

SYMMETRY IN EDUCATION

CREATIVE CEREBRAL ASYMMETRY

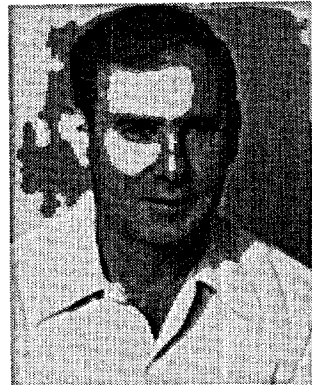
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QUESTION 1 **THE ORIGIN OF CEREBRAL ASYMMETRY**

what is
symmetry?

My cultural circle is science, namely, the human effort to define order in the kaleidoscope of phenomena. My specific field of study within the scientific system is the investigation of the cognitive function of the human brain, in particular the investigation of the cerebral mechanisms related to the cognition of mathematics and of ontology.

One of the major discoveries in this field of study is the discovery of the functional asymmetry of the human cerebral hemispheres. The left hemisphere is somewhat specialized in the analysis of details, in perceiving temporally ordered phenomena, in lingual functions, and in supervising the order of performing the movements of the hands, in particular of the right hand (each hemisphere controls the contralateral part of the body). On the other hand the right hemisphere is somewhat specialized in perceiving spatial and simultaneous phenomena, like forms. This specialization of the hemispheres is a specific human quality (it appears in a smaller degree also in humanoid apes).

Luria (1966, p. 577) suggested another, though rather similar, functional dichotomy of the human brain. Luria suggested that the central part of the left hemisphere is related to serial synthesis, while the posterior part of the left cerebral hemisphere is related to simultaneous synthesis. Luria's dichotomy can be applied in an attempt to explain the development of the hemispheric asymmetry in human beings.

It is well known that the central part of both hemispheres of the mammalian brain differentiates between temporally close auditory signals. This differentiation is so delicate that a mammal can determine the direction in space of a sound source by the minor temporal difference between the arrival times of the sound to the left and right ears. The details of a series are perceived by us one after another temporally, therefore it is natural that the central part of the brain has a role, at least a partial role, in serial synthesis. On the other hand the posterior part of both hemispheres of the mammalian brain receives visual information which is presented spatially; space is the synthesis of many simultaneously presented details. Therefore Luria's observation may be related to the rather temporal mode in which we perceive auditory data versus the rather spatial (and therefore rather simultaneous) mode of perceiving visual data.

It is possible that this functional difference between the central and posterior parts of the brain developed evolutionarily into the left-right asymmetry in the following process. When the pre-human ancestors of humanity left the trees and began to make their living by hunting, they applied their hands, which originally evolved for climbing trees, to produce and use weapons. Simultaneously they developed speech which enabled them to communicate and organize hunting parties. Both these tasks involve fine temporal analysis of muscular movements, moreover, the producing and understanding of speech involve temporal perception. Therefore they are executed by the central part of the brain.

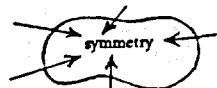
These new neural tasks apparently require a relatively large quantity of energy. The human body is not entirely symmetric, the heart is in its left side. Therefore the blood supply to the brain is not symmetric and the left hemisphere receives a larger supply of blood than the right one, see Harris (1985, pp. 242-243). Therefore these new serial tasks developed more in the left hemisphere than in the right one, i.e., there was an evolutionary advantage to individuals who executed these tasks in the left hemisphere. The cerebral hemispheres supervise the contralateral side of the body, which accounts for the preference of the right hand for motoric tasks.

The right hemisphere specialized in the less energy consuming tasks of spatial and simultaneous perception which are related to visual perception. However, some simultaneous tasks remained in the posterior part of the left hemisphere as Luria observed, see Kinsburne and Warrington (1962).

Thus the human brain developed with two cognitive functions located asymmetrically. However, the term "cognitive duality" seems to be more appropriate than the term "cognitive asymmetry" for characterizing the two cerebral functions cognitively.

QUESTION 3

CHARACTERIZATION OF THE CEREBRAL ASYMMETRY WITHIN THE CULTURAL KALEIDOSCOPE



Before answering the second question, we shall answer the third one: how the cultural surroundings influenced the meaning given to the left-right duality in human cognition by brain researchers.

There are several dichotomical characterizations of human culture. Some brain researchers tried to characterize the duality in human cognition related to hemispheric asymmetry. Bogen (1969) prepared a list of such cognitive and cultural dichotomies, which their authors related to the hemispheric *asymmetry*. He also prepared another list of dichotomies which he related to the hemispheric asymmetry, though their authors did not. Orenstein (1977) suggested a far-reaching theory that western culture is related to the left hemisphere while far-eastern culture is related to the right hemisphere. We shall see now how such dichotomies can be obtained from a single dichotomy, that is, the multi-axis asymmetry of the cultural and cognitive kaleidoscope can be reduced into a one axis asymmetry.

Bogen and Gazzaniga (1965) suggested that the functions of the left and right hemispheres are related, respectively, to the dichotomy of *verbal* versus *visuospatial* aspects of cognition and culture. This dichotomy is obtained from the observation that the left hemisphere is related to the performance of verbal and lingual functions, while the right hemisphere is related to the visual perception of forms and to the orientation in space. May be that this dichotomy was noticed since it presents two modes of transferring information, by words and by pictures; two modes of art, literature and plastic arts; and two modes of thinking, verbal versus image-creation.

Carmon and Nachshon (1971) suggested another dichotomy of the left and right hemispheres: temporal versus spatial perception, respectively. This cognitive dichotomy too is related to culture, for example temporal history versus spatial geography.

The verbal versus visuospatial dichotomy can be obtained from the temporal versus spatial dichotomy as follows: each of the lingual functions is performed one step after another temporally; speaking and hearing are performed one syllable after another, writing and reading are executed one letter after another and one word after another. Thus we see that the lingual functions are performed applying temporal analysis of one item after another. On the other hand forms are perceived visually in space.

Levy-Agresti and Sperry (1968) suggested a dichotomy based on information theory, namely, analytic processing of single items at one time by the left hemisphere versus synthesis of a new whole from several data by the right one. For example, right-brain damaged patients draw pictures with accurate details, but they cannot organize these details into forms. On the other hand left-brain damaged patients can draw general forms, but cannot draw details. Thus this dichotomy can explain the existence of two styles of drawing. The first is detail-oriented like the pictures

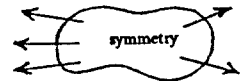
of Breughel, while the second emphasizes the form and structure of the entire picture, like the Italian Renaissance artists.

Ben-Dov and Carmon (1976) obtained the temporal versus spatial dichotomy (and through it also the verbal versus visuospatial dichotomy) from the analytic versus synthetic dichotomy of Levy-Agresti and Sperry. They argued that the analysis of details is necessarily performed one item after another temporally. On the other hand spatial presentation is the synthesis of many details presented simultaneously. Ben-Dov and Carmon argued further that when data from the external world arrive at the brain, the left hemispheric data processing mechanism presents them to consciousness organized one after another temporally, while the right hemispheric mechanism presents the same data to consciousness simultaneously, and therefore they are perceived spatially. Ben-Dov and Carmon concluded that space and time are no more than the subjective modes in which the hemispheric mechanisms present the just arrived data to consciousness. Thus they obtained from neurology Kant's doctrine about the subjectivity of space and time.

Ben-Dov and Carmon (1976) argued also that since the left hemisphere does not perceive any datum between two consecutively perceived data, time cannot be continuous and it must be quantized. This idea originates from the quantization of various entities in modern physics.

Thus Ben-Dov and Carmon (1976) showed that the whole of cognition can be explained by the analytic versus synthetic data processing dichotomy of Levy-Agresti and Sperry.

QUESTION 2 CREATION OF AN EPISTEMOLOGICAL KALEIDOSCOPE BY THE ASYMMETRIC BRAIN



Now we shall answer the second question: how the duality in human cognition, which was discovered by brain research, influenced, or can influence, the scientific and philosophical cultural spheres.

We are surrounded by an enormous number of data, which emerge into our consciousness from the physical world through our senses. The brain processes this kaleidoscope and orders it. According to the model of Ben-Dov and Carmon (1976) the construction of all our cognition is performed by the analytical and synthetic data processing mechanisms. The modes by which we perceive this information are defined by the metaphysical realistic ontologies. There are two such ontologies, namely, nominalism and Platonism. According to nominalism the phenomena are perceived as concrete and discrete details, while according to Platonism phenomena are properties which can be distinguished in the kaleidoscope of phenomena.

It was suggested by Fidelman (1985; 1987a; 1988a; 1989a) that the two ontological modes of perception exist as a result of the existence of the two hemispheric data

processing mechanisms. This concerns perceiving phenomena of experience as well as perceiving mathematical objects and ideas.

The left hemispheric mechanism extracts details out of the external information and presents them to consciousness in the nominalistic mode of perceiving phenomena. On the other hand the right hemispheric mechanism perceives wholes which are synthesized from the details. Sets, the elements of which are the details, are such wholes. According to Frege the concept of a set and the concept of a property characterizing the elements of this set are virtually equivalent, therefore we may relate the cognition of the Platonistic concept "property" to the right hemispheric mechanism.

We may conclude that while the left hemispheric mechanism perceives phenomena as discrete details, the right hemispheric mechanism perceives the same phenomena as physical properties. Therefore we may consider nominalism and Platonism to be our two only possible subjective modes of ontological perception of phenomena. Thus we extend Kant's view that while the things in themselves exist, space and time, the two extensional modes of perceiving experience, are subjective. Thus we also extended the idea of Ben-Dov and Carmon (1976) that Kant's extensional modes of perception are, indeed, subjective and related to the hemispheric mechanisms, to the ontological modes of perception.

An experiment in which this idea was tested is described by Fidelman (1989a). Subjects with a dominant left hemisphere preferred a nominalistic ontological approach to the physical world, while subjects with a dominant right hemisphere preferred a Platonistic ontology.

It was argued by Fidelman (1987a; 1988a) that the duality in physics also originates from the two cerebral mechanisms. The left hemisphere can perceive only concrete and discrete objects, and it interprets the physical world as comprising particles. Moreover, according to Ben-Dov and Carmon (1976), time is necessarily quantized. Since the left hemisphere can perceive phenomena only one after another in time, this quantization of time is necessarily extended to every physical dimension. On the other hand the right hemisphere cannot perceive discrete details, therefore it necessarily perceives phenomena as continuous and spatial. The dual perception of physical phenomena as quanta or elementary particles on one hand and as continuous waves on the other hand, are two possible interpretations of the things in themselves by the hemispheric mechanisms. May be that these two mechanisms evolved evolutionarily so that they suit the thing in themselves, but we cannot know for sure.

Interactions between the hemispheres construct complex cognitive structures. According to the model of Ben-Dov and Carmon (1976), the output of each hemispheric mechanism is available to the other one as an input. For example, children who learn to read perceive each letter as a form by the right hemisphere, which integrates it from its details. Then the left hemisphere perceives the letters one after the other temporally. At the next stage the right hemisphere integrates a word from several letters. Then the left hemisphere reads the words one after another temporally, and so on.

The letters, initially perceived by children through the right hemisphere as forms comprising details, are perceived by experienced readers through the left hemisphere as single details. The same is true regarding the reading of whole words. Shanon (1982) found that reading words in a foreign language is related more to the right hemisphere than to the left one, while reading words in a native language is related more to the left hemisphere than to the right one.

Another example is the cognition of music. While notes are perceived one after another temporally by the left hemisphere, the synthesis of chords and melodies is performed, according to Kimura (1964) and Gordon (1970), by the right hemisphere of ordinary persons. This example manifests that the ordinal synthesis related by Luria to the central part of the left hemisphere is, indeed, a combination of ordinal (in time) processing in the central part of the left hemisphere and synthesis of the series by the right hemispheric mechanism. However, according to Bever and Chiarello (1974), professional musicians perceive chords and melodies as individual items through the left hemisphere.

We may formulate a universal principle: familiarity with a whole, perceived by the right hemisphere, causes it to be perceived as a single item by the left hemisphere.

In a paper by Fidelman (1988a) this principle was applied to explain how the cosmos, i.e., the whole of phenomena, is perceived by the human brain. The holistic concept "cosmos" (which according to Einstein is finite) is integrated by the right hemisphere from details which are phenomena. However, we may expect that professional cosmologists, who deal with this concept frequently, will eventually treat it as a new single item through the left hemisphere. When this happens, the consciousness of the trained cosmologist needs a stage, i.e., extensional modes of perception, in which the new item will exist. Therefore this finite cosmos is perceived within some spatial continuum. This continuum may be inhabited by additional such items, namely additional cosmoses. It was suggested by Fidelman (1988a) that the cosmological "bubbles theory" of Linde (1983a; 1983b) may emerge from such a neuropsychological process. It should be noticed that Linde's extended cosmos emerged outside the domain of experience. Therefore the neuropsychological model for the creation of such a theory indicates the possibility that, though the dual models in physics may be alternative presentations of the things in themselves to our consciousness, the extension of experience may exist only mentally, having an ontological status similar to that of some mathematical structures.

We observe that the cerebral functional asymmetry explains epistemologically scientific theories. This explanation may have an impact on our appreciation of scientific theories and on the future evolution of science.

The hemispheric asymmetry concerns also another domain of human cognition: mathematics. There is a duality in mathematics: ordinal numbers versus cardinal numbers, series versus sets, and potential infinity versus actual infinity. A natural assumption is that the first item of each pair is related to the left hemisphere while the second item is related to the right one. This assumption was confirmed experimentally in Fidelman (1984; 1987c; 1989b).

A duality exists also in the philosophical approaches by which mathematics is founded. Kant (1964) obtained mathematics from the two modes of perceiving

experience, space and time. According to Kant ordinal arithmetic is obtained from the intuition of one after another in time, while geometry is obtained from the intuition of space. Two dual schools emerged from Kant's approach. The first is Brouwer's intuitionism which establishes all of mathematics on the intuition of one after another in time, the other is Frege's late approach of establishing the whole of mathematics on the intuition of space alone. It is explained in Fidelman (1987a; 1988a) how the temporal approach to mathematics is related to the left hemisphere, while the spatial approach is related to the right one. Therefore Kant's approach is related to both hemispheres.

There are also two extreme metaphysical realistic approaches to mathematics. The first is Frege's logicism which is Platonistic and demands the existence of sets, but not of atomic elements. The second approach is Hilbert's formalism which is nominalistic and establishes mathematics on the intuition of concrete and discrete individual objects. It is explained by Fidelman (1987a; 1988a) how Frege's logicism is related to the right hemispheric mechanism, while Hilbert's formalism is related to the left one. There is also a dualistic approach, namely, Russell's theory of types. It is a logistic approach too, but unlike Frege's approach, Russell demanded the existence of both atomic elements and sets. Therefore this approach is related to both hemispheres.

Frege's logicism collapsed because Russell's paradox was formulated within it. This paradox can be formulated as follows:

Let s be a set having the property P that all its elements are sets which do not include themselves as elements. Then s itself has this property P , otherwise one of its elements, namely, s itself, does not have the property P . Let R be the set of all sets having the property P . Since R itself has this property, it must be an element of itself. But this implies that R has not the property P , which leads to a contradiction.

It was shown by Fidelman (1987b; 1988a; 1988b) that the cognition of this and other foundational paradoxes in mathematics can be explained as follows:

The set R is integrated by the right hemispheric mechanism. Then this product of the right hemisphere is treated by the left hemisphere as a new individual element, which is not one of the elements integrated by the right hemisphere into the set R . But this element too has the property P , contrary to the definition of R . That is, the left hemisphere continues to extract additional elements having the property P after the final construction of the set of all the elements having this property by the right hemisphere. Thus the left hemisphere causes the disintegration of the set R which was integrated by the right hemisphere. This leads into a cognitive conflict. This cognitive model was confirmed in experiments described in Fidelman (1987b; 1988b).

We conclude that the foundational paradoxes are the product of a lack of coordination between the hemispheric mechanisms regarding infinite sets. In this case the hemispheric interactions have a destructive role. However, a similar hemispheric-interaction related cognitive conflict has a constructive role in proving the existence of certain infinite mathematical structures by negating their non-existence. Cantor's diagonal process is an example for such proofs.

We found that the dual hemispheric mechanisms are the source from which mathematical knowledge originates. Moreover, they construct more and more compli-

cated mathematical structures by hemispheric interaction, i.e., integration of sets by the right hemisphere from elements presented by the left hemisphere. According to Fidelman (1987c; 1988b), there is some evidence that when the right hemisphere integrates a series presented by the left hemisphere in a potentially infinite process which does not terminate, the integration of the series by the right hemisphere involves a termination of the process. This lack of coordination between the hemispheres involves a cognitive conflict which was described by Zeno in his paradoxes of *Achilles and the tortoise* as well as *the runner*. In this conflict the integration of the infinite set is accomplished when the right hemisphere "overcomes" the left one, while in Cantor's diagonal process, as well as in set theoretical paradoxes, the left hemisphere "overcomes" the right one.

In conclusion, mathematics originates from the hemispheric mechanisms, hemispheric interactions are responsible for the construction of mathematical structures, and they are responsible for the collapse of too comprehensive mathematical systems, as happened to Frege's logicism. This observation may have an impact on our understanding of what mathematics is.

The experimental results regarding the relation of mathematics to the hemispheric mechanisms can influence culture considerably through its possible application in mathematical education. In an experiment described by Fidelman (1984), grown up students studied Peano's ordinal arithmetic first, and then Frege's cardinal arithmetic. The students were examined on both arithmetics and participated in tests for the performance of the cerebral hemispheres. The results were that students with a dominant left hemisphere succeeded more in learning ordinal arithmetic, while students with a dominant right hemisphere succeeded more in learning cardinal arithmetic.

In a similar experiment described by Fidelman (1987c), students studied calculus simultaneously in the ordinary "standard" approach of potential infinity, and in the "non-standard" approach of actual infinity according to Keisler. The results were that students with a dominant left hemisphere succeeded more in the "standard" approach, while students with a dominant right hemisphere succeeded more in the "non-standard" approach.

These results can be applied to the classification of children before entering school in order to teach them arithmetic in a mode which suits their brain. It is proposed that children whose left hemisphere is more efficient than the right one will learn reading letter by letter and arithmetic based on ordinal numbers. On the other hand children whose right hemisphere is more efficient than their left one will learn reading in the global method, namely, a whole word as one form, and arithmetic based on cardinal numbers. A similar classification can be tried with university students who study calculus.

The existing educational system imposes a uniform method of learning on all the students without considering individual differences. Thus students whose brain function does not suit this method have a smaller chance of success. The contribution of the discovery of the brain's functional asymmetry to education may be the establishment of a dual educational approach which adapts the method of teaching to the student.

This educational approach will increase the knowledge level of the entire population through more efficient teaching, but it may be most important to students with an extreme difference between the efficiencies of their hemispheric mechanisms. In many cases they can hardly succeed with a method of learning which does not suit their aptitude, and they may be considered, sometimes, by their teachers as lacking learning capability. Nevertheless, they may succeed in an educational method which suits their brain. After graduating, these students may apply their unique and extreme modes of thinking and contribute unusual achievements to culture.

May be that Einstein's extreme geometrical conception of the physical world is a result of an extremely larger efficiency of his right hemisphere relatively to his left one (indeed, he was not considered by his teachers to be a brilliant student). Similarly, may be that Frege's early Platonistic approach and his later geometrical approach to mathematics are a result of a relatively more efficient right hemisphere, while Brouwer's extreme temporal and ordinal approach to mathematics is a result of a relatively more efficient left hemisphere.

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