Symmetry: Culture and Science
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SYMMETRY IN EDUCATION

ON SYMMETRY IN SCIENCE EDUCATION

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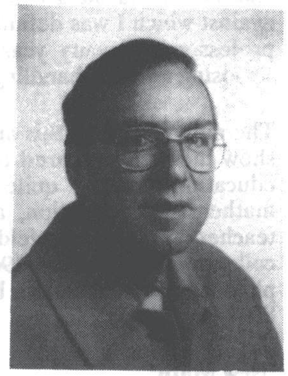

Fields of interest: History and philosophy of education (esp. 18th and 19th ct.), interdisciplinary education, philosophy of science, history of art (esp. architecture); collects graphics (fantastic baroque architecture), 20th ct. pottery.

Publications (see also list of references): Rousseau und die Verhaltensbiologie. (*Vierteljahrsschrift f. Wissenschaftliche Pädagogik* 57, 1981); Principles of environmental education as a basis of ethnospecific education. (ICSU-CTS-Conference "*Science and Technology Education ...*", Bangalore 1985, Health 32); Science theory -- Conceptual "versus" empirical study of scientific progress. (*Berichte des 11. Internat. Wittgenstein-Symposiums*, Wien 1987)

Books in preparation für 1989: *Symmetry im Unterricht; Bausteine zur Anthropologie der Wissenschaft*. (coll.papers on philosophy of science); (ed.) *Joachim Jungius und die Traditionen und Tendenzen einer "Praktischen Logik"*.

I. SYMMETRY AND CAUSALITY, OR: HOW TO BECOME AN EDUCATIONAL SYMMETRIST

Preprofessional Preludium

Since I was thirteen in 1953, the year that Wernher v. Brauns "Station im Weltall" appeared, it was certain for me that I would become a "space scientist", and I lived with these wellknown imaginations of limitless journeys through space in the same naive sense as v. Braun himself did in those days. In addition, thus preparing the next step of development, I sketched buildings of space centers, very dramatic, but completely disfunctional high tech fantasies.

In 1957 -- that admirable, sensitive starting consciously into culture age of seventeen -- I met with Frank Lloyd Wright's architecture. His spectacular structures, playing elegantly with technical and spatial fantasy, his convincing relation between repeated basic elements and their asymmetric grouping -- in fact a constant of modern architecture, but there related schematically to common reception mechanisms, with him an expression of individual, slightly zynical game -- since has continued to be a passionate love of mine. No doubt, I would become an architect! -- But it is hard to compete with F. Ll. Wright, and my own sketches rapidly demonstrated me my limits.

So when things became earnest at the end of school, the space scientist regenerated, now turned into a serious plan: to study physics (with its common obligatories mathematics and chemistry, and besides philosophy, education, and -- what remained from architecture -- history of art). When coming close to the end of this quite normal curriculum I had the good fortune to meet again, from the side of physics, with those problems I felt well attached to: with the laws of regular space and its irregular peculiarities, as they appear in optics and solid state physics. Esp. my diploma's thesis, that dealt with electron microscopy of crystal defects in semiconductors, strongly developed my feeling for physical consequences of symmetry arguments, and for their limits in crystal defects by thermic statistical laws, too. Happy enough, my institute -- led by the Clemens Schaefer disciple Johannes Jaumann -- laid great emphasis on understanding and solving problems by sophisticated common sense thinking, by symmetrical plausibilities and general laws guided trends, instead of just getting effects by formal calculation (not to speak of computers and the accompanying syndrome of letting them think instead of yourself, against which I was definitely immunized from the very beginning by my mathematics professors). Twenty years later I met with the same mixture of problems when opposing modern handling of selforganisation.

The meaning of all this autobiographic stuff, justification of its being told here, is to show me well prepared to realize the interdisciplinary importance of symmetry for education, when I quite surprisingly found myself as an assistant professor for mathematics education, after I had modestly just asked my former mathematics teacher, Kurt Honnefelder, what professional chances there were at teachers' colleges; that was in 1965. (Apart from this I prepared my doctoral thesis in philosophy of education, but this has turned out to become an independentstring of life -- seemingly).

"New Math"

In 1965, I rushed right into flourishing "New Math" in school, a fashion that had been imported to Germany from America with a few years' delay. Let us remember its two roots!

The first, which is not so important for us here, was the postulate that there was a gap between mathematics in science and mathematics in schools that should be bridged by a basically new, a truly modern concept of mathematics in school. The reasons should be national welfare (esp. successful competition with socialist states) and individual emancipation by means of what was pretended to characterize modern civilization, science; the way to do so was approximation of the topics and styles of school mathematics to those of scientific mathematics. The latter was found to be grounded on certain basic structures and constructed from them deductively, whilst the topics of old fashioned school calculating took a rather peripheral place within this modern construct, which appears systematically seen very "late", too. Exp. the set of "Bourbaki's" structures were taken as the representative stream of modern mathematics, and its basic structures like sets, topological and arithmetical relations seemed to fit well into the aim of school mathematics to construct a system of knowledge that starts with simple basic ideas.

In fact, the latter idea marks the second root of New Math, and it will lead us immediately to symmetry, too: For the Bourbaki structural development of mathematics seemed to have a parallel in the development of the minds of children, as developmental psychology claimed, of whose various streams the works of Piaget had become the most prominent.

With respect to his findings, it was not so surprising, I think -- in spite of what some educators said -- that these psychologically dealt structures could be expressed in terms of mathematics; for *if* you are in search of the structures of mind, and *if* mathematics is understood as the systematical elaboration of the structures of mind (as many philosophers ofmathematics take it), it might be *expected* that these structures of mind can be expressed in terms of their own systematization, mathematics.

Of really startling interest for educational purposes, however, was the pretended finding that there was a parallel between the *systematic* order of the construction of mathematics, in the sense of Bourbaki, and of the *chronological* order of the development of children's formal mind, as developmental psychology had pointed out. Now I called this idea an only *pretended* finding because in fact it was more something like a plausible hypothesis and an analogy, thus a program for future empirical research. Nevertheless, doubtful as it was in both respects -- really describing scientific mathematics or psychological facts(?) which both might be open for historicalchange? -- in spite of these doubts this hypothesis could have served as a basis for imaginative and fruitful curriculum construction in a normative sense just if it does *not* describea bold factum.

But we all remember what really happened instead: neither the one nor the other path, but a disastrous third one, that turned New Math a verbal formal drill, thus producing that the structural concept of school mathematics as a general concept has been omitted again and since has become a taboo on the conceptual level, though on the practical level it urged some reforms in classical syllabus of calculating, mainly towards techniques of problem solving.

I watched this misdevelopment with sorrow, for I felt very well what should have happened instead. If the structures of mathematics express the structures of mind, and were to promote them during the phase of their development (which in fact never ends, however), the development of the formal side were to happen by mechanisms of dealing with real objects -- which for other topical reasons should be "learned" -- but which parallel to this activity urge the development of formal structures in mind which may afterwards be abstracted and treated isolated if time has come to do so in a fruitful way. (This depends on age -- later on the autonomy of formal aspects might be more effective.) The educational task would have been to select, or construct, suitable objects, natural or artificial, that would promote this process best, and to accompany and steer pupils during their development. Symmetrical objects which render their structures open to the eyes immediately appear as eminently suitable for this general aim.

I met the same misunderstanding of formal objectives as topics of material learning later, when I returned to my own profession, physics, and to its education. Here, the stream of modern science for primary education had generated a lot of science-based curricula that bore the seed of formalistic misdevelopment, of "Superherbartianism" (Carl Schietzel) and verbal learning-fetishism within its very selfunderstanding -- with one exception (which nevertheless did not escape it, too); I mean the curriculum project "Naturwissenschaftlicher Unterricht in der Grundschule" of the Kai Spreckelsen group. It intended learning of scientific topics on the ground of certain basic and very general concepts (such as preservation, interrelation, corpuscularity) that characterize aspects of the resp. topics and give them an intellectual pattern that refers to that special unity of structures of mind and structures of empirical objects where it is hard to say whether these originate from one or the other side. We see: it was the same intention as that of New Math, and it seems clear -- but nevertheless did not work -- how to take place in educational reality. Structures do not "exist" autonomously, but only as the structures of those *realities* the structure of which they are, and our mind has evolved to be capable of them implicitly and work with them, and so, by working with them, happens their development, in a basic sense during all live; (furthermore however -- what makes him a *human* mind -- he is able to become conscious of these structures by abstracting them from reality and to systematize them, the result of which will be [explicit] mathematics).

Already during my solid state physics studies I had experienced the stimulating relation between topical considerations and formal "play", between strict calculation and problem solving by illustrating "with hands and feet". In mathematics education, my professor broadened this understanding by his love of group theory and its application *to* -- and vice versa its generation *by* -- ornaments. "The Wolf-Wolff" (1956) was continuously at hand and exhibited the wide horizon of symmetry in art and architecture, in biology, chemistry and formal symmetries in mathematics. We tried to implement this feeling within our students. But as far as we could see, most of them "learned" it as just another topic. So -- was it a suitable concept only for an elite, or is it of common importance? The question is still open, but the concept seems convincing, yet it has never been tested seriously.

Now an ideal is valid beyond the limits of possible reality. Symmetry as a ferment of holistic mental development -- perhaps I am the only educator who still is convinced of the charm and chances of "New Math" for school: as a possible concept to unify instruction, inner development and beauty.

Self-organisation

Meanwhile we live with another international fashion that has become a melting pot of hope for comprehensive unification of sciences under a general idea: "self-organisation", that gets of increasing importance in education, too.

By calling self-organisation, as a topic of contemporary science, a "fashion" I refer to its claims to be an new scientific stream, rendered possible only by new, during the last years developed methods (non-linear processes, "chaos"-theories), or even by complete new sciences ("synergetics"), and that it appears in the public with all features of a fashion show.

The recent discussion in the field is still quite complex and characterized by lack of clearness -- at an international workshop I participated last year, the term "non-linear" was used in at least five different meanings, without any attempts of discrimination --; if we try a bold résumé we might say that the term "selforganisation" refers to spontaneous processes of generation of order that happen far from thermodynamic equilibrium (Prigogine), thus conflicting with the 2nd thermodynamic law, that is said to postulate a general spontaneous increase of disorder in nature, which contradiction now may be explained by using new ideas of thermodynamics of irreversible processes.

In education, these ideas have become of increasing importance; they are said to have farreaching consequences as for the implementation of new topic, so for the treatment of classical topics like thermodynamics (change in valuing the basic laws), in biology (with respect to life as a spontaneous process gaining order like genetic code, metabolism...), and of course in -- what never will be far! -- philosophical "Weltbild".

I certainly will not try a systematic criticism in what is just an autobiographical sketch. I will only mark some aspects of astonishment that I developed "spontaneously" and which relate to our topic, symmetry, and are as I hope evident immediately.

- In contrast to the opinion that selforganisation is a recent topic rendered possible only by developing special (of course: computerbased) methods, modern (post-Galilean) science to a large extent just has developed *by* dealing with such processes. In certain respect we might even say that science is identical with the attempt to explain processes of selforganisation. Indeed, if we try "to explain what may happen in nature in a natural way, effected by those laws that constitute nature", this looks like a description of selforganisation, but really quotes 13th century's Albertus' Magnus definition of *science*. And if we illustrate this general scientific aim by examples like the explanation

    - of structure and genesis of planetary system (Keplers laws, Newton-Kant-Laplace theory),

    - of crystal structure and growth (see next chapter),

    - of evolution of genetic code (Watson/Crick), of life (Darwin), of culture (Herder) --

we see that they are characterized by aspects of selforganisation -- and by symmetry to a great extent.

- It seems to me a strange enterprise to try to explain processes of generation of order (whatever that may be) it is not clearly elaborated a term, anyway!) just by means of a scientific approach which explicitly was developed to install most general laws for processes of disorder; why not better start with methods constructed for order? I certainly do not say the reverse attempt must fail -- this would be contradicted by the doubtless success with suitable problems -- but it is an unnatural attempt that raises a false image in the public.

- Beyond the beforementioned problems of the style of performance, the used relation between disorder and the second law is only an analogy, not a scientific finding, because one may *never* get scientific extensions from common sense interpretations of a scientific law, but only more or less vague analogies, which afterwards need scientific proof -- or suffer refutation.

- In this case the "disorder" interpretation is wrong, for this interpretation originally has been used by Boltzmann only to render a didactically simple illustration of the simplest case in statistical thermodynamics, namely the behavior of a system of *independent* particles. Processes of selforganisation, however, do not happen with independent particles but with such that are related to each other by *forces*. It is possible, indeed, to specialize the laws of thermodynamics that way, that they comprise also the processes of interacting particles, and this certainly becomes a sportive caprice of scientists, the stronger the forces will be.

- But starting with methods specially designed for *interacting* particles, the *general* consequence will be the very *reverse* of the common interpretation of entropy's law: *The general tendency in nature are processes of self-organisation*, that are guided by the symmetries of forces of the interacting particles, differentiated by the regular variations of these symmetries, and only *disturbed* by the statistical laws of the particles' temperature dependent movement.

Thus, selforganisation leading to symmetrical structures, is an *expression of causality in nature*.

II. NIELS STENSEN AND THE SYMMETRY OF STRUCTURES, OR: THE CRISIS OF ATOMISM IN 17TH CENTURY

I want to go back now to the roots of the symmetry -- selforganisation -- relation on the occasion of a recent event. On October 23rd, 1988, Niels Stensen (Nicolaus Steno) has beendeclared a "Beatus" (that means a Saint with provisionally regional veneration) by the Roman Catholic Church. Of course, this honor was dedicated to him mainly in appreciation of his deeply religious life esp. after his conversion from protestant belief and for his merits as a catholic priest and finally as bishop in the "mission" areas of Northern Germany after Thirty Years' War. But quite naturally, this event also draw the attention of scientists on Stensens scientific merits again.

Born in Copenhagen in 1638, Stensen is well recommended as the founder of paleography, of modern geology, and of scientific crystallography, but his first, and fabulously rapid, career was in comparative anatomy, in which field he traced basically new paths in understanding the innersecretory system and the mechanisms of muscles, the brain, and the reproductive system.

In dissecting a shark's head in Florence in 1667, he noticed the relation between the shark's teeth and certain similarly looking findings ("glossopetra") from landscapes far from the sea, that hitherto had been understood as a "play of nature", but which he now tentatively identified as fossile relics of equal animals that had come to that strange place "somehow" (as Hooke in his "micrographia" had vaguely stated already two years before).

Stensen on his side used this first step of insight for travelling through the environments of Florence for the next two years, investigating the conditions of petrefaction and the laws of landscape's development and change. He published his results in a preliminary paper, the "Prodromus" (Stensen 1669) that contains the basic principles of modern geology (Scherz 1971) -- but those of crystallography as well. For Stensen, investigating the form and laws of growth of crystals, is the first to make *measurements* with crystals, and thus gets the law of constant angles between the crystals' surface planes, that, as is well known, became the cornerstone of phenomenological crystallography (other steps on its way to structural science are: the law of rational indices about 1800, the zone law, crystal systems, group theory of crystal symmetry, Laues law 1912; for details of the chemical strain see Schmidt 1987, p 147ff).

One would expect that Stensen, as some of his forerunners had already tried (or later on Haüy in fact did), would have used atomistic ideas in order to explain his law, but strange enough he did *not*, though he was very well informed about the atomistic ideas before and around him, as Schneer (1971) has pointed out. This strange fact still needs sufficient explanation, since Schneer in doing so stopped half way, and we will try to do this here, because it will give us interesting hints for the use of symmetry in science.

Christoph Meinel recently (1988) has described the development of atomistic theories through seventeenth century, changing from a deadly personal risk, for its common understanding as "materialistic", to a commonly accepted theoretical background in chemistry and natural philosophy. The arguments in favor of atomism referred to philosophical epistemology as well as, mainly, to empirical findings, but these arguments must be considered of mere declamatory character, since Meinels critical examination of them comes to the conclusion, that the epistemology was classical, the empirical facts well known since long times, and the "derivations" of atomism lacked inductive strictness.

Now, Meinels call for *strictness* in inductive arguments, for "proofs" of atomism, certainly is too strong a demand. That induction in science is no logical method in a common opinion of methodologists; rather it is a complex texture comprising deductions, plausibilities, fashions, innate laws and historical ideas, which usually leads to conviction, but rarely to a true experimentum crucis (Feyerabend 1975, Klein 1983). So the next step in understanding should be to see "by what exact mechanism the corpuscular theory, despite the obvious lack of experimental support, was able to win so many adherents among those who considered themselves empirical scientists" (Meinel 1988, p.103), but this explanation lacks, too.

Meinel in his considerations excludes the stream of crystallographical attempts towards atomism, but we should mention it here, because here things were even worse: atomism raised true difficulties which could not be overcome till late 19th century. Already in 1611, Kepler tentatively had given the well known atomistic explanation of snow crystals: a honeycomb -- like structure of densely packed spheres indeed would have given the observed hexagonal angles of snowflakes, but Kepler refuted this idea as a "nothing" not only for philosophical reasons, but also because his explanation met the problem, how then to explain the many *different* angles of other crystals, and this continued to be the problem for all similar attempts through 17th century by Descartes, Bartholin, Hooke (references see Schneer 1971), and still Dalton for his atomistic ideas had to confine himself to the laws of constant and multiple proportions, but did not manage to derive any truly explanatory structural images of molecules.

So there was a "decline of corpuscular theory during Newton's age" (Heller 1970, p. 86); nevertheless "both theories existed side by side: a doubtful atomism, and a theory of immaterial forces. The unity of physics had gone lost though physicists weren't conscious of it" (Heller 88). "Corpuscular ideas kept their popular validity, because they seemed conspicuous and simple, but their scientific power was small, because they could not be expressed in mathematical terms, and hardly could be combined with mechanical principles". (Heller 87, my transl.) This continued to be the case until late 19th century when the marked limits of mathematics as well as those of mechanicism were overcome.

Stensen was the first to move into that direction, esp. for overcoming mechanistic limits. He did so by means of his characteristic combination of sensitive empirical confinement and elegant imagination that is so typical for his scientific personality. He was too careful a methodologist and hat too much truly crystallographic knowledge about crystals (Schafranowsky 1971) as to be consent with a merely declamatory explanation of what he had *seen*; and with stupendous structural instinct he concentrates on the *surfaces* of crystals in order to explain their regular growth (Stensen 1669, ed. Scherz 1967, 68f). By considering both, the influence of the inner forces on the already existing crystal surfaces, and the possible material transport in the surrounding solution, and illustrating what happens by a "magnetic" analogy, he comes to a "hypothesis of oriented forces" to direct the growth of crystal surface (Schneer 1971, 293).

What is the meaning of this idea? Stensen, in need for an explanation of regular surfaces, neglects all those too simple atomistic suggestions of his contemporaries which he knew very well. Instead, he gives a kind of explanation which we might call a "structural phenomenological" one, and that we might declare to be the first step towards a field concept of matter -- in what respect?

Up to Stensen, the question what atoms "look like" was a matter of geometric arguments, metaphysical ones with the postulate of regular (platonic) bodies, or descriptive ones resulting hard spheres; anyway, atoms were supposed to be solid bodies in the sense of Descartes' metaphysics: hard, impenetrable substantial portions, occupying a certain spatial portion. But here, arguing had to stop, for there were no further arguments based on experience to differentiate these ideas, and there was no sensual evidence, since atoms were defined as "invisibly small". So in fact it was a correct decision to neglect all this and to confine himself to a theoretical image of qualities directly accessible to observation. Mere correctness however turned out to have been ingeniousity, since Stensen emphasized just that aspect, orientation of *forces*, which later on in fact became the basis to overcome mechanicism; but it lasted for another two centuries -- and needed all empirical data gathered since for elaboration -- until atomism could be integrated into that "Field" concept. Atoms then lost their character of being "a-tomos", i. e. indivisible entities, but became empirical "minimalia", portions of matter with soft surface and internal structure: no longer defined by sharp boundaries, but by the sphere of influence; no longer impenetrable bodies, but centers of interacting forces; no longer geometrical objects, but defined by oriented forces; in brief: defined by the *symmetries of their fields*.

Let us try some educational consequences. If we examine school text books of physics or chemistry for secondary I level on how they deal with atoms, we will find the same kind of argumentation -- or: non-argumentation -- as in 17th century science (Klein 1975, part I): They play the same role as a background theory of general validity, but as a mere information which the pupils are talked; into plausible indeed in some sort of simple persuasion by means of lack of alternatives, but far from being gained as general ideas by inductive guessing on base of firm experience, and reversely, also far from making use of these ideas by deducing macroscopic phenomenological effects.

Usually, this kind of basic atomistic information is continued with informations about scientific knowledge of the *structures* of atoms which suffer from the same insufficiency: not to be gained by active imagination about already existing knowledge nor being used for the prediction of effects; especially not accompanied by a feeling for the various aspects of a "symmetry of forces" concept of the atom.

The analysis of Stensen's problem showed us the copiousness of a restrictive yet imaginative "play" with general ideas that is not limited by what we "know for sure". Of course, the genius of Stensen, as of every great scientist, cannot be imitated; careful help of the teacher has to take its place to set pupils' mind free for critical imagination.

III. "GAMES WITH RULES", OR: SYMMETRY AS FERMENT OF MENTAL DEVELOPMENT

I want to sketch now some ideas for implementation of symmetry in education. From the beginning of my work in science education I have developed such ideas and tested them on various school levels. Only few fragments have been published yet (e. g. Klein 1980, 1986), but I hope to present a book on the subject in summer, 1989. Part of the concept recently has been integrated into the Hamburg Comprehensive School, Secondary I General Science Syllabus (Freie und Hansestadt Hamburg, 1988). The full range of possible interdisciplinary role of symmetry cannot be treated here for reasons of briefness; I want to confine myself to some remarks on primary mathematics and secondary science.

Symmetry in school should not be considered as just another topic of instruction for learning by heart; it will show its full richness only when taken as a general horizon of mental development, a vivid and multifunctional ferment of mental formation ("Bildung"). With respect to science education, two further views should act as general background ideas:

- The idea that our mind, as the medium of understanding the world, is the product of its interaction with the world; thus mind receives its shape by the experience of the world during its development, individual or general (in evolution); but reversely, that the mind also shapes the conception ofthe world, by applying its structures to the world (the idea of Kant).

- The consciousness that the empirical order in nature (e.g. symmetry), and the control of natural processes towards this empirical order by natural laws (selforganisation) are just two different aspects of the same thing.

Finally, considering the realization in school, the basic postulate would be to keep away from mere verbal instruction, but to aim for a participation of the whole personality of pupils. Esp. in lower classes, this may best be achieved, artless and very suitable to the topic, by true manipulating work with appropriate materials and techniques. Preferable exp. for scientific modeling would be types of material which do not command a *special* interpretation (e.g. "this is a NaCl-model"), but those types which, by its various "bricks", "building elements" or whatever, just show certain symmetrical qualities, but otherwise are free for concrete interpretation (to my knowledge, the most versatile material in this respect is "Geomix" from Ratec/Frankfurt).

The advantage of this feature is that constructing and modelling activities get the character of games that follow certain rules, yet have an earnest background (it is not necessary to declare a certain activity to be "art" or "mathematics" or "chemistry", not even in the mind of the teacher!) A vivid variation of the different levels of meaning seems a most important method to promote both, formal development, and topical learning.

These four basic axioms in mind, symmetry might be considered an outstanding medium of education, since it fulfils nearly all formal requirements which might be marked for fertile topics of instruction:

- It aims at *interdisciplinary* approach since it deals first with formal conditions of understanding applicable to all possible objects of experience;

- It renders *basic, constituting* objectives, thus relating structures of mind with structures of objects;

- It relates objects of learning to each other, thus rendering possible *shaped, understanding* learning;

- All formal laws raise from and remain closely related to *sensual experience*;

- A deep feeling of comfort is raised by having symmetrical orders open to our senses; this affects our sense of *beauty*;

- Learning with symmetries may be based on *action*, and action will continue to give a basis of understanding for complicated problems: the promoting unity of action, of sensual and intellectual activity in understanding will be experienced;

- These activities may be abstracted and *formalized* towards mathematics, simple enough, yet basic, thus *evolving* the mathematical interpretation of the world;

- On the other side, they are applicable for detailed *concretization*, but with basic principles as promoters;

- These problems also cover *all school levels*, but its typical subjects are not too far from traditional school topics, so that implementation would not raise serious difficulties: it is more a *new spirit* than new contents that are aimed for.

Of course, there will be, and shall be, some subjects which school hitherto does not deal with , or subjects, that will be introduced "too early", and teachers might be in fear of additional stress, and of enlargement of already overcrowded curricula. This would be an unnecessary fear, since, you remember, the intention is not to introduce problems artificially, but that the problems *introduce themselves*, so that it would be an artificial obstacle to keep them from affecting pupils' minds: the intrinsic logic and inner dynamics of the symmetries of the used materials, methods ... should (nearly) always generate the problems being treated, thus creating a sort of *selforganisation of problems*, so that the result, though it demands hard motoric and intellectual work, should be joy and relief.

I will now give a survey of topics I treated in schools on the various levels I had access to. I should illustrate them by pictures, but where to stop, or which to select! On the other hand, this restraint could be an advantage: instead of being limited, your imagination could be inspired -- so feel free in imagining!

Preschool Level

Activities should *prepare* symmetry exercises by concentrating on *topological* aspects of action and imagination in space, such as:

- Orientation in space (up -- down, left -- right, before -- behind, inclined, diagonal, through, around ...)

- Spatial relations between different objects.

- Connect different objects ("trains", garlands), let them close (chains, necklets, fences), cross each other (bridges, warps), encircle areas, erect boundaries ...

- Study qualities of simple geom. bodies: balls (look alike from all directions), bricks (different types: cubes, blocks, pyramids, bars; different views: surfaces, edges, corners); stable and unstable positions

- Generation of lines by stringing points, of planes by shifting bars (or whirling bars around your fingers, or around axes, or ...), covering space by shifting planes, asf.

Do not drill the children to do certain exercises, but let the challenge of material do the job; vary problems; embed them into "meaningful" games, into narratives, artistic activities; or, at different occasions, abstract them and make them conscious. Very important: make them speak about what they do, this gives implicit selfreference; but do not intend a canonic, or even "professional" terminology, on the contrary: intend a versatile common language, rich with associations, yet precise.

All these are well-known axioms of preschool teaching; just make them rich, and be sure they will effect implicitly what you intend.

Primary Level

The New Math reform of primary mathematics, in spite of its condemnation, in fact resulted a considerable enrichment of classical geometry and of calculating towards activities with symmetry and structures. Apart from regional differences, one nevertheless might criticize in general:

- that they do not go far enough; especially the symmetries of space usually are lacking -- for bad reason: aversion of teachers against true actions ("no time", "no materials") and the difficulty of illustrating them in textbooks. The latter however is the reason for their importance: symmetry operations in space would need a fourth dimension -- the simulation of line-symmetry in (dim n) by a true operation, turning around an axis on (dim n+1), is no longer possible -- so action in space needs also imagination.

- A further criticism is that symmetries are fixed on future *mathematical* use, but neglect the colorful enrichment they could gain by looking on biology, on arts, on music and dancing; in reverse direction, artistic activities, or the beauties and functions of biological objects, lack the deepening by structural insight, which would lead immediately into mathematics. (In general, the split into the "two cultures" caused a completely unwarranted fear of mathematics teachers to lose "strictness" by artistic fantasy, of teachers in arts, to hinder "creativity" by disciplined reflection on the process of creation -- numerous books on ornament construction in art nouveau, many of which have been reprinted recently, show the contrary.)

- Finally, the innermathematical use of symmetries in school is too poor, too: the study or the construction of ornaments is shown immediately to lead to classical geometrical problems, whilst the autonomous value of symmetry problems, e.g. first steps into group theory, are estimated low -- grace to the flop of New Math, but without reason, since group theory denotes the structures of our space of experience.

To turn to the subjects of primary instruction now, symmetry would appear especially as a matter of "mathematics", but with respect to the *media* by which it should be introduced, mainly as a matter of "art".

In fact, at this level an approach of a certain systematic character should introduce the various types of symmetry operations. First would be finite symmetries: line symmetry and rotational symmetries; their combinations constitute the full range of rotational structures and fulfil the axioms of cyclic groups. When studying them (in the sense of: playing with them), notice how originally non-symmetry problems come into consideration, too: rotational subgroups and submultiples; line symmetries of rotations, and odd and even numbers; order of rotational symmetry, and fractions and angles.

To add shifting operations (translatory symmetry) will cause infinitely extended ornaments: strings in one dimension, mosaics in the plane, finally lattices in full space. It is an enormous step for children of age eight or nine to understand what really it means to say just "and so forth", and to operate with it -- this needs careful elaboration. New types of symmetry operation will arise from this addition, fulfilling the group axioms again, now resulting the two types of (Felix) Klein's four element group; so the structure of space, insofar it is constituted by symmetry operations, is group theory on Klein's group.

Now construct regular figures by means of rotational symmetry: you get regular polygons; extend this to spread over the whole plane: mosaics; or into third dimension: polyhedra (regular or semiregular, -- the latter are at the intellectual limits of primary children).

Take the simplest and most general one of the polyhedra, a parallel-epiped, and shift it translatory into three independent direction over whole space, and children will get first experiences with lattices; construct regular aggregates of bricks, of matchboxes, cells; for oblique epipeds, take ball -- and -- rodlets-structures, they immediately render experience with symmetry centers, mirror type symmetry in space and the crystallographic law of rational indices -- this happens nearly automatically.

"Automatically" -- that is the key-word for the suitable *methods* of dealing with all these things; for indeed by no means it is intended to go ahead by direct, systematical learning; instead, all these subjects should evolve intrinsically from the structures of materials and methods, most of which, as already mentioned, with classical categories in mind would fall under the topic of "arts".

The intention with all of them is that the resulting symmetries are automatically generated by the used techniques, so that the more or less surprising effect causes the question why that is so, and what are the details. Well known in this respect, and possessing a lot of literature, generating line symmetry is paper folding, gluing, blotting; for rotational symmetry is folding through centers, puttering with straw ... Regular mosaics are constructed with "elementary ornaments" (paste board or wood) which are encircled and added to each other by continuously repeated rules of symmetry operations. Translational symmetry of all types is most easily achieved by constructions with bricks or spheres regularly pierced and tied together by rodlets (Geomix-type).

Of course, all plane ornaments may also be drawn or painted freely on paper, but then it is a conscious *decision* to regularity instead of an instrinsic automatism of elements or techniques. In some techniques, one finds even both; in Javanese batik, for instance, the restraint to regular design in spite of free "painting" technique serves as a medium of contemplation (Haake), whilst the modern, semiindustrial technique of stamping the pattern with a "cap" represents instrinsic regularity, the laws of which must be considered during production and afterwards are fixed (the same holds for European "blue print" on textiles with wooden stamps, whilst wide spread "potato print" allows both types of design.) The various techniques of weaving, by its different rules of interweaving warp and welt, render convincing examples how a mere technical process implies different symmetrical textures.

Of course, these *active* methods should be paralleled by studying an contemplating *given* ornaments and works of art, by pictures or better with the originals. Take them from murals or textiles, from potteries, weapons, furniture or stained glass. To combine the study of textures with meaning, for instance, when enjoying a Gothic window for its fine edging, tell the narrative of the pictures in the fields, too -- imagine why the medieval artist has combined them!

Secondary Level

This is the appropriate age to experience the dependance of structures in nature from symmetries, and this will last as an "open end" situation -- its ours, too.

You may start with "lattices", on basis of linear and plane translatory symmetry, as mentioned in the preceding chapter; this would be adequate if an extended phase of primary level work with symmetries lacks (as I presumed for the Hamburg syllabus).

You may also start with particles of highest symmetry, spheres, which would restitute the 17th century atomistic approach, with anticipated success today, however, since knowledge increased since to steer this process of learning. In Germany, there is a lot of conceptual work in this direction (Grosser/Bauer 1985, Schmidt 1987) which one might call a "starting chemistry with structures"-concept ("Strukturorientierter Chemieunterricht").

The principal idea is just to *put together a lot of spheres* of provisionally same size (made from styropore, wood, or metal), and to look what will happen. There will be densely packed, hexagonal plane layers, which when stacked up will result densely packed spherical lattices; there are two of them, the plane centered cubic and the hexagonal densest lattices (plus the energetically interesting bodycentered cubic).

The spheres of course are meant to represent particles of spherical symmetry, i. e. particles which commit equal forces into all directions. This is the case for electrically charged particles, ions (with stripped or added electrons) and metals (with some electrons delivered to the "electron gas").

The three mentioned lattices indeed are the typical structures of metals, and the study of lattice gaps and configuration groups will get a lot of chemical problems, while plausible arguments on stacking faults, dislocations and particle exchange lead immediately on problems of plastic flow and of alloys, thus to metallurgy and solid state physics.

Ions, to their side, are characterized by usually different particle radii, and in fact, when throwing together spheres with different size (preferred ratio of radii are 1 : 2 [for NaCl-lattice] and 3 : 4 [for CsCl-lattice]), the main types of salt crystals will immediately result.

Isolated molecules are constituted by atoms which commit directed forces, so their symmetry is low, and also shows a great variety of subtypes. But the very representative of them is the *tetrahedron* configuration, since the symmetry of four equivalent forces, which is preferred as the symmetry of the four electron pairs of the rare gas shell, shows this shape (Klein 1980). Tetrahedral forces may be represented by all carbon models, or, as a "merely symmetry"-particle, with Geomix by a tetrahedron brick or a tetrahedral pierced sphere. Consequent use of symmetrical orientation of neighbors in Tetrahedral configurations will automatically lead to wide regions of organic chemistry. Nearly all subjects of basic organic chemistry are enclosed, but also subjects which rarely if ever will appear in school -- like cyclo-alcanes or polyhedral alcanes -- and modern important materials like silicates and silicone come into view by the automatisms of modelling by means of symmetry.

The tetrahedral configurations may also be extended infinitely into three dimensions, so that there will result the structures of diamond, semi-conductors and ice, with structural approach to their interesting problems.

Parallel to these structural games, and their interpretation as and application in "classical" themes of science education there should be rich experiences and activities with topics connecting phenomena with structures as Keller (1980) proposed for crystal growth.

Let this be enough for a gross sketch of the fruitful problems which may be generated and mastered by symmetry considerations in science education. I have confined myself to natural science here for reasons both of briefness as of competence; I hope the comprehensive role of symmetry was clear enough, though.

This comprehensive role does not mean, however, that it should happen mainly within comprehensive -- so called "interdisciplinary" or "integrated" -- courses; though frequently favored, this aim usually can result only a more or less diffuse unit of a whole. In contrast, education on secondary level should make use of the full clearness and differentiation of knowledge and problem-solving that is rendered possible by the discursive nature of mind and which happens most effective "classically", namely in different topics.

"Different", however, must not mean "isolated". It was my main intention here to stress the understanding that also during those phases of parallel strings of instruction, these strings should not forget their common roots.

Symmetry then may act as a background organizer of learning that holds students' minds together and protects them from the stupidity of learning into separated boxes.

Thus, instead of becoming more or less efficient instruction, learning would keep its character as a possible medium of true cultural formation. By its intrinsic beauty, symmetry would connect knowledge with morality -- and that would be the *humane* task of Symmetry in education.

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