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SYMMETROSPECTIVE: A HISTORIC VIEW

SYMMETRY AND GEOMETRIC PSYCHOLOGY

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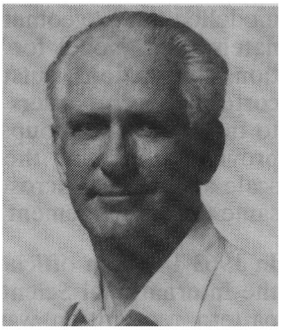

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Abstract: *An informal history is sketched of how the author came to discover the Lie group theory of neuropsychology (L.T.G./N.P.) – so-called 'Geometric Psychology'. Support for the elements of the theory is also outlined.*

Co-editor of *Symmetry: Culture and Science*, Dénes Nagy has kindly invited me to describe how it was that I came to discover the Lie transformation group theory of neuropsychology – so-called 'Geometric Psychology'.[1] A Lie group consists of a group and a differentiable ('smooth') manifold (think of a surface) wherein the group action upon the manifold – like that of a flow – is continuous and consistent with the manifold structure. Such a mathematical structure seems at first glance to be as far as possible from anything psychological or neuropsychological. Yet, recall Heraclitus dictum: *Panta rhei* – everything flows, and the physical

[1] 'Geometric' because it relies heavily on the Geometry of Systems (Mayne and Brockett, 1973). The latter is a methodology for studying systems in terms of modern differential geometry and topology, in particular, by use of Lie transformation groups.

science view (Feynman, 1965) that symmetry in its broadest sense consists of invariance under transformation.

The story goes that when Lie and Klein were graduate students together at Göttingen they decided to divide the world up between them, Lie to take the continuous groups and Klein the discrete ones. Though this was said at the time as a joke, in the intervening century or so this postulated world structure seems to have actually become realized. The role of Lie groups in physics and chemistry is an old story; that it may be no less true in the life sciences (Hoffman, 1973) and psychology has been the burden of my researches in this area for the last couple of decades.

Helmholtz was perhaps the first to see that group-theoretic actions must somehow be involved in perception. This insight was followed by further studies by Cassirer, Rashevsky, and others of group structures for perception. A major paper was that by Pitts and McCulloch (1947) utilizing finite transformations upon visual percepts modelled after "geometric objects in the sense of Kähler and Cartan." Unfortunately their model for the group actions, involving only translations and dilations/contractions, outstripped the total number of neurons available in the entire cortex by several orders of magnitude. It remained for the author (Hoffman, 1966) to introduce Lie groups into the theory of perception. The Lie group approach provides a unifying theory uniting perception at the macroscopic, psychological scale and at the microscopic, neuronal scale in a self-consistent way that is at the same time in agreement with the neuropsychological realities.

In 1963 I was an official USA delegate to the XIIIth General Assembly of URSI, the International Scientific Radio Union, in Tokyo. At that meeting the topic of pattern recognition was very big – the French, the Russians, the English, the Americans, and the Germans were all very active in this field. But the more I listened, the more what I heard seemed like statistical discrimination functions, such as those that Sebestyen and others were pursuing at the time.

So I resolved that when I returned to the United States I would look into the matter, and upon my return I wrote a report on pattern classification. I was pleased to note that in some quarters, at least, this term came to replace 'pattern recognition', as then practiced.

But then I began to wonder just how the brain actually does do visual pattern recognition. The outcome is a nice example of dialectical reasoning: the apposition of opposites, leading to a commonality – a synthesis – at perhaps a higher level.[2] This was way back in the mid-1960's, and computer models of the brain had not yet taken over.[3] In going to the literature of *visual* pattern recognition, I ran across three major themes. One of these was the psychological constancies, much

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[2] In recent work the author has shown that *symmetric difference* operation provides a seemingly perfect model for Klaus Riegel's Dialectical Psychology, and so the way that human cognition comes about.

[3] The brain is not a digital computer. In the CNS the neuronal response is graded, not all-or-none. The latter is appropriate to the case of muscle end-plate. What the brain does it does by flows along neuronal processes which are connected in intricate ways that correspond to the mathematical structure of equivalent dynamical systems, not digital computers. The latter has been referred to as "the binary fallacy".

emphasized at the time by the Gestalt school of psychology. The other psychological one was Attneave's tricoordinates. The latter are in essence a point and a direction associated with it – in mathematical terms, a bound vector field – for the elements of the contours that comprise the visual field of view. At about the same time Hubel and Wiesel had found in their electrophysiological research the so-called 'orientation response' associated with neuronal response fields of the visual cortex. The 'orientation response' is a physiological one consisting of a retinotopic point-mapping in the visual field plus a direction associated with it. Again, the character of a vector field.

The burden of the visual constancies is that such vectorfields, and the associated integral curves (visual contours) have to be invariant under the transformations imposed by viewing conditions. One recognizes Mr. Anyman down at the end of the hall just as well as close up (*size constancy*) and even when he is up-or-down, right-or-left or rotated (*shape constancy*) in the field of view, or in motion (unless he is travelling "too fast for the eye to follow"). A classic textbook of the time (Krech and Crutchfield, 1958) put it this way

> ... our perceptual experience of our worlds is characterized by an enormous degree of constancy in the over-all characteristics of objects. Mr. Anyman is perceived as Mr. Anyman with his characteristic body proportions whether he is seen lying down, standing upright, or balancing on his head. A dictionary is seen as the same massive tome from every angle of observation. ...

As noted by Von Fieandt, without the constancies, we would be living in a surrealistic world of perpetually deforming rubbery objects. Furthermore the penalty for memory storage is obvious: Every object would have to be remembered in *all* its possible views arising out of the distortions imposed by viewing conditions. Our brains would have to be the size of skyscrapers to provide the necessary memory storage. This, by the way, is the real trouble with so-called neural nets. Every time the weights on the intermediate linear transformation in a 'neural net' are changed for a new perception all previous memories are lost unless stored elsewhere. The brain does it, on the other hand, by means of Lie's fundamental theorem (Cohen, 1931). This has two advantages. It replaces the perhaps badly nonlinear transformations of form perception with a locally linear transformation, at the microscopic, neuronal scale. Secondly, its (mathematical) basis consists of the Lie derivatives defining the Lie group. The overall Lie derivative that generates any particular instantaneous perception is thus a linear combination of these Lie derivatives perpetually stored in neuronal morphology and cortical cytoarchitecture. The coefficients of the linear combination consist of those momentary psychophysical parameters that are generated by whatever stimulus is then present. The psychophysical parameters – retinal excitations, binocular convergence, accommodation, teleceptor inputs, etc. – are physiologically intrinsic to the stimulus; the Lie derivatives, on the other hand, are embodied in neuronal morphology (Hoffman, 1968, 1970, 1971, 1977, 1978, 1984, 1989), and do not change with the stimulus.[4] Only the flows change, in accord with Heraclitus' observation of many centuries ago.

So, when I began work on the L.T.G./N.P. (*the Lie group theory of neuropsychology*) there were these three things: the psychological constancies, Attneave's tricoordinates (psychological), and Hubel and Wiesel's 'orientation response' (neuropsychological). There was also the Figure-Ground Relation: Visual objects appear as visual contours that emerge from background. Visual contours are the things that remain invariant under the distortions imposed by viewing conditions and which permit economical visual pattern recognition. The basic visual contours of the constancies (Hoffman, 1966) are exactly the figures generated by stimulating the exposed visual cortex (Penfield and Rasmussen, 1950).

So it was natural to take the *states* of the visual system as the visual contours that define visual objects and the *constancy transformations* that permit invariant recognition of vectorfield-generated visual contours under the distortions imposed by viewing conditions as *Lie transformation groups*. The latter act at the local level on the visual vector field (or its dual differential form) that generates these visual contours. The cortical correlates of the visual vector field are thus to be found in the Hubel and Wiesel neuronal response fields and in standard neuronal morphology (Hoffman, 1971, 1977, 1978, 1984, 1989). What the brain does is *simulate* such computations by means of flows through appropriate neuronal morphologies, those of the corresponding local phase portraits.

Associated with the Lie derivatives that define a Lie group is a Pfaffian dynamical system. The solutions of such Pfaffian systems provide the invariants of the Lie groups – visual contours in the present instance. Such a visual contour presented to the visual system is **annulled** by the appropriate Lie derivative. Visual pattern recognition is thus more dependent upon annulment (inhibition) than registration (excitation) as such. The non-specific recruiting response from the Ascending Reticular Activating Formation floods the cortex a few milliseconds before the stimulus itself arrives, and the latter essentially cancels the former (Li, Cullen, and Jasper, 1956). The non-specific recruiting response provides the *black* blackboard for the visual stimulus. The latter appears as the *white* chalk that cancels the *blackboard*. Such an arrangement – cancellation rather than registration – provides the answer to the dilemma raised by Hume several centuries ago: How can the brain avoid becoming saturated by the tremendous flood of sensations that continually come bombarding in? Without this cancellation (inhibition) system, as opposed to a registration (pure excitation) system, sleep and anaesthesia would be impossible.[5]

[4] For many years I have resisted the quantum theory of perception, mainly because of the direct analogies that the proponents of the theory have attempted to Fourier series and the wave equation (this, in a badly inhomogeneous medium consisting of neuronal cell bodies and processes, neuroglia, clefts, and the blood flows through the latter). Yet what we do have in Lie's fundamental theorem is in essence a finite linear combination *with an appropriate basis* that is like the functional analysis representations of quantum theory. However, the Lie group action takes place along orbits (neuronal processes), not as waves through such an inhomogeneous medium as the neocortex. This inhomogeneity of the medium also rules against hologram models of cortical function. Holograms require a very high degree of coherence.

[5] In sleep and anaesthesia the non-specific recruiting response is absent.

With this mathematical model of the visual system[6] at hand I was simply able to turn the mathematical crank and a plethora of (confirmed) interpretation hypotheses followed in rapid order. All the powerful mathematical apparatus of modern differential geometry, differential topology, and the theory of dynamical systems immediately became available. Prolongations of the Lie derivatives of the constancies generate higher form vision (Hoffman, 1970, 1978, 1984) in the same way as is done in areas 18 and 19 (the psychovisual cortex) after area 17 (the visual cortex) has pre-processed a visual stimulus for constancy distortions. The differential topology of perception provides a nice explanation of the visual Gestalt (Hoffman and Dodwell, 1985). Functorial maps on the category of fibre bundles determined by the L.T.G./N.P. lead readily from form perception to the information processing theory of cognition (Hoffman, 1980, 1985). Further, the manifold properties of Riegel's (1973) Dialectical Psychology cognitive theory are well expressed by the operations of the symmetric difference (Hoffman, 1990), whose combinatorial topology is isomorphic to that of the simplicial complexes basic to information processing psychology.

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[6] Here we follow Brockett's (1977) definition of *system*: A system is a quadruple (E, X, Y, h), where $E = (M, E, p)$ is a fiber bundle; M is the state space; $X: E \to T(M)$ is a (tangent) vector field such that, for all u in $p^{-1}(m)$, $X(m, u)$ is in $T_m(M)$; Y is a differentiable manifold; and $h: M \to Y$ is a smooth map. The system inputs take on values in a fiber-over-a-given-state, i.e., in $p^{-1}(m)$. An input and a state together fix a vector field via the function X. Each point m in M determines an output $y = h(m)$ in Y.

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