; A = Augmentation; D = Diminution; R_1 = Retrograde (contrast); U = Unification of notes to form a chord scheme)

(d) and (e) Unification of the entire song to form the scheme **I V I**, and 2^n.

In systems that replace the tonal system, in part or in full, symmetry, as manifested formally in the numbers themselves, is especially important. For our purposes, let us stress that, despite the difference between tonal and atonal systems and their realizations in music, they all share the same categories of operations to express various manifestations of symmetry. For example, in the dodecaphonic system, the original 'row' is the scheme, and the operations that refer to them explicitly are *contrast* (here, only the 'mirror' for the pitch and time axes); *shift* (real, i.e., transposition); *substitution* (realizations of the pitch class in various octave states); *division* (of the line into subunits). In music theories based on set theory (e.g., Vieru, 1993; Conner, 1994), the tonal schemes are replaced by various 'sets' of notes, each with a range of realizations achieved through the various operations. Of course, what one hears is also important. An artificial organization that ignores psychoacoustic and cognitive constraints (and, especially, the meaning of similarity and difference from the auditory stand-point) is meaningless in terms of the musical structure.

3. CONCLUSION

Symmetry is most familiar, direct, and obvious in the visual domain; in music it is apparently an artificial product, so to speak, because symmetry in sound does not exist in nature. In practice, however, it is a complex natural phenomenon obtained indirectly from the relationship between musical rules and cognitive regularity. Moreover, much evidence indicates that there are numerous natural analogies between the elements of visual material and the elements of auditory material in terms of perception of them and the reaction to them (Bregman, 1990). In other words, the relationship between symmetry in music and the 'nature' external to the human being is indirect; it is reached through the relationship with the internal nature (we are part of nature) – cognition.

As we saw, symmetry can occur in schemes and transformations on various levels of the raw material, the compositional rules, and the piece itself. It is based on difference and similarity, two fundamental variables that are vital to any organization and that serve as a foundation for all cognitive activity.

Symmetry plays a part in the shaping of musical units and in predictability regarding the continuation of the musical progression, and thus also plays a part in shaping the sense of directionality, which is an important feature of the stylistic ideal. Different types of symmetry contribute to different types of directionality and thus play a part in determining how we experience music.

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