# *WASAN* (OLD JAPANESE MATHEMATICS): ART AND SCIENCE

Dénes Nagy
Institute of Applied Physics, University of Tsukuba, Japan 305
and Symmetrion, Budapest, P.O.Box 4, H-1361 Hungary
E-mail: nagy@bk.tsukuba.ac.jp and h492dar@ella.hu

## 1 Briefly about *wasan*

*Wasan* (from *wa-* Japanese and *san* calculation, mathematics) was developed in isolation from the European mathematics (*yôsan*), which did not become dominant in Japan until the 1870s. Although *wasan* had Chinese influence in the beginnings, but the Japanese developed a special interest in some problems, especially in densest (close) packing of circles in larger figures, that are unparalleled in Chinese and other cultures in the same level. Another interesting feature of *wasan* geometry is the custom that many of the problems and their solutions were offered to gods. These problems were written on wooden tablets, called *sangaku* (from *san* mathematics and *gaku* framed picture), with beautifully colored figures. Among the circa 900 tablets that survived roughly twice as many appear in Shinto shrines than in Buddhist temples and these are distributed in both rural and urban areas (Fukagawa and Pedoe, 1989, p. xii). Considering these facts, *wasan* is obviously a relevant part of Japanese culture and may provide some interesting questions not only to mathematicians, but also those who are interested in religion or art. It is also an interesting topic for comparative studies: we may see various analogies between the Pythagoreans and some *wasan* schools, i.e., the unity of mathematical, aesthetical, and religious ideas, as well as about secrecy. In the Western world the Pythagoreans (6th-5th cc., B.C.) significantly contributed to the birth of modern mathematics: they provided the basic sources to various parts of Euclid's comprehensive work (3rd c., B.C.) that presents mathematics as a deductive system with rigorous proofs. It would be interesting to see the reasons of the missing 'Japanese Euclid' who could transfer the rich mathematical knowledge of *wasan* to major breakthroughs, instead of going to a deadlock manifested in the 1872 decree by the Ministry of Education that eliminated *wasan* from the curricula and required teaching exclusively of Western mathematics. Ravina (1993) gave some explanation of the failure of *wasan* focusing on the lack of its application to physical problems. Note, however, that the 'pure mathematics' of Euclid and his followers remained dominant through the ages without initiating any connection with physics or other practical problems from their side.

Despite this setback of *wasan*, it is still a significant source of the history of ideas and requires international attention. We have here a quite unique possibility to see another type of mathematics. *Wasan* may also have an importance in mathematics education. The ethnomathematical approach, i.e., using mathematics-related ideas of various cultures, instead of just referring to Western examples, has a success in education (cf., D'Ambrosio, Gerdes, Harris, Zaslavsky). Last, but not least, we also believe, that *wasan* - especially the geometry problems, practised widely in both urban and rural areas - had an indirect influence on Japanese culture after the Meiji restoration (1868): the skills gained in *wasan* could contribute, at least in some circles, to the quick adaptation and utilization of Western science and technology that requires some mathematical backgrounds. This traditional knowledge should not be forgotten in our age, but rather we ought to use it to widen our knowledge in the cultural history of mathematics and to enrich the often boring problem books used in mathematics education.

## 2 *Wasan* as science and art

Mikami (1913, pp. 166, 169, and 177) formulated an interesting thesis that *wasan* did not exist as science with rigorous proofs, but as art. However, very recently Horiuchi (1994, pp. 16-18) emphasized that the general characterization of *wasan* as art does not properly consider the developments in this field. The mathematical results by Takakazu Seki (1642?-1708) and Katahiro Takebe (1664-1739) are important tools to describe, for example, trajectories of astral bodies. The conclusion of the book emphasizes that the general techniques developed by Seki and Takebe in algebra and trigonometry gave a scientific status to *wasan*.

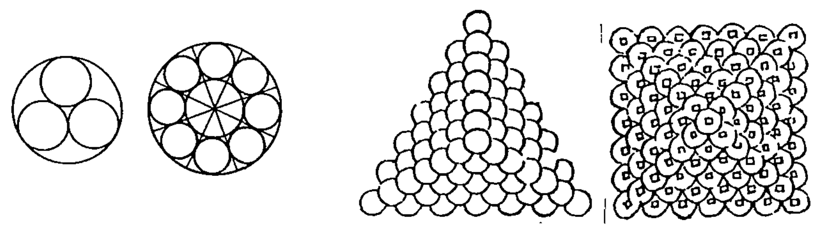

**Figure 1:** Densest packing of circles in larger circles (from *Sanpô futsudankai*, 1673) and densest packing of equal spheres (*Kokon sanpôki*, 1671). The systematic study of such questions in Western mathematics were started by Fejes Tóth in the late 1940s; also see the works by Coxeter, Rogers, and Delaunay [Delone]. An interesting result of these studies is the fact that density may induce order in a chaotic figure system. This topic also has a special importance in crystallography.

In this paper, making a compromise between the opposite views by Mikami and Horiuchi, we argue that *wasan* is both science and art. Horiuchi is right that Seki and his school lead *wasan* to such a level that it reached scientific status. Some of their results are beyond the level of the best results of contemporary Chinese and Western mathematics. However, we think that *wasan* was partly continued in an artistic way. We use this statement in a very positive sense: we believe that an artistic imagination is a very important aspect of mathematical creativity, especially in geometry-related fields. There are various confessions from leading mathematicians and physicists - from Norbert Wiener to Paul Dirac - that link their fields to art or to the search for beauty. Although these statements can be considered metaphoric, in the field of geometry it is obviously more meaningful. For example, G. D. Birkhoff dealt with aesthetic measure of shapes, George Pólya and Andreas Speiser initiated the group theoretic analysis of ornamental art in the 1920s. This was continued by the monographs written by Shubnikov, Fejes Tóth, and more recently by Crowe and Washburn. All of these books are illustrated by a large number of art works.

However, *wasan* started this practice earlier. The fact that the results are often communicated in shrines and temples includes a special desire for beauty. We will refer here to some topics that are strongly connected with symmetry.

## 3 Some symmetry-related topics in *wasan*

### *3.1 A field of* wasan *that is unparalleled in other cultures: densest packing of circles, spheres, and other round objects*

(See Figure 1.)

### *3.2 Theory of tilings*

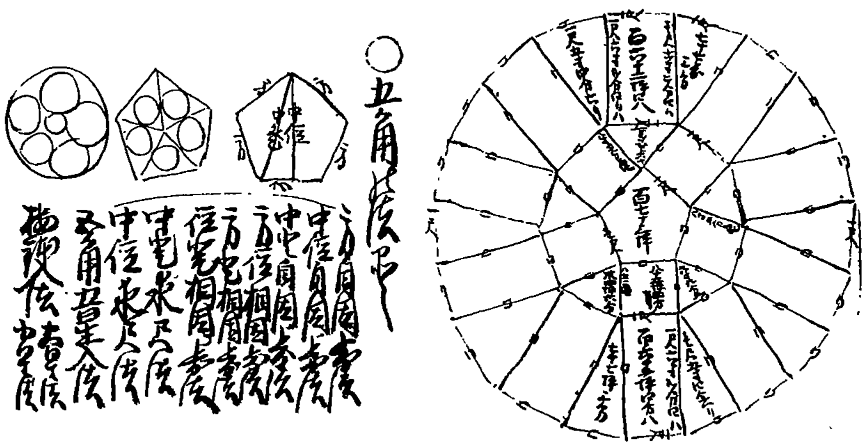

**Figure 2:** A tiling related to packing of circles (Dirichlet-Voronnoi cells) and an interesting pentagonal tiling (from a manuscript book of 1798, Symmetrion Library, Budapest). The topic of tiling (tessellation, mosaic, paving) has a great record in the history of culture, but its systematic study from a mathematical point of view was started in the West only in the 20th century (Heesch in the 1940s and more recently Grünbaum and Shephard).

### *3.3 Surfaces and polyhedra*

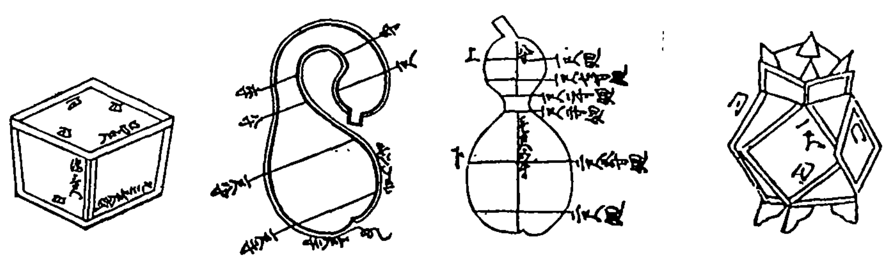

**Figure 3:** Surfaces and polyhedra, including a figure that resembles the Klein-bottle (from *Sangenki*, 1657). Although the roots of this topic go back to the Greek mathematics, but the systematic studies in the West were not started until the 19th century.

## 4 *Wasan* and Japanese culture

What were the sources or inspirations of this strong interest in packing problems and other symmetry-related topics? This question was raised by some distinguished historians of science, but they could not locate any relevant preliminaries in mathematics. Both Smith and Needham refer to Chinese connections; Needham adds, however, that he does not see the intermediate channel, and refers to some relation to the origin of calculus. Although a limited number of packing problems were discussed by Chinese mathematicians, but they never had such a strong interest in these questions as the Japanese. Here we suggest such aspects that are outside the mathematical activity in a narrow sense, but we may call it ethnomathematics:

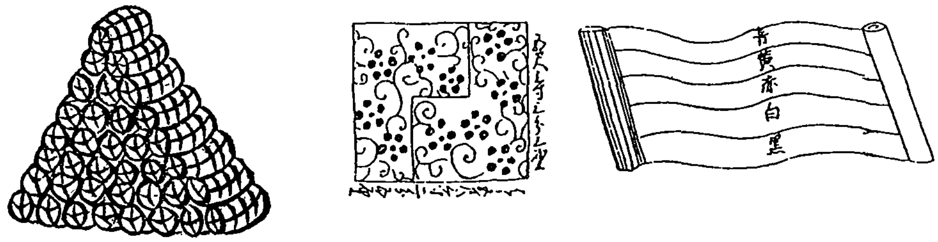


**Figure 4:** Practical problems in *wasan* books associated with densest packing of figures (see in many works) and patterns in art (from *Sangenki*, 1657).

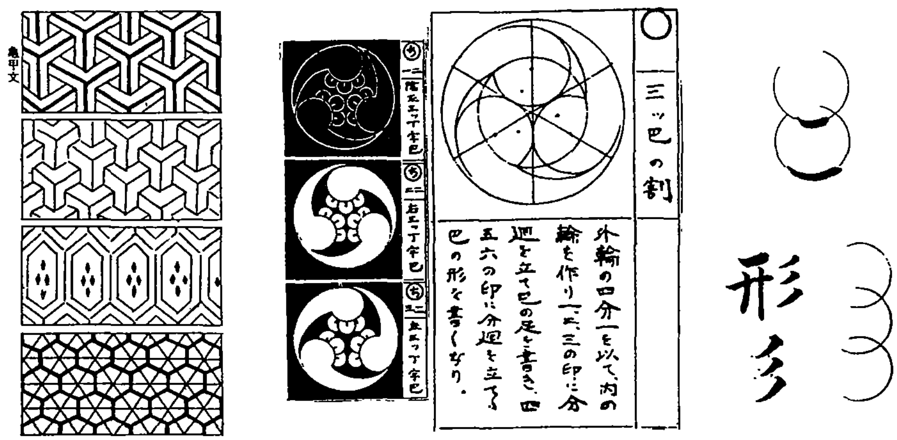


**Figure 5:** There are various fields in Japanese culture that could inspire related questions, including (1) geometric ornaments, (2) design of family crests, (3) the 'hidden dimension' of calligraphy (here the character *katachi*, i.e., form, shape, figure, is explained).

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