The Role of Symmetry in Mathematical Problem Solving: An Interdisciplinary Approach

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Symmetry is one of the more important concepts of mathematics. It plays a central role in art and nature, it has a strong aesthetic aspect (Evered, 1992; Geddes & Fortunato, 1992; Zaslavsky, 1990, Zaslavsky, 1994; Jones, 1991; Graff & Hodgson, 1990; Harris, 1988), and it portrays advanced mathematical thinking. Symmetry, in its broadest meaning, reflects a view of the nature of mathematics, its values, and its goals, and is a means to bridge between various branches of mathematics such as algebra, geometry, probability, and calculus. In this sense, symmetry is a unique interdisciplinary concept within mathematics (Reesink, 1987; Eccles, 1972; Whitman, 1991; Zaslavsky, 1994; Allendoefer, 1969; Ellis-Davies, 1986, Tobias et. al , 1981)

Although there are many opportunities to use symmetry in secondary school mathematics, symmetry is hardly explicitly included in the curriculum The purpose of the study to be presented at the conference is to enhance the use, the appreciation, and the understanding of symmetry by facilitating a problem solving approach in high school mathematics based on the potential of symmetry considerations as an interdisciplinary problem solving strategy.

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The important role of symmetry in problem solving is expressed by Polya (1981) and by Schoenfeld (1985):

"We expect that any symmetry found in the data and condition of the problem will be mirrored by the solution." (Polya, 1981 p. 161)

"The circumstance that the condition of a problem involving several unknowns is symmetric with respect to these unknowns may, if recognized, influence the course of, and generally facilitate, the solution." (Polya, 1981. p. 153)

" Special cases of symmetric objects are often prime candidates for examination."

(Schoenfeld, 1985: p 81)

In order to achieve our main goal, a careful analysis of the curriculum has been done to locate problems related to different mathematical topics that could be solved with the use of symmetry The study is conducted within the framework of a series of workshops for mathematics teachers dealing with applications of symmetry to problem solving. In these workshops the participants are presented with problems that can be solved by at least two different ways -- one more "standard", and the other based on the use of symmetry. The workshops are designed to document processes which teachers encounter in order to follow their preferences and tendencies regarding the use of symmetry.

The following four problems are examples of such problems. At the conference we will analyze solution strategies and difficulties encountered by secondary mathematics teachers and student-teachers related to these and other similar problems.

Problem #1 (algebra):

Solve the following system of equations:

$$\begin{cases} 4x+2y+3z=36 \\ 3x+4y+2z=36 \\ 2x+3y+4z=36 \end{cases}$$

Problem #2 (geometry):

Let ℓ be a straight line and A and B two points that are not on the line ℓ. Find a point C on the line ℓ such that the sum of its distances from points A and B is minimal.

Problem #3 (probability):

Two men Alan and Bill toss N+1 and N coins respectively. What is the probability that Alan gets "Head" more times than Bill does?

All three problems can be solved based on symmetrical considerations of various types The characteristics of each type will be discussed.

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