The geometric stability of highly symmetric tensegrity structures

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A tensegrity structure is a collection of incompressible struts held rigidly in place by a collection of cables. Mathematically it can be modelled as a collection of points where certain pairs of the points are constrained not to get closer together (the struts) and another collection of pairs are constrained not to get further apart (the cables). A basic question is to tell if this structure is (globally) rigid (or stable) in the sense that the constraints determine the configuration up to congruence. One way to do this is to calculate a certain n-by-n symmetric matrix S, the stress matrix, where is the number of points is n. If S is positive semi-definite of maximal rank, then the tensegrity structure is stable. If further the tensegrity structure is so symmetric that the group of rigid congruences of the structure is transitive on the vertices, then S decomposes into much smaller symmetric matrices corresponding to the irreducible representations of the group of symmetries. In fact it is possible to catalog all the configurations of such highly symmetric tensegrities for all the point groups of three-space for all the combinatorial choices of connecting the cables. We will show some pictures of these stable tensegrities when the group is the group of rotations of the cube. They are quite pretty.