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APPLICATION EXAMPLES OF THE METHOD OF SECONDARY GRID DEFORMATION

Janusz Rębielak

Architect, educator, (b. Bierutów, Poland, 1955).

Address: Department of Architecture, Faculty of Building Structures, Technical University of Wrocław, ul. B. Prusa 53/55, 50-317 Wrocław, Poland, E-mail: rebielak@plwrtu11.bitnet .

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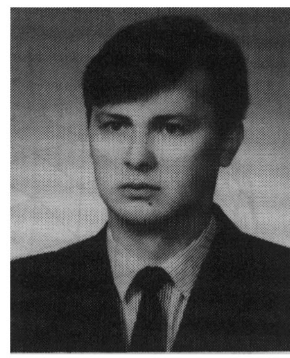

INTRODUCTION

The spherical form, considered as the perfect form, was in the past and still is applied in the architecture of buildings of great social importance. The spherical surface is a surface of two curvatures and this geometrical quality induces great rigidity allowing large structures to be constructed with relatively smaller quantities of construction materials. For these reasons the sphere is often used for large span constructions. Symmetries of this form are used in the design process of the construction and architectural form of the shell. Early domes were built with so-called traditional materials such as timber, stone, or brick. With the development of metal construction in the middle of the 19th century dome shells could sometimes obtain spans in excess of 100 meters using prefabricated elements. The application, at first, of iron and later steel has required a very economic design of the whole structure and its constituent elements. Typically the spherical shell is a bar structure where the bar pattern determines the manner of force distribution in the whole shell. Triangular grids, because of their constructional features, are most often used to describe the curvatures of the sphere surface. Grids based on other patterns are elaborated on the basis of triangular grids. A triangular spherical grid should

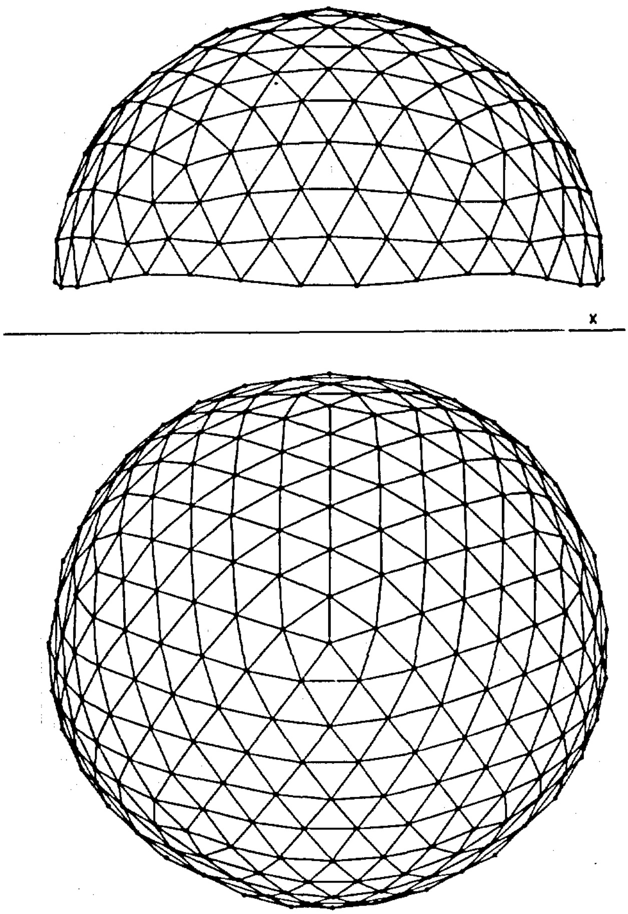

Figure 1: Exemplary shape of a geodesic dome.

describe, in the previously designed degree, the curvatures of the sphere surface, and it should consist of segments of minimum length differentiations, and the number of segments of different lengths should be as small as possible. By taking advantage of spherical symmetries the surface can be decomposed into modular elements and satisfy these requirements. In the 50s of this century geodesic domes were developed by Buckminster Fuller and attained great popularity (Fig. 1). In the design process of highly modular domes, the grid plays an important role in determining the use of prefabricated units. Due to the topology of the spherical surface symmetry must always be taken into consideration in developing these types of space grids. The process of deriving the geometric measurements in triangular spherical grids consists of the appropriate subdivision of chosen modular spherical triangles that comprise the faces of regular polyhedra, like the icosahedron and dodecahedron. The most often applied methods of deriving triangular subdivisions are presented in the paper (Makai, Tarnai, 1987).

SHORT DESCRIPTION OF THE METHOD

The method of the secondary grid transformation was developed to determine the geometric dimensions in the spherical grids spaced over various types of base projections. The inspiration for this method comes from the patterns of deformation in a crystal lattice brought about by crystal voids and interstitial atoms in metal (Penkala, 1977). The essence of the method consists in appropriate deformation of a triangular grid on the flat face of a polyhedron so that after its projection to the corresponding sphere triangle one should obtain the triangular spherical grid in

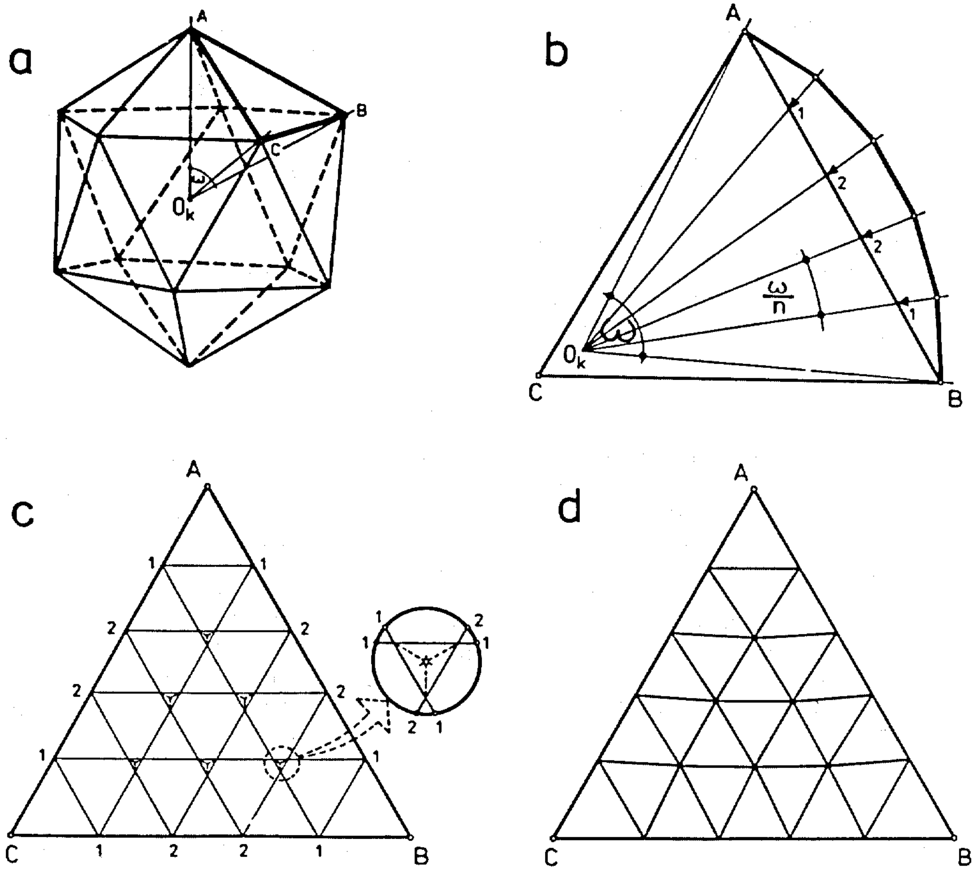

Figure 2: Scheme of determination procedure of the deformed triangular grid on the flat face of the icosahedron.

which segment lengths are slightly differentiated (Rębielak, 1983). The concept of the method of secondary grid deformation is presented on the basis of the icosahedron (Fig. 2a), (Rębielak, 1990). This polyhedron consists of twenty faces which are equilateral triangles. Therefore spherical grids worked out on its basis are built of segments of small differentiation of their lengths. The procedure of this method is as follows. In its first step the arc of great circle of the sphere, circumscribed about the icosahedron and limited by two adjacent vertices, should be divided into n equal segments (Fig. 2b). The point O_K, as the centre of the sphere, and successive points of equal subdivision of the chosen arc will determine directions of some additional segments which will define a new arrangement of icosahedron edges. This arrangement is appropriate for principles of central projection. The triangle ABC is equilateral therefore all its edges will be divided in the same way. In the next step of this method the appropriate points of edge subdivision will be joined by means of segments parallel to one of base edges of the triangle ABC (Fig. 2c). Then it is necessary to determine gravity centres of small triangular fields located between appropriate sets of segments. These points will determine position of nodes of the flat triangular deformed grid located on the face of the icosahedron (Fig. 2d). Its deformation is caused by central projection properties therefore after projection of this grid from the centre of the polyhedron we will get a spherical grid of a small degree of differentiation of its triangular fields and segment lengths. Differentiation of segment lengths is defined by the size of the quotient of the longest and the shortest segments. This quotient is marked by means of symbol η. In the presented method by using central projection this quotient, for very dense subdivisions, will approach the dimensions of edges of one of the isosceles triangles which create the regular pentagon. These regular polygons are located around each vertex of the basic polyhedron, in this case in the form of the icosahedron. Therefore the quotient η will approach the limiting value:

$$\eta_{\lim} = 2\sin 36° \sim 1{,}1756$$

It is the smallest differentiation of segment lengths possible to obtain in all of the methods of triangular subdivision of spherical surface. Therefore the method of secondary grid deformation makes possible to determine very regular spherical grids. The method of secondary grid deformation, in its construction, is convergent to the procedure presented in the paper (Clinton, 1971), which is used to create the multi-faceted polyhedrons. Applying the method of the secondary grid deformation and using the principles of the central or perpendicular projection from the plane to the spherical surface it is possible to determine the dimension relations of spherical networks spaced over any base shape (Rebielak, 1988 and 1990). Spherical grids, obtained in this way, are characterized by a small differentiation of segment lengths, creating these grids. Moreover these segments, situated in the belts "parallel" with each side edge of the basic spherical triangle, are approximately of the same lengths. For that reason the presented method can be ranked among the group of methods described in the paper (Makai, 1987).

EXAMPLES OF SUBDIVISIONS OF THE SPHERICAL SURFACE SPACED OVER A SQUARE PLANE OF BASE FORM

In this chapter there are presented proposals for applying the method of secondary grid deformation to spherical grids spaced over polygonal shapes of base projection. The first three of the proposed ways are shown on the examples of grids determined on the sphere surface, the geometrical parameters of which correspond to the parameters of the sphere circumscribed on the hexahedron (cube) and spaced over one of its square faces. There are many different ways of subdividing the part of the sphere spaced over a square base. Each way consists of choosing the repeatable form of spherical triangle or triangles in the area of the mentioned cup. It is also very important to choose the appropriate type of division for the side edges of the chosen form of triangles. In this presentation of new forms of the method of secondary grid deformation a modification of this method has been applied which uses the properties of the perpendicular projection to the plane. Constructions according to the proposed ways are naturally possible to apply in a modification of this method which uses the properties of the central projection and only needs choosing of appropriate arcs and forms of angles included between the relevant points and segments. Schemes of some elementary procedures are shown in Figure 3.

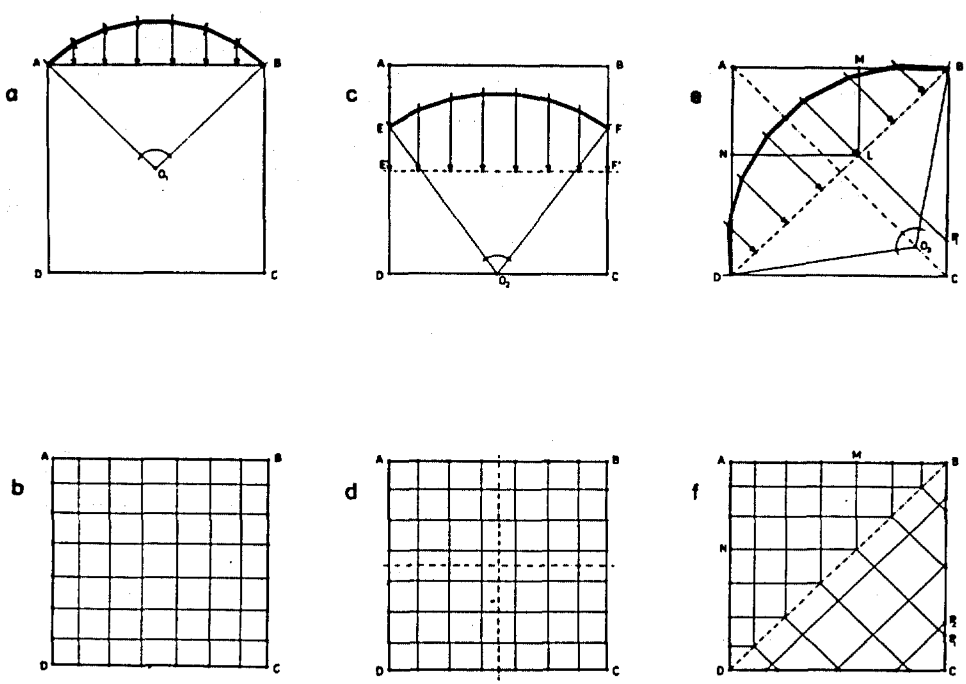

Figure 3: Examples of elementary procedures proposed for determination of the deformed grid in square plane of the base form.

The scheme of one of these procedures consisting of subdivision only of the side edges of the sphere sector spaced over the square *ABCD* into n equal segments is shown in Figure 3a. The points of equal subdivision of the arc will determine, in the perpendicular projection, the new division of the edges. The appropriate points of the edge division will be linked by means of segments parallel to each side edge. In this way a quadrangular grid of different sizes of quadrangular fields is determined onto the base plane. After determining the node localization of this grid (Fig. 3b) and its projection onto the sphere an orthogonal spherical grid is obtained. This grid can be applied as basis for defining at least two types of triangular spherical grids.

The second of the elementary procedures consists of arc divisions, which in the horizontal projection correspond to the localization of the central, orthogonal axes. The points of equal subdivision of these arcs determine, in the perpendicular projection, a new arrangement within the square form of the base projection (Fig. 3c). This new division of symmetry axes can be applied to the determination of an orthogonal grid in the field of the square *ABCD* (Fig. 3d). Nodes of this grid, by means of perpendicular projection, will be projected onto the cup spaced over this base and they will create a simple form of quadrangular spherical grid. The triangular spherical grid will be obtained through the linking of appropriate nodes by means of segments of previously chosen directions.

In the third of the elementary procedures only the arcs, situated over diagonal symmetry axes of the square *ABCD*, are subjected to the equal subdivision (Fig. 3e). A new arrangement of segments on these arcs can be used for the determination of an other shape of orthogonal or diagonal grids in the base plane (Fig. 3f). In order to obtain the diagonal grid, having only the division of diagonals of the base square *ABCD*, it is necessary to determine the middle points of segments included between points e.g., P_1 and P_2 which are situated on the boundary edges of the base square (Fig. 3f). These middle points should be the nodes of more compound form of a quadrangular grid in the base plane.

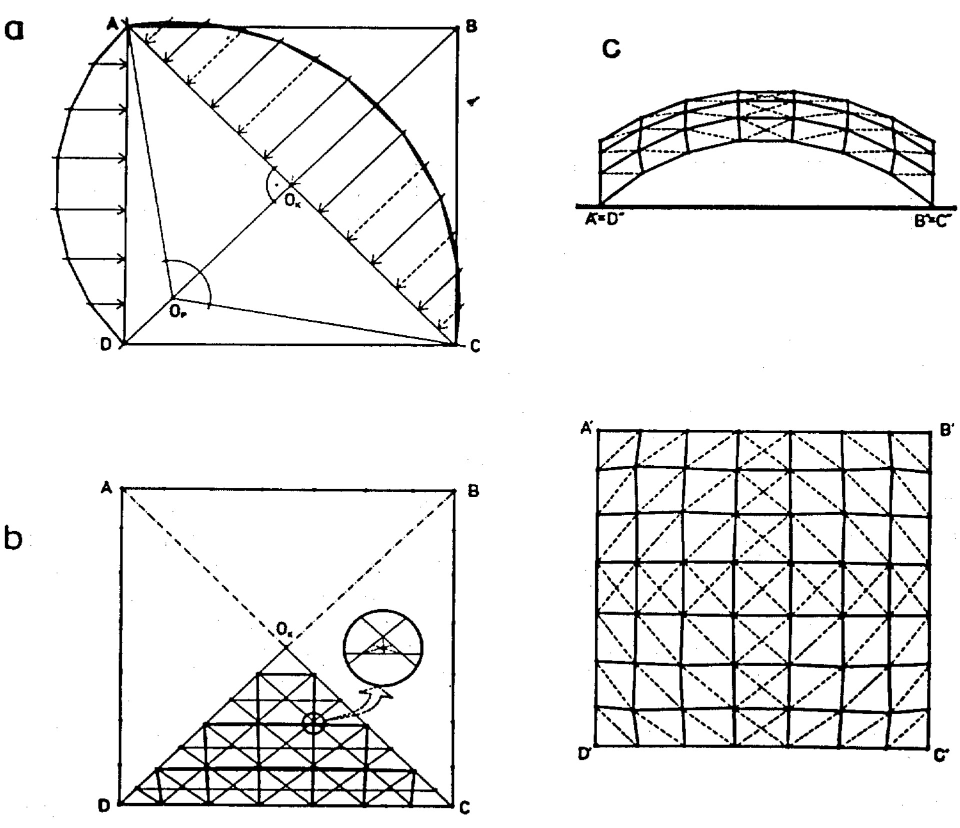

Figure 4: Spherical grid example obtained by applying two of the elementary procedures.

The method of secondary grid deformation, applying each of these three elementary procedures, can determine simple forms of spherical grids. The sphere subdivision, obtained in this way, can consist of segments of considerably great differentiation of their segments. Therefore it would be advantageous if in this method two or three of the elementary procedures could be used which would make possible the limiting of length differentiation of segments of the obtained spherical grid. The determination procedures of the spherical grids spaced over the quadratic base projection, described in (Rębielak, 1988 and 1992) provide relatively regular grids. But these grids have a somewhat complex segment system on the spherical surface. Therefore efforts were undertaken to search for other procedures of the node determination of the deformed grid in the base plane, which would make it possible to receive regular spherical grids of a clear and simple system of segments.

The first proposed method, which in its construction combines two of the elementary procedures, is shown in Figure 4. In this case it is necessary to divide the arcs, corresponding in the perpendicular projection to the positions of the side edges of the square *ABCD*, into any number n of equal parts. The arcs, corresponding to the diagonals of this square, should be divided into the number of $2n$ equal parts (Fig. 4a). In the next step the points of equal subdivision of all these arcs are mapped by means of the perpendicular projection to the base plane. After their perpendicular projection these points of the equal arc subdivision will determine the division points of the side edges of the flat triangles included between diagonals and edges of the base square form. The points of this subdivision are joined by means of straight segments. Small triangular fields are located between these segments. Now it is necessary to determine their gravity centres taking into consideration only the triangular fields, which are situated between chosen pairs of segments and between every second segment, parallel to, e.g. the edge *DC* (Fig. 4b). Figure 4c shows the shape of the spherical grid obtained in this way.

Figure 5 shows the scheme of one of the possible ways of subdivision which uses three of the elementary procedures. On account of the symmetry of the base shape the determination process of node localizations in the deformed flat grid can be restricted only to $1/8^{th}$ part of the base plan, e.g., to the field of the triangle EO_1D (Fig. 5a). In the first step of the proposed procedure it is necessary to divide the arcs, which in the perpendicular projection correspond to the location of diagonals, central segments and side edges of the square *ABCD*, into any number n of equal parts. Since these arcs are different, therefore we shall obtain at the beginning three arc segments of different lengths, which will form a part of designed spherical grid. With this respect the grids spaced over the base plan in form of an equilateral triangle are characterized with the smallest number of segments of different lengths. In the next procedure step the points of equal division of each of these arcs are mapped, by means of perpendicular projection, to the segments situated in the base plane. These points will divide the edges of the triangle EO_1D into segments of different lengths. In the area of a chosen flat triangle EO_1D one should connect the appropriate subdivision points of its side edges with straight segments in the manner, which is shown in Figure 5b. The grid created in this manner is composed of segments with small triangular fields situated between them. Having determinated the gravity centres of these small fields (Fig. 5b) we can locate the node positions of the deformed triangular grid in the base plane. The deformation degree of this network depends on the distance between the sphere surface and the base plane. It is greater for the grids spaced on the spherical surfaces which are situated in the relatively greater distance from the flat form of the base. These remarks obviously refer to all the deformated grids obtained by the method presented. When the nodes of this flat grid are projected, by means of perpendicular projection onto the spherical surface, we can get the grid of relatively small differentiation of its segment lengths. Figure 5c illustrates top and front views of such a resultant spherical surface from the example, in the patterns of which the quadrangular bar system was selected.

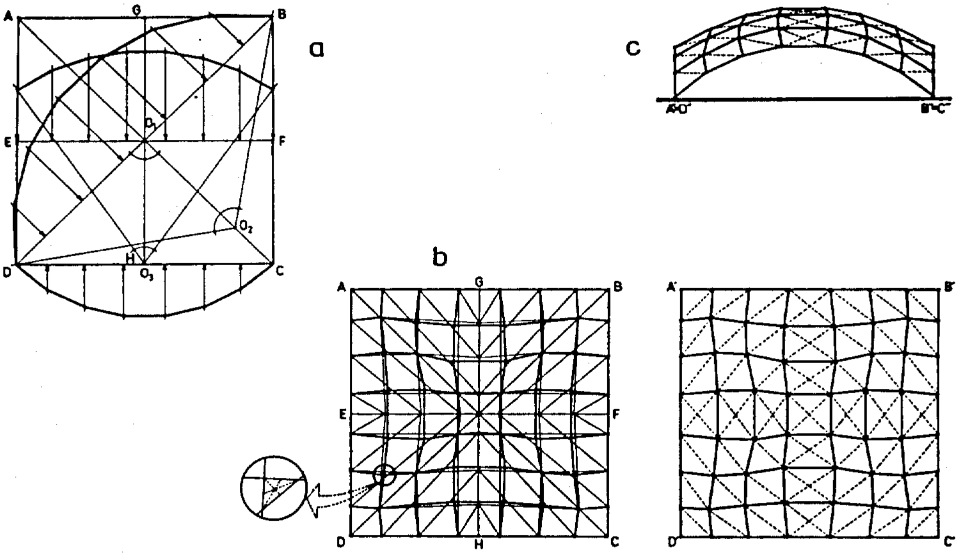

Figure 5: Determination scheme of a spherical grid using three elementary procedures.

SPHERICAL GRID SPACED OVER A CIRCULAR PLANE OF BASE FORM

Each of the two previously described methods, make it possible to generate spherical grids spaced on the surface of quadrangular cup bounded by edges in the shape of arcs located in planes perpendicular to the base plane. The form often used in practice is the shape of spherical cup spaced over the polygonal form of base projection. These polygons usually approach the shape of a circle. The construction of the method of secondary grid deformation enables determining of dimension relations of various types of grids located on spherical surfaces bounded by edges of various forms and spaced over different shapes of base projection.

Figure 6 shows the scheme of a procedure proposed for the determination of nodes in the sectors of the sphere surface. These sectors can complement the quadrangular spherical cup to the full form of the spherical cup spaced over the circular base projection. The radius of the base projection equals the radius of the circle circumscribed on the base of square ABCD. The quadrangular spherical cup has the same geometric parameters of its surface and the same density of its subdivision as in the example form presented in Figure 5. The side edges have the arrangement resulting from the perpendicular projection of the points of equal subdivision of appropriate arcs into n equal parts. In this case $n = 7$ and the arcs of the base circle, which are limited by vertices of the square $ABCD$, should be divided in the same degree. It is obvious that the angle $B'O'_PC'$ equals 90° (Fig. 6a) and its subdivision into seven equal parts will determine the division of the arc of the radius $r = O'_PB'$ into seven equal parts. This arc is located in the base plane and is bounded by points B and C. The length of the arc O_KP should be defined in the next step of the proposed procedure. This arc is a part of the great sphere circle, perpendicular to the base plane. It is limited by point O_K, which is located in the middle of the arc BC, and the position of the point P, which lies on the base plane. Point P is situated in the centre of the arc spaced between points B and C. If the number of subdivisions is an odd number (e.g., $n = 7$) the arc O_KP should be divided into a number of equal parts, which can be defined by the following relationship: $[(n - 1)/2] - 1$; which in our case equals the number two. It should be assumed that for the even number of n the proposed relationship will take the shape $(n/2) - 1$. The relationships given above have the character of recommendations and for a greater number of n these relationships should be verified. The verification will consist in carrying out the whole procedure and in the acceptation or rejection of its results, referring particularly to the degree of length differentiation of the segments obtained in this way (Rębielak, 1992).

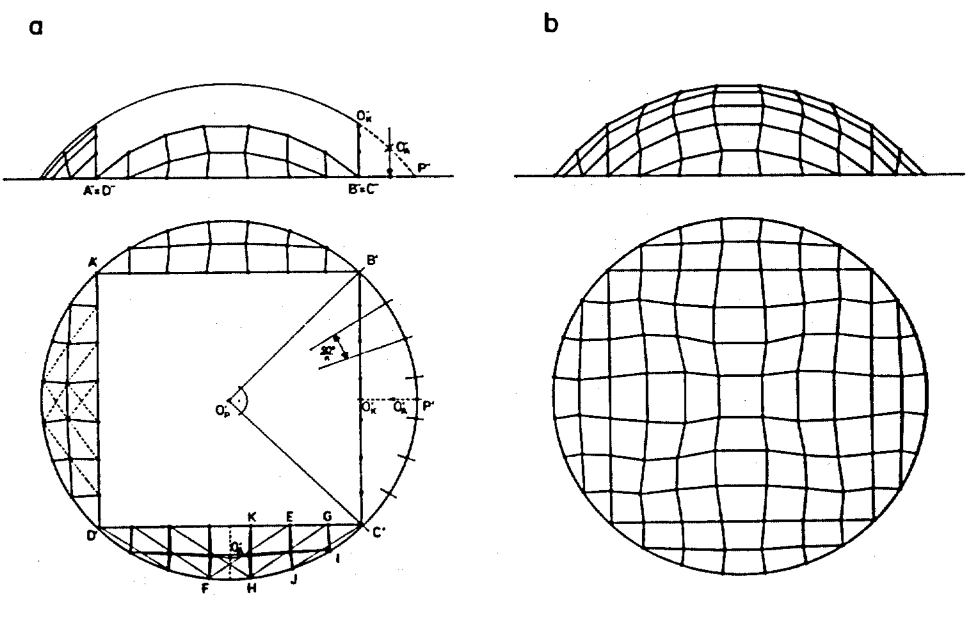

Figure 6: Example of application of the method of secondary grid deformation for determination of spherical grids spaced over the circular form of base projection.

In the next step of the proposed procedure the point O_A of the equal subdivision of the arc O_KP, determines in the perpendicular projection the localization of the point O'_A. Having the localization of this point we can now mark by letters the successive points of the edge $D'C'$ (Fig. 6a). These points are marked by letters G, E and K. The following points of equal subdivision of the arc DC are marked by letters I, J, H and F. The first group of assistance segments consists of segments located between nodes C and J, nodes G and H and between nodes E and F. The second group of segments will be included between points G and I, points E and J and points K and H. The third group consists in this case of only the one segment which is situated between nodes I and O'_A. By determining the centres of gravity of the appropriate triangular fields, except for the field in the vicinity of the I node, we can define the nodes localization of the deformed grid in the base plane (Fig. 6a). The node I after carrying out the proposed procedure remains as the same node of the new grid, which is a consequence of the type of subdivision. The node localization on the sphere surface designed will be made by means of the perpendicular projection. Figure 6b shows the results of all these stages of the procedures.

The pattern of the whole spherical grid is a result of the complement of the quadrangular spherical grid (Figure 5c) by grids obtained on the boundary areas. Also in this case, the quadrangular grid should be filled up by means of diagonal segments of appropriate chosen directions in each part of this form of spherical grid. The third of the presented ways allows designing of spherical grids in the shaping of which there can be taken into consideration not only the construction requirements but also the aesthetics aspects, referring to the architectural view of the designed cover.

CLOSING REMARKS

The method of secondary grid deformation was worked out on purpose to define regular bar grids onto sphere surface for needs of single or double-layer space structures. An appropriate computer program will make possible fast and easy calculation process of segment lengths for any type of spherical triangle subdivisions conducted by means of this method. The method of the secondary grid deformation, presented in this paper through examples of various procedures of node determination of the deformed triangular grid in the base planes of the designed domes, can be used for a wide scope of the segment length determination of the spherical grids spaced over various forms of the base.

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