THE SUBSTITUTION PROCEDURE: PERIODICITY AS A NOTABLE CASE OF APERIODICITY

P.Bellingeri[1], M.Dedol[1] and P.M.Ossi[2]

[1] Dipartimento di Matematica, Universital Statale di Milano,
via Saldini 50, 20100, Milano, Italy
[2] INFM-Dipartimento di Ingegneria Nucleare, Politecnico di Milano,
via Ponzio 34/3, 20133 Milano, Italy

In the study of aperiodicity, inflation, composition, substitution, are usual terms. However, they are often used with different meanings and the lack of a standard notation introduces further confusion. A systematic treatment of such concepts is useful to classify quasicrystals, and it is fundamental tocorrectly link aperiodicity to periodicity.
In this work we present the concept of substitution. Simply, the substitution is defined as follows: take a tiling T, admitted by a finite protoset P = P_{1}, ...,P_{n}}, and suppose the tiles can be recomposed to make a tiling G built by a protoset >P,={ PI_{1}, ..., PI_{n}}, where PI_{i} is similar to P-{i}, with a fixed similitudine relation: if such a composition procedure can be iterated ad infinitum, then T is defined "substitution tiling". At present substitution is the most used and most rigorous technique to prove that a given protoset is aperiodic. It is useful to force, according to defined rules, the procedure of prototile matching, so that the tilings admitted by the matching rules are substitution tilings. We show that the substitution procedure, which operates also in periodic tilings, leads to two only possible results: first, there is a unique tiling and it is periodic; second, all admitted tilings are non periodic and they are an uncountable set. Therefore, periodicity is indeed, for periodic tilings, local equalities ("locala notable case of aperiodicity: isomorphism") become global equality. We provide some examples of physical interpretations and modellings which can be deduced by the present approach in the study of aperiodicity. The same approach leads to new kinds of problems, such as i) the existence, of protosets that admit more "disordered" tilings than those presently known, and ii) the possibility to classify tilings and protosets according to their degree of order.