GEOMETRY OF ROD PACKINGS

Yoshinori Watanabe, Yoshinori Teshima and Tohru Ogawa
Institute of Applied Physics, University of Tsukuba, Tsukuba, 305 Japan
E-mail: watanabe@kafka.bk.tsukuba.ac.jp
teshima@kafka.bk.tsukuba.ac.jp
ogawa@bk.tsukuba.ac.jp

1 Introduction

A *rod packing* is an assemblage of contiguous rods such that each rod contacts more than three other rods. Rods are congruent rigid bodies whose shape is cylindrical and of infinite length. The directions of rods are restricted to n kinds, where n is an integer and then the structure is called *n-axes structure*. A configuration of infinite number of rods is called *self-supporting* if any of these rods cannot be displaced without displacing infinite number of rods (Ogawa et al., 1995, 1996). The structures can be either periodic or nonperiodic (Ogawa, 1993, Hizume, 1993, 1994). It is useful for the geometrical model of the composite materials or the three-dimensional fabrics (Hijikata and Fukuta, 1992). In addition, certain crystal structures can be described simply by rod packings when the rods are replaced by a linear or zigzag chain of atoms or groups of atoms (O'Keeffe and Anderson, 1977).

The maximum density of any space packing with equal circular cylinders of infinite length is $\pi/\sqrt{12} \simeq 0.907$ (Bezdek and Kuperberg, 1990). The maximum density is obtained when all of the cylinders are parallel. However, it is rather two-dimensional and more interesting are essentially three-dimensional cases.

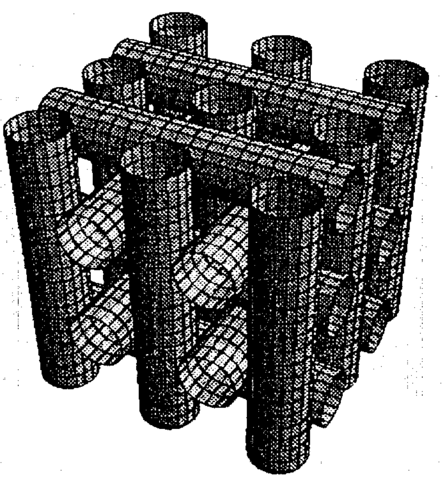

Figure 1: The periodic structure of 3-axes

2 The periodic structures of 3-axes and 4-axes

Let us take three directions as $(1,0,0),(0,1,0),(0,0,1)$ and summarize them symbolically as $\langle 100\rangle$. These correspond to the directions of 4-fold rotational axes of a cube. Fig.1 shows the densest packing of 3-axes. The packing density is $3\pi/16 \simeq 0.589$. In this case, it is easy to see the structure and easy to construct.

In the case of 4-axes, four directions are taken as $\langle 111\rangle$, which correspond to 3-fold rotational axes of a cube. The densest packing of 4-axes is a little complicated but it is not difficult to construct since one direction can be symmetrically put to the previous 3-axes structure (See Fig.2). The packing density is $\sqrt{3}\pi/8 \simeq 0.680$. We can construct another periodic structure of 4-axes, shown in Fig.3, which is less dense but isotropic. It should be noted that the highest density and the highest symmetry are realized in two different configurations.

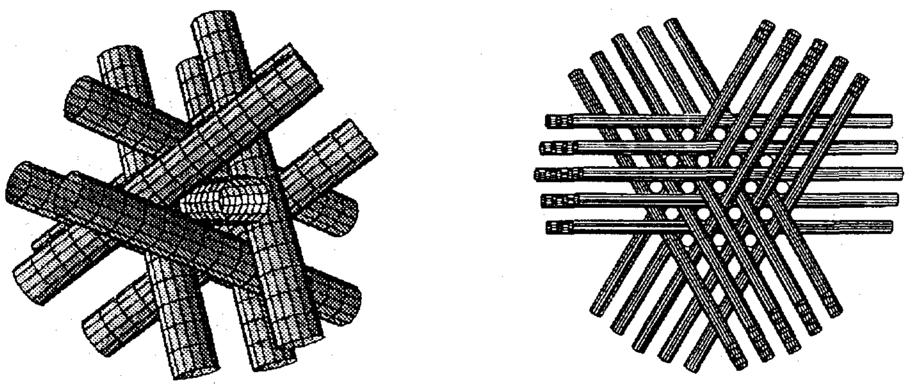

Figure 2: A fragment of the periodic structure of 4-axes (high density).

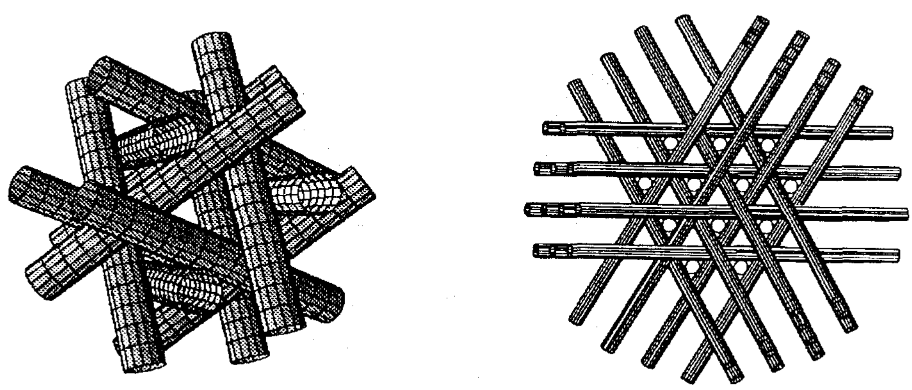

Figure 3: A fragment of the periodic structure of 4-axes (high symmetry).

3 The 6-axes structures

In the periodic case of 6-axes, six directions are taken as $\langle 110 \rangle$, which correspond to 2-fold rotational axes of a cube. Any of the six directions has a dual direction among other five which make right angle with each other. The angle between nondual pair is $60°$. There are several periodic structures in which any rod is supported by rods of four nondual directions (See Fig.4).

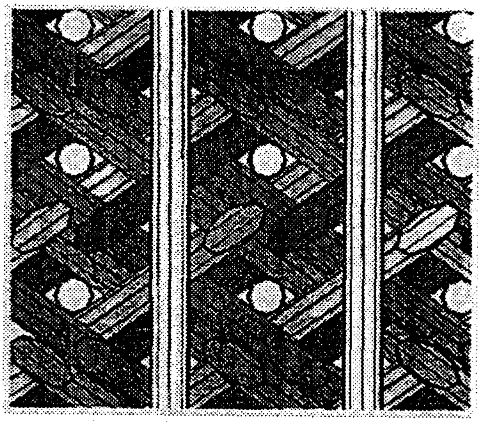

Figure 4: The periodic 6-axes structure projected on the plane which is perpendicular to one of the six directions $\langle 110 \rangle$.

Fig.5(a) shows one of the periodic 6-axes structures with the packing density $\sqrt{2}\pi/18 \simeq 0.247$. The other structure, shown in Fig.5(b), has the same density as the previous one. They are distinguished by the rod-separation of three dual pairs. The 6-axes structures of the double density $\sqrt{2}\pi/9 \simeq 0.494$ are also possible (See Fig.5(c)).

The most isotropic six directions in three-dimension are not $\langle 110 \rangle$ but $\langle \tau 10 \rangle$ directions, where $\tau = (1+\sqrt{5})/2 \simeq 1.618$ is golden mean. These correspond to 5-fold rotational axes of a regular dodecahedron. The 6-axes structure of $\langle \tau 10 \rangle$ type directions are also possible (See Fig.5(d)). They are inevitably quasiperiodic with some close connection with quasicrystals and Penrose tilling.

References

Bezdek, A., and Kuperberg, W. (1990) Maximum density space packing with congruent circular cylinders of infinite length, *Mathematika*, **37**, 74-80.

Hijikata, A., and Fukuta, K. (1992) Structure of three-dimensional multi-axes composites, *Nihon Fukugō Zairyō Gakkai Shi* (in Japanese), **18**, 231-238.

Hizume, A. (1993) Hosikago (Star Cage) (in Japanese) in *Katachi no Bunka-shi 1: Asia no Katachi wo yomu*, Kōsakushya, 208-225.

Hizume, A. (1994) Star Cage; New dimension of the Penrose lattice, *Forma*, **9**, 259-272.

Ogawa, T. (1993) Rittai Tajikuori (3-D multi-axes weavings) (in Japanese) in *Katachi no Bunka-shi 1: Asia no Katachi wo yomu*, Kōsakushya, 226-235.

Ogawa, T., Teshima, Y., and Watanabe Y. (1995) *Nihon Fukugō Zairyō Gakkai Shi* (in Japanese), **21**, 165-173.

Ogawa, T., Teshima, Y., and Watanabe Y. (1996) Geometry and crystallography of self-supporting rod structures, in *Katachi* ∪ *Symmetry*, T. Ogawa, K. Miura, T. Masunari and D. Nagy (eds.), Tokyo: Springer-Verlag, 239-246.

O'Keeffe, M., and Anderson, S. (1977) Rod packings and crystal chemistry, *Acta Cryst.*, **A33**, 914-923.

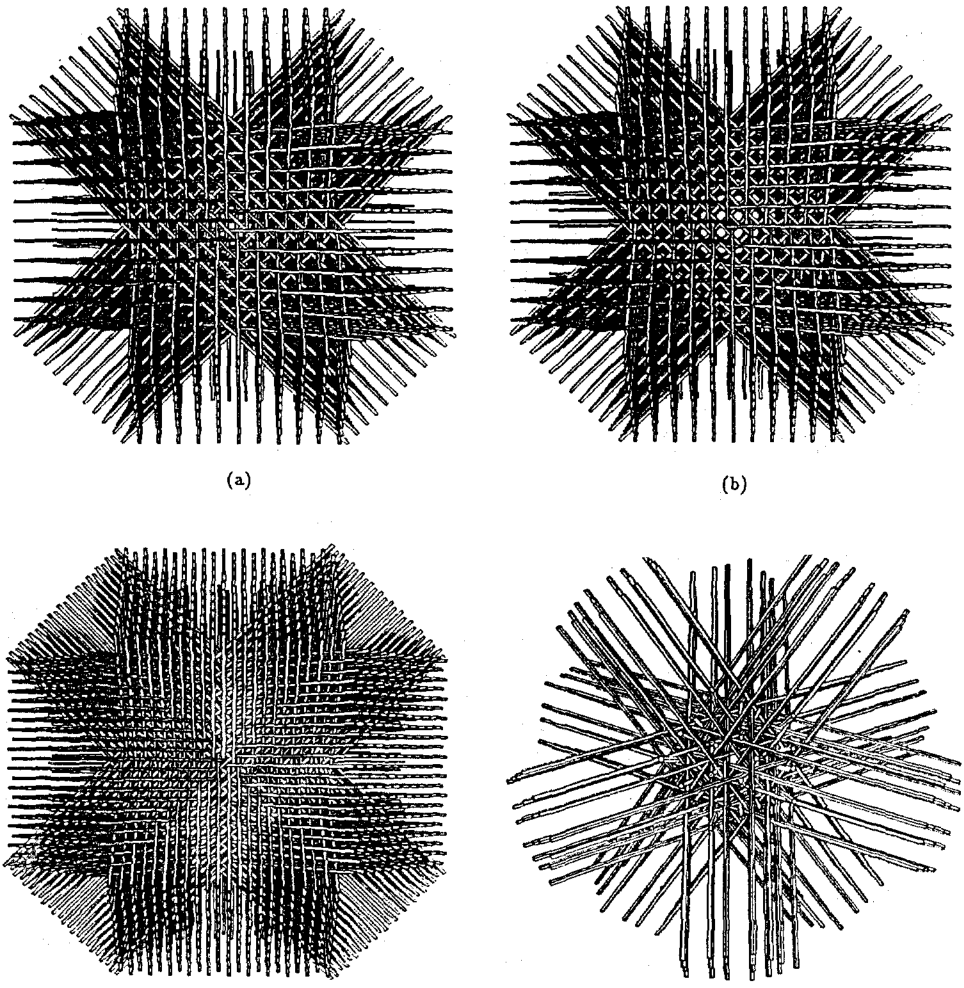

Figure 5: Examples of 6-axes structures. A fragment of structure viewed down (a)(b)(c) 4-fold rotoinversion axis; (d) one of 5-fold rotational axes.