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RESEARCH PROBLEMS ON SYMMETRY

RESEARCH PROBLEM 1

Two-sided wallpaper groups of periodic origami patterns

Origami is useful to make infinite periodic patterns, more precisely, to represent a finite part of them. These periodic patterns are not 2-dimensional in a strict sense, because they are not totally flat, but rather occupy a narrow layer. We may call them 2.5-dimensional patterns, continuing Coxeter's joking remark that the 7 frieze-groups (strip-groups) are 1.5-dimensional.

The 2.5-dimensional periodic patterns can be analyzed with two-sided wallpaper groups (layer groups). These were firstly enumerated by C. Hermann (1929): there are exactly 80 types. In the same year L. Weber (1929) illustrated all of them by black-and-white patterns where the colors refer to the front and the back sides, respectively (see, e.g., Shubnikov and Koptsik, 1972, where Weber's figures are reprinted). From these 80 types only 46 ones are really interesting, while the remaining 34 types are degenerate cases (i.e., the 17 strictly 2-dimensional wallpaper patterns, as those cases where only one side is used; and those further 17 cases where the above mentioned 17 patterns are mirrored to the back side). These 46 patterns are also known as two-colored (black-and-white) patterns. This interpretation with colors is useful in both fields: crystallography (where the colors refer to different physical properties, e.g., positive and negative magnetism) and design. Indeed, these patterns were discovered and studied not only by the mentioned crystallographers, but also by the textile engineer H. J. Woods (1936). His patterns are reprinted in a more recent monograph on pattern analysis of ornamental arts (Crowe and Washburn, 1988).

Question: How many of these 46 types can be represented by origami without using cuts?

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