

SYMMETRY: SCIENCE & ART

**THE GOLDEN FRACTION $\tan [(1/2) \text{ARC TAN } 2]$:
A TRIBUTE TO ERNŐ LENDVAI**

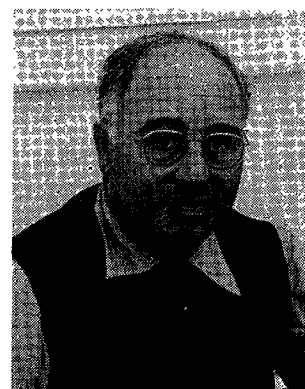
Arthur L. Loeb

Crystallographer, chemical physicist, visual artist, choreographer
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Fields of interest: Mathematical and systematic crystallography;
architecture and urban design; court and country dance; music
(performer on Renaissance and Medieval instruments); history.

Publications: *Color and Symmetry* (1971) New York: John Wiley &
Sons; Preface and contribution to R. Buckminster Fuller's
Synergetics (1975) New York: Macmillan; *Space Structure, their
Harmony and Counterpoint* (1976) Reading, MA: Addison Wesley
Advances Design Science Collection; *The Design Science
Collection* (series editor); *Crimson Heather: twenty-one Scottish
Country Dances* (written for the Harvard Scottish Country Dancers).



The Golden Fraction Φ is defined by the equation

$$\Phi = 1 / (1 + \Phi) \quad (1)$$

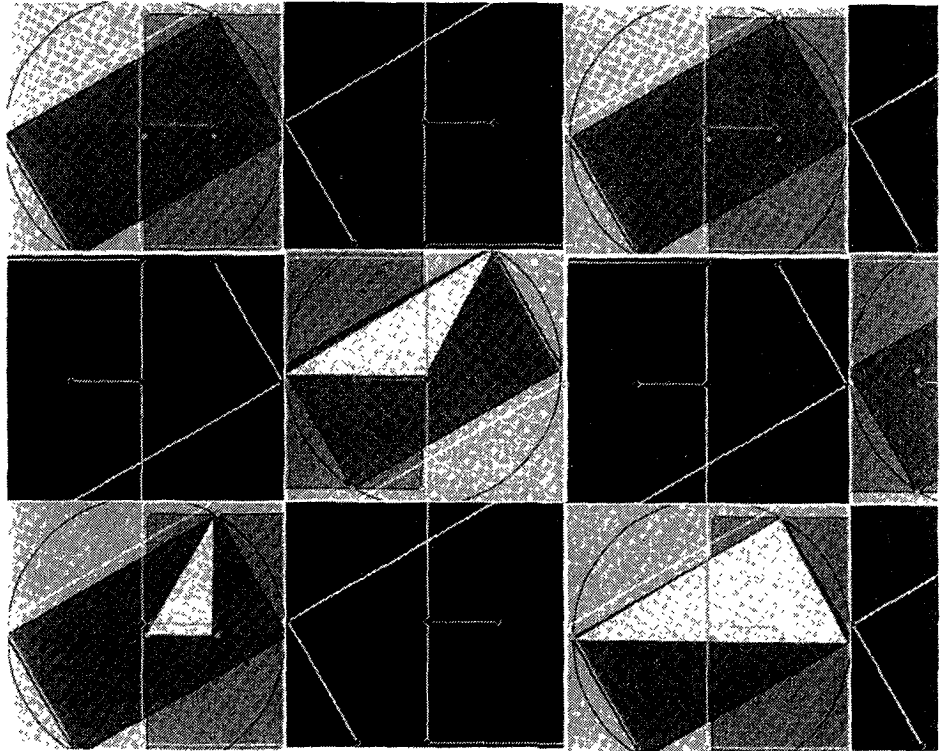
This fraction, approximately 0.618 and its reciprocal $(1 + \Phi)$ were of special interest to Ernő Lendvai. An Interesting symmetrical solution to equation (1) is:¹

$$\Phi = \tan [(1/2) \text{ARC TAN } 2] \quad (2)$$

¹ Loeb, A. L and William Varney: "Does the Golden Spiral exist, and if not, Where is its Center?", in *Spiral Symmetry*, Clifford A. Pickover / István Hargittai (eds.), Singapore: World Scientific, (1992) 47-61.

This solution is used graphically in Figure 1, a visual tribute to Ernő Lendvai.

The grey panels show a slanted golden rectangle, whose sides have lengths in the ratio equal to the golden fraction. In the lower right-hand grey panel a right triangle is highlighted whose smaller acute angle equals $(\arctan \phi)$. In the lower left-hand grey panel a right triangle is highlighted whose larger acute angle equals $(\arctan 2)$. The central grey panel shows that $(\text{ARC TAN } 2) = 2 \text{ ARC TAN } \phi$ in accordance with equation (2)



$$\tan \left[\left(\frac{1}{2} \right) \arctan 2 \right]$$

The black panels display the initials L(endvai) E(rnő). Comparison of the grey and black panels shows that the initial L is constructed of two adjacent sides of the golden rectangle, whereas the initial E is constructed from a 2 by 1 rectangle. The all-over design (discounting the highlighting) has twofold symmetry, and continues indefinitely to the eternal glory of Ernő Lendvai whose memory we honor.