

Topology and geometry of bar constructions made from regular 20-hedron

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The centres of 20-polyhedrons edges determine the halfregular 32-polyhedron. Vertices of the 20-polyhedron and 32-polyhedron determine the 80-polyhedron, which topology is presented on fig.1a. The centres of 32-polyhedron's edges are vertices of the 92-polyhedron. Placing on the walls of the 32-polyhedron pyramids, which vertices are lying on the common spherical surface, we get the 180-polyhedron.

If on the spherical surface circumscribing the vertices of the regular 20-polyhedron are projected the centres of its walls, we get the 12-polyhedron. Each edge of the 20-polyhedron fits in one edge square with one of the 12-polyhedron. The vertices of the 20-, 12-, and 32-polyhedron compose the vertices of the 240-polyhedron having 360 edges, which topology is shown on fig.1b. The vertices of the 20-polyhedron and 12-polyhedron compose the vertices of the 60-polyhedron.

From the given polyhedrons can be formed, in turn, bigger ones for instance - in the first kind the 180-polyhedron is converted into the 540-polyhedron, 240-polyhedron into 720-polyhedron, the 320-polyhedron into 960-polyhedron etc., in the second case the 80-polyhedron is converted into 320-one, 180-polyhedron into 720-one, 240-polyhedron into 960-one, etc. In the third case the 240-polyhedron is converted into 320-one, 540-polyhedron into the 720-one, the 720-polyhedron into the 960-one, etc.. These 3 kinds of conversion enable 3 quite different constructions.

Treating the edges of each polyhedron as bars and vertices as nodes we get one layer space truss. Taking the opportunity of converting of one polyhedron into the second one, at their concentric setting, we can join together two one layer structures, forming in this way two-layers bar constructions.

Fig.1c presents the topology of the two-layers construction consisting from the bar layer fitting in the 80-polyhedron /thick lines/ the bar layer fitting in the 240-polyhedron /thin lines/ and the layer of joining bars /punctate lines/. The calculated angle coordinates of nodes for 1/20 part are presented in tab.1.

TABLE 1

NODE	COORDINATES	
	φ	δ
1	0°	0°
2	0°	16°28'19,92"
3	36°	31°43'02,908"
4	0°	37°22'38,525"
5	23°33'13,02"	50°39'04,9"
6	0°	58°16'57,091"
7	36°	63°26'05,815"
8	19°15'55,88"	69°28'58,31"
9	0°	79°11'15,659"
10	36°	79°54'25,755"
11	18°	90°

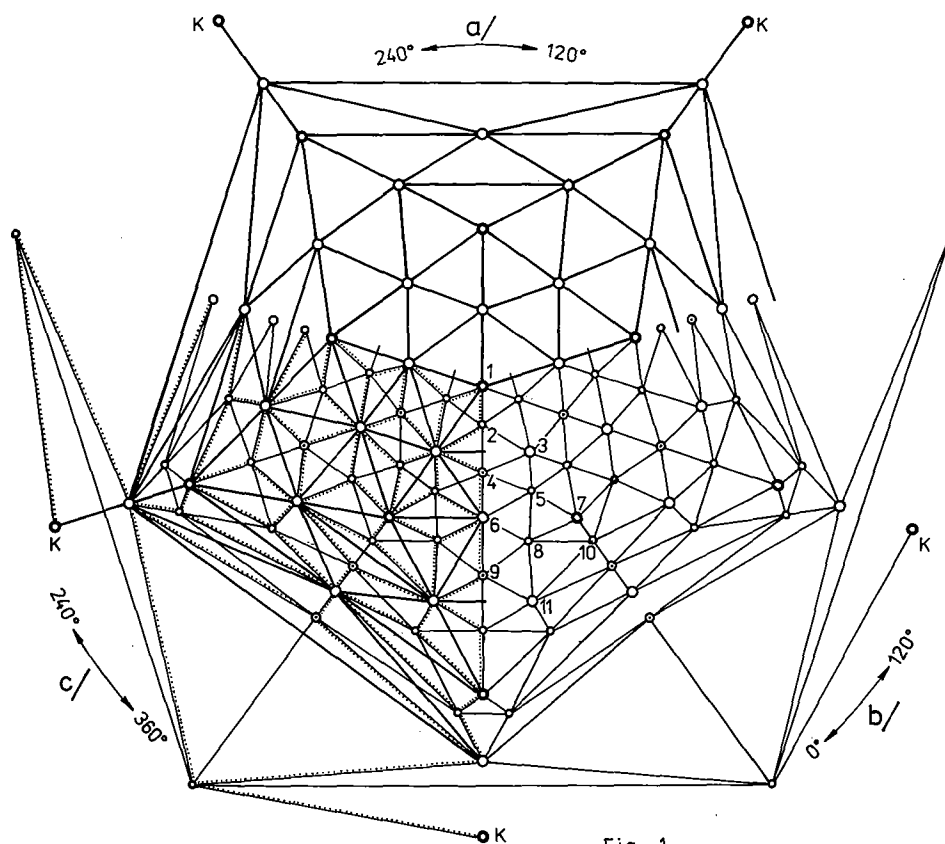


Fig. 2 shows the two-layer bar construction and at the same time reflects the conversion of a polyhedron into a second one. The bars of the internal layer are the edges of the 180-polyhedron. The double line is marking the mother network of this 180-polyhedron, which is the halfregular 32-polyhedron. The external layer bars are the edges of the 92-polyhedron which walls are: 60 isocles triangles, 20 regular hexagons. Transposing polygons into triangles we get 240-polyhedron. It's the conversion of the 3d kind.

Fig. 3 displays the two-layers bar structure based on the conversion of the internal 60-polyhedron into the external 240-polyhedron changed here into mother network of the second type /60x4/, being 92-polyhedron. The double line is marking the network of

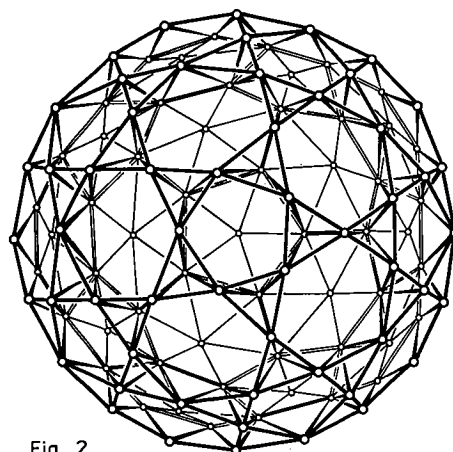


Fig. 2.

The nodes of two-layers constructions were described by independent concentric spheres and changed the reciprocal relation of their radii calculating geometric characteristics of the constructions.

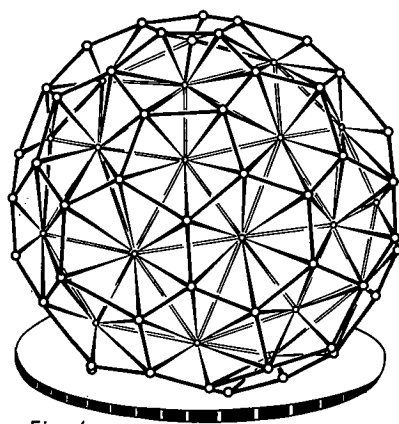


Fig. 4.

the 60-polyhedron. Completing the polygons with triangles we get the 240-polyhedron.

Fig. 4 presents the two-layer bar construction based on the conversion of the internal 80-polyhedron into the external 240-one changed here into mother network of the first kind /80x3/ closed by 12 pentagons and 30 hexagonals.

Fig. 5 presents the view of the 240-polyhedron.

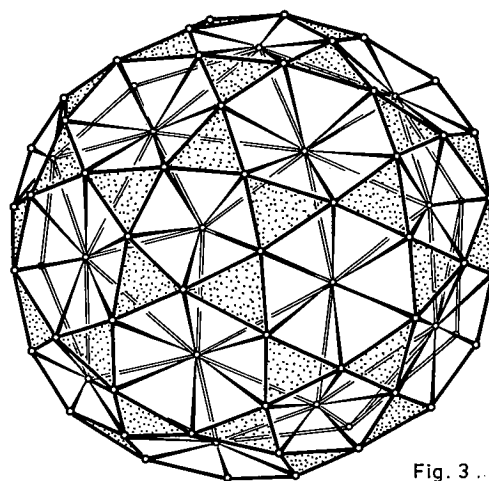


Fig. 3.

Fig. 6 illustrates an example of a change of bars length according to radii ratio R_{180}/R_{60} is the radius of sphere describing construction nodes coming from the 180-polyhedron, R_{60} is the radius of sphere describing the nodes of construction coming from the 60-polyhedron. The lines 1 and 2 illustrate the changes of length for bars of the

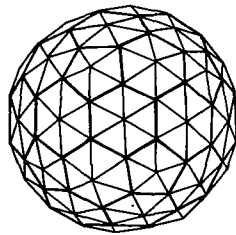


Fig. 5.

construction described by sphere with the radius R_{60} ; the lines 3, 4 and 5 illustrate changes of length bar groups for the construction described by the sphere with the radius R_{180} ; the lines 6, 7 present the change of groups of bar length joining both layers.

Similarly were marked the lengths of bars for two-layers bar construction based on 80-polyhedron and 240-one, and the results are presented on fig.7.

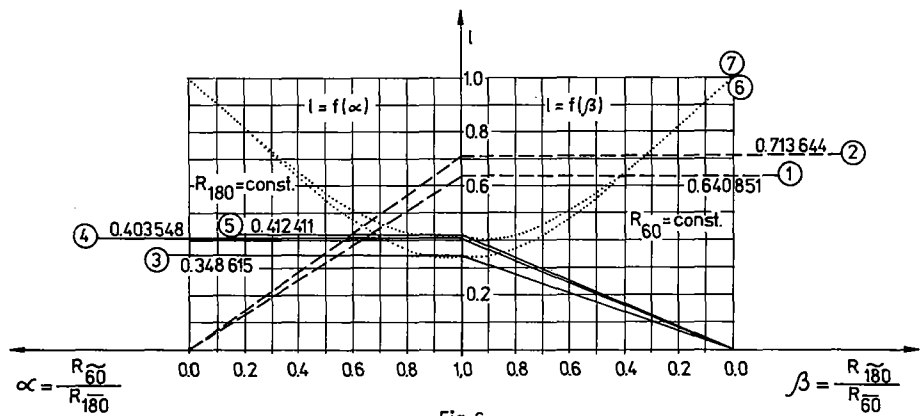


Fig. 6.

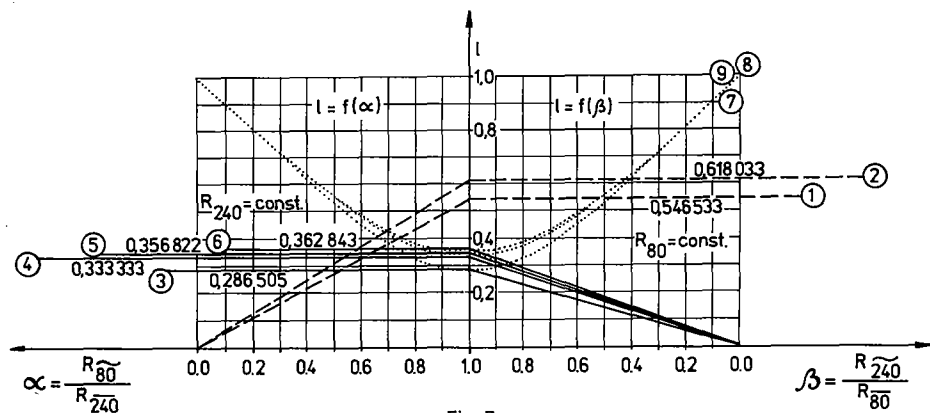


Fig. 7.