THE E.P.R. APPROACH TO SUBDIVISION OF SPACE INTO TOWN IDENTICAL SUBSPACES

Ami Korren & Prof. Michael Burt

Technion - Faculty of Architecture and Town Planning

2-manifolds, which subdivide the space into two identical subspaces, represent partitions between two identical - dual lattices.
The lattices in question represent two congruent subspaces, called " complementary dual spaces".
The subsequent result of the a.m. congruency is the existence of symmetry operation that takes one subspace into the dual - identical one. This symmetry operation acts also on the space of the 2-manifold, which separates between the two subspaces and is of a two-fold order.
We have chosen to discuss 2-manifold with 2-fold (1800) rotation symmetry axes. These axes generate a space lattice, which should be called "the 2-fold axes lattice'@ This lattice is periodic, and as such has all the periodicity generating properties, meaning an 3-dimensional repetitive cell unit and elementary periodic region (E.P R.).
The relation between the 2-manifold and the two dual lattices separated by it are associate; the discovery of the one will lead to the exposure of the other.
The relation between the 2-fold lattices and the 2-manifolds and the lattices is of a different nature.
Although any two identical-dual lattices and the related separating 2-manifold have a specific 2-fold lattice of the'm'-own, any of the 2-fold lattices in question may correspond to topologically different dual lattice pair and the subsequent 2-manifold.

The E.P.R. Approach
The E.P.R. is the smallest space unit derived from the Euclidean space by means of the symmetry group that acts on this space
The E.P.R. includes representation of every periodic element, which implies the existence of certain E.P.R.'s' representing the 2-manifold - partitions of the space into two identical subspaces, the dual lattice pair and the 2-fold lattice.
Identification and exhaustive enumeration of the E.P.R.'s is based on the fact that the number of the symmetry groups is finitei
From all the E.P.R.'s we will identify those, which contain 2-fold axes that rotate them into themselves. This group of E.P.R.'s will contain 2-fold axes lattice, two identical dual subspaces and the 2-manifold piece separating between them.
Exhaustive enumeration of this group (of E.P.R.'s) will lead to the exhaustive enumeration of the 2-fold lattices.
The elementary unit of the said lattices is enclosed between the 2-fold axes and appears in two distinct forms: one - a closed spatial perimeter with a variable number of edges (more than 3), representing the 2-fold axes. The other - in the form of a split perimeter. Composed of two plane polygons (triangular or quadrangular), with an h-undefined distance in between.
Dipping of the elementary (wire perimeter) units in soap solution generates **seven,** topologically different manifolds.

This unique property differentiates this group of 2-manifolds from the other surfaces that have discovered lately.
Along a research effort to exhaustively define the 2-manifolds an affect was, discovered that a specific 2-fold lattice might correspond to different manifolds. Hence the conclusion that more than the already seven mentioned manifolds, mav exist, and as it turned out, such a manifold was discovered, corresponding to an new identical dual pair of lattices, topologically different from the seven already mentioned pairs. This discovery brought a break- through and subsequently for a search and discovery of more of such networks.
At this stage we are looking for a way to find additional 2-manifolds with similar properties.
A question arises: is the number of these dual lattice pairs finite or infinite?
The search after the new manifolds is done also through the E.P.R.
In the specific E.P.R.'s that contain 2-fold axes, we are looking after additional identical dual lattices, with new, unknown yet, manifolds.
The method of searching after identical dual lattices is by placing vertices and edges on and within the E.P.R. and through 2-fold rotation generating its dual taking care not result in a mutual intersection.
Since the number of edges and vertices to be inscribed within an E.P.R. is not limited, so is the possible number of different dual pairs of lattices.
The empirical conclusion, at this stage, of the applied method is that there are infinite number of identical dual pairs of lattices, which are topologically different and, consequentially, infinite number of topologically different 2-manifolds subdividing space into two identical subspaces.