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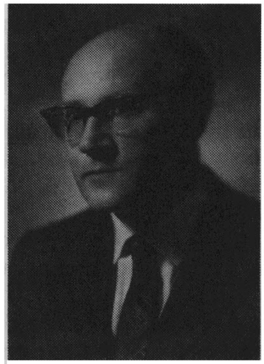
WHAT IS SYMMETRY

Vladimir A. Koptsik



Crystallographer, (b. Ivanovo, USSR, 1924).

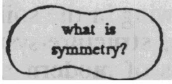
Address: Department of Physics, Moscow State University, 117234, Moscow, USSR.

Fields of interest: Crystallography, crystal physics, generalized symmetry and phase transitions theory (also systems theory, structural linguistics, visual arts).

Awards: Fedorov Prize of the Academy of Sciences of the USSR, 1973.

Publications: *Shubnikovskiye gruppy* [Shubnikov's Groups, in Russian], Moskva, 1966; *Symmetry in Science and Art*, New York: Plenum Press, 1974, [Translation of *Simmetriia v nauke i iskusstve*, Moskva: Nauka, 1972] (coauthor); Color symmetry and scaling in phase transitions and critical phenomena theory, In: *Group-Theoretical Methods in Physics*, Hamburg: Harwood Academic Publ., 1986; Generalized symmetry in crystal physics, *Comyputers and Mathematics with Applications*, 16 (1988), 407-424; Super-Fedorov symmetry of molecular modulated crystals, In: *Modern Crystallography*, New York: Nova Science Publ., 1988.

QUESTION 1



In the second part of the 20th century the classical theory of symmetry has been enriched by many new fields, e.g., antisymmetry, colored symmetry, the symmetry of multidimensional spaces, etc. The use of symmetry theory in the natural sciences – physics, chemistry, and biology – in their many ramifications has become broader and deeper. The methods of symmetry have acquired a philosophical significance: they have become some of the most general and effective methods of theoretical investigation in contemporary natural philosophy as a whole. The fundamental significance of these methods arises from their capacity to reveal the invariants of transformations and to describe the inner structure of material and ideal systems – the objects of scientific and artistic research.

The concept of symmetry has two opposing aspects: transformation (change) and conservation (invariance). That which is conserved during a change is an invariant; the set of transformations which keeps something invariant is its symmetry group or semigroup. The theory of symmetry considers that all transformations of a system are executed at the level of a certain set of elements which are equivalent in some particular respect. The set of elements and their structural relationships, forming the complete system, are conserved as a single whole.

Different ways of distinguishing the structural sublevels associated with one particular object lead to different definitions of its symmetry groups. We therefore define symmetry as the law governing the constitution of structural objects or, more precisely, as the group of automorphisms conserving the qualitative completeness of the systems under consideration.

The foregoing and the following words of my three essays were taken from the book *Symmetry in Science and Art* by A. V. Shubnikov and me (1972). A. V. Shubnikov was my teacher who taught me and many others to love symmetry in all of its manifestations.

Academician A. V. Shubnikov was one of the most brilliant scientists of the 20th century who worked in the field of the theory of symmetry and its applications. He was the first to comprehend that the concepts of symmetry of natural structures and processes were, in principle, approximate and relative and he was the first to reconstruct all of the science of symmetry.

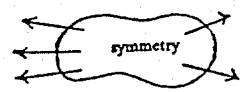
In his study he has merged together the symmetry of discrete structures, which has been of interest for classical crystallography, and the symmetry of continuous and semicontinuous media and fields. He deduced the groups of antisymmetry of finite figures, of one-, two-, and three-dimensional discontinua, continua, semicontinua and the groups of similarity symmetries. In a single real object he could see an entire hierarchy of symmetries, which are active at different structure levels, by considering them in their unity with the dissymmetries with respect to the principal groups of possible embracing transformations. At the present time, the pioneering works by A. V. Shubnikov on non-crystallographic and irrational-rotational symmetries of rods, and two- and three-dimensional semicontinua look very modern because they anticipated the present-day research on descriptions of real crystals, incommensurable crystal phases, and biological macromolecules and structures.

Growing in the soil of structural biology (and its precursor, structural crystallography), structure-system methods of investigation have gradually spread to other fields of modern natural science and art. The great generality of these methods follows from the fact that structure is a common form of the existence and development of matter. This treatment rests on the famous remarks of Lenin as to the inexhaustibility of the electron, which may also be interpreted in the guise of the inexhaustibility of the structural levels of matter.

Structure, understood in the broad sense of the word, is an invariant aspect of any integral system at a specific stage of its development. The movements of matter should be represented as a dialectic unity of moments of change and conservation. The mathematical apparatus of group theory provides an excellent reflection of this specific feature of motion. In order to define a group of transformations, in fact, we have to specify an invariant object (or system of objects), the internal structure of which consists of elements which are equivalent in a certain specified sense. On the other hand, each of the groups of transformations has its own system of conserved quantities.

After all that has been said, the reader will not be surprised to discover that Soviet philosophers treat the concept of symmetry as a philosophic category. In the theory of knowledge, symmetry enters as a method of observing and describing invariant laws (laws of conservation) arising in the form of specific modifications of the principle that matter can neither be created nor destroyed. The principles of symmetry have this to say to us: "In every material process, find the governing laws, the interrelationships, the quantities, which are conserved!" At the same time, if a model of the phenomenon in question is created, these principles make it clear to us precisely what properties are forbidden in the system so specified. It is in these properties of the principles of symmetry, reflecting causal relationships, that their heuristic role lies.

QUESTION 2



Let us repeat once more the definition of symmetry as it was given in the book *Symmetry in Science and Art* (Shubnikov and Koptsik, 1974): we define the symmetry of an object as the largest possible group of automorphism transformations mapping any integral structural object (consisting of elements equivalent in the sense of relative equality) onto itself. In this way we define symmetry as the law of composition (assembling or construction) of structural objects, or, more precisely, as the group of permissible one-to-one transformations preserving the structural integrity of the systems under consideration. A still more general concept of symmetry semigroups arises for the case of mappings of an arbitrary set X onto itself when the mappings $X \longrightarrow X$ are not necessarily one-to-one. Under certain conditions we may speak also about generalized symmetry operations such as groupoids or quasigroups of symmetry when the law of multiplication of symmetry transformations has 'multiple identities' or is not associative.

Nature knows no structureless objects. The essential generality of the category of structure, arising as an invariant aspect of a system, and the possibility (in principle) of distinguishing equivalent (in one sense or another) parts within the whole, make the concept of symmetry of just as much general significance for contemporary natural science as for art. We can consider here not just the symmetry of material objects, but also the symmetry of systems of concepts and theories reflecting the structure of the real world. This was well expressed by Newman in his introductory commentary on the excerpt from H. Weyl's book *Symmetry* that appeared in *The World of Mathematics* (Newman, 1956):

> "Symmetry establishes a ridiculous and wonderful cousinship between objects, phenomena and theories outwardly unrelated: terrestrial magnetism, women's veils, polarized light, natural selection, the theory of groups, invariants and transformations, the work habits of bees in the hive, the structure of space, vase designs, quantum physics, scarabs, flower petals, X-ray interference patterns, cell division in sea urchins, equilibrium positions of crystals, Romanesque cathedrals, snowflakes, music, the theory of relativity."

Ideas of symmetry (literally proportionality) arose among the ancient Greek philosophers and mathematicians in connection with their study of the harmony of the world. The ancient sculptors, artists, and architects created their masterpieces in accordance with the canons of harmony (the concept of which changed with the centuries). The theory of symmetry took its modern scientific form, however, only after the development of the concept of groups (Galois in 1830) and then its generalization. At the beginning of the 20th century the theory of crystal symmetry was the most highly developed, taking the form of the classical space group of rigid transformations (Fedorov, Schoenflies in 1891). After the development of crystallography and crystal physics, group-theoretical methods were applied in physics and other natural sciences. The method of symmetry has become a powerful and effective instrument of theoretical research in modern science. The problem of classification is a primary one for every science, so that symmetry, which establishes structural invariants, constitutes an essential technique for all of them.

In dealing with natural objects forming integral systems, we first note the complex composite character of their structural organization. Any material object is characterized by interpenetration, by a specific coordination or subordination of substructures. This is true, for example, of the 'elementary' particles, the internal structure of which is a problem of prime interest in contemporary physics, and of the structure of atoms (once considered 'indivisible'), which consists of a nucleus (in turn composed of nucleons) and electrons distributed in shells. Atoms and ions form the structure on the next levels of organization: molecules, crystals, structures of biological polymers. The structures of a wide variety of macroscopic objects, right up to the structures of planetary and stellar systems, galaxies and metagalaxies, are composite. A complex composite character is encountered in biological systems, beginning with those of the atomic and molecular type and ending with complete organisms and societies of organisms.

We can associate with every substructure of a system its own group of automorphisms which transforms the elements of that substructure only into one another. The relationship between the symmetry groups of substructures and the symmetry group of the system as a whole is determined by the generalized Curie-Shubnikov principle. The process of forming a composite system from nonequivalent parts is accompanied by the dissymmetrization of the system, since it leads to a reduction in the symmetry of the whole in comparison with the symmetry of the parts (Curie principle of the superposition of the groups). The opposite process, symmetrization, occurs on forming composite systems from equivalent parts (Shubnikov principle). We have combined both of them into a single principle and considered the relationships between the symmetry of causes and the symmetry of consequences in detail in a number publications.

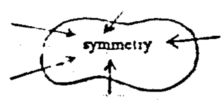
The extension of the geometric principle of dissymmetrization to physical phenomena is due to Pierre Curie. It was he who originated the famous phrase "Dissymmetry makes the phenomenon". In this connection, we should note that scientists are most disturbed by cases of so-called violation of conservation laws. However, in every case it can be shown that there is no violation of the logical axioms or of the actual principle of invariance; it is simply that research workers have discovered new, previously undiscovered relationships and that a transition has occurred to extended heterogeneous or contracted homogeneous systems. The abstract symmetry of isolated systems is preserved under the phase transitions as in the case of quasicrystals or incommensurable modulated crystalline phases.

In the course of mechanical vibrations, the particles of a crystal are displaced from their equilibrium positions, as a result of which the symmetry of the system at any particular instant is described by one of the subgroups of the space group. The space group itself emerges as a union of symmetry and dissymmetry – the result of the time averaging of the dissymmetries realized during the vibrations. This conclusion is quite general. The symmetrization-dissymmetrization processes of the instantaneous states of systems occur in every isolated physical system.

The entire process of a phase transition obeys the astonishing law discovered by philosophers (N.F. Ovchinnikov, 1966) – the symmetry compensation law: If symmetry is reduced at one structural level, it arises and is preserved at another! Metaphorically speaking, the structure of the old phase continues to exist in the new phase, but in a different form. On the other hand, the elements of the structure of the new phase already exist in the old and only appear at a phase transition.

The relationship between the conservation laws and the symmetries of physical laws, in the words of Richard Feynman, "most physicists still find somewhat staggering... These connections are very interesting and beautiful things, among the most beautiful and profound things in physics". For any physical theory in which the laws for the phenomena studied are formulated in the language of differential or algebraic equations, we can find groups of symmetry transformations of the equations and the systems of their solutions.

QUESTION 3



The concept of symmetry arises in the theory of art through the concept of structure. Art, as a depictive form of the cognition and modeling of the world around us, should reflect, and indeed does reflect, the structural aspect of that world. Structuredness is truly a wide-ranging law, a form of existence and motion of matter, and the products of scientific and artistic creativity are also subject to this law. Artistic products – literature, poetry, music, painting, architecture, etc. – have a complex artistic structure, presenting an organic interweaving and interpenetration of a variety of substructures constituting individual components of artistic expressiveness.

The extension of the concept of symmetry greatly broadens the range of its applications in art. In fact the laws of perspective in painting and drawing are none other than the laws of the groups of projective transformations, which may be expressed as similarity or colored similarity transformations at the levels of the corresponding substructures. The harmonic proportionality of antique sculptural and architectural forms is the result of the old masters' strict observance of the canons of projective groups. Groups of transformations in four-dimensional space-time, together with colored groups, define the specific laws of composition for those arts which incorporate time: music, poetry, dancing, and motion pictures.

The application of the extended concept of symmetry in art is associated with finding specific equivalence relations between the elements of artistic substructures and finding groups of artistic automorphisms which keep a distinct structural level invariant. Transformations and their invariants – these are the most important concepts in both science and art! The deep-seated unity of poetry and music is founded on their common temporal nature, their structural peculiarities, and the existence of certain common laws of composition. These laws may be reduced to three basic principles: the translationally identical (or similar), the contrasting (antisymmetrical), or the varied (colored) repetitions of structural elements in time (or space).

The four lower, 'nondescriptive', levels (geometric composition, graphical outline, light and shade, coloring) determine, to a greater or lesser degree, the decorative qualities of the artistic canvas. At the same time, being elements of the composite semiotic structure, they are also material carriers of specific aspects of content. If we provisionally withdraw our attention from the content aspect, we can associate specific groups of symmetry transformations with each of these levels. As in network patterns, orthogonal groups with vertical symmetry planes will define the general static characteristic of the composition. With orthogonal axial groups we shall associate cyclic motion enclosed within the space of the picture. The idea of infinite motion is brought into the artistic structure by projective groups defining linear perspective and translational rhythms associated with space groups of similarity symmetry. Special antisymmetry groups may be associated with the distribution of light and shade at the level of the topological equivalence relations between 'shapeless' spots. In exactly the same way, the generalized colored groups determine the color matching between topologically equivalent colored spots. In all cases, symmetry is not followed with mathematical accuracy, but only to some degree of approximation.

Prejudice against the use of scientific methods for studying artistic phenomena has still not been finally overcome among the creative workers. It is based on an incorrect idea, as if even an acknowledgment of laws in art might in some way interfere with the direct perception of artistic compositions. A. V. Shubnikov stated in 1927 that artists had "a horror of the words law, order, symmetry, geometry; they prefer harmony, beauty, style, rhythm, unity, although the true meaning of these words differs very little from that of the former. Words, of course, are not really the point; the essence of the hostility of art towards science lies in the conviction that a fully discovered law will introduce triviality into poetry. This might indeed be true to a certain extent, but it is certainly not entirely so: Only one who is prepared to feel, and as far as possible understand, the laws of art can really enjoy it."

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