

THE DUALISTIC SYMMETRY BETWEEN PLANE- AND POINT-BASED SPATIAL STRUCTURES.

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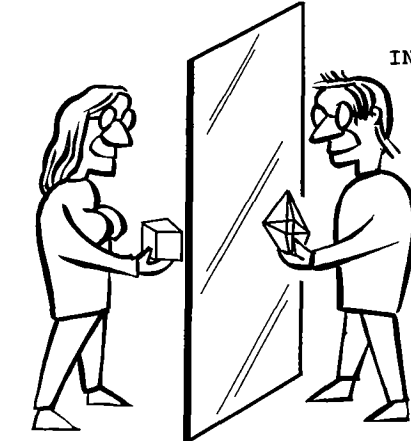


Fig.1

INTRODUCTION

The 3-D structural system based on points and their lines of connection (i.e. trusses, space frames, braced and lattice structures etc) are very well known as efficient and material-economic structures. This, the most basic structural system, creating only axial forces (tension and compression), turns out to have a counterpart, which is its diametrical opposite, its symmetrical mirror image, its polar unambiguous dual system -the plate structure-. This structural system, based on plane thin plates interconnected by shear resistant "lines of support", has traditionally been regarded as a secondary

kind of structure, mainly stabilizing buildings for wind forces, but it can with the knowledge presented here, be dealt with in a mere complex and direct way as the lattice structure. It might seem odd to relate the unrefined plate structure to the highly sophisticated lattice structure. However, it has turned out that between these two types of structure there is a connection so fundamental, that they are mutual geometrical and statical duals. To the author's knowledge it is the first time the concept of genuine dualism and complementarity well established in physics is introduced into statics. In fact this theory explains in a very simple way, not only the statical behaviour of all five Platonic polyhedra, but of any arbitrarily plane faceted surface. As the space for this abstract is very limited, only a concentrate of this dualism will be described, and references for more detailed information are given. To make the analogy as clear as possible, the dual qualities will be put directly opposite to each other in the following. The part dealing with lattice structures is generally elementary, and it is included merely to emphasize the analogy to the plates.

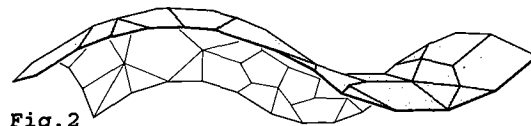


Fig.2

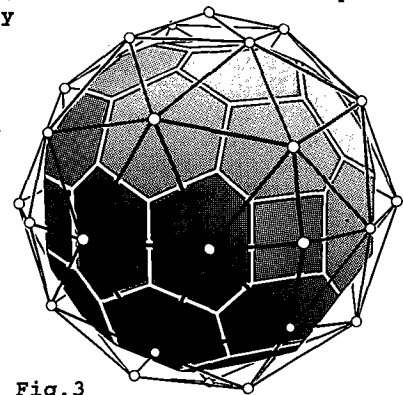


Fig.3

SUBSTANCE OF THE DUALISM
Basic considerations and definitions
Statical basis

LATTICE	:	PLATE
Nodes - Defined with three coordinates - Plates (*)	:	
Bars interconnecting the nodes	:	Lines of support interconnecting the plates.
	:	Transfer of only
axial forces	:	shear forces
	:	between the
nodes	:	plates
To achieve spatial structural stability for an additional node	:	plate
	:	an extra number of
3 bars	:	3 lines of support
	:	is necessary

This basic minimum relation is of course very well-known for lattice structures, but obviously this is not the case for the plate structures. As it is seen, nodes and plates may be interchanged if bars are changed to lines of support and axial forces to shear forces (Wester, 1984 & 1987a).

(*) A plane is defined by the coordinates to the terminus of its position vector (the vector from origo to the nearest point on the plane of the plate). This point is called a "plane-point", while the point of the node is called a "point-point".

Geometrical basis

A geometrical type of net, containing all the active elements mentioned for both kinds of structure is the spatial 2-dim. net (fig.2), consisting of the following elements: a) Planes. b) Lines of intersection between two planes only. c) Vertices as points of intersections between three or more planes. This kind of net also follows Euler's theorem on relation between the number of faces, vertices and edges in space.

Combination of requirements for stability and topology

If the relations mentioned for basic stability for spatial structures and topology are combined, the result is quite surprising. If a spatial net of the type shown in fig.2 & 4 is either unlimited or singly connected like a simple polyhedron, its statical behaviour is unambiguously related to the geometry of the net (Wester, 1988b).

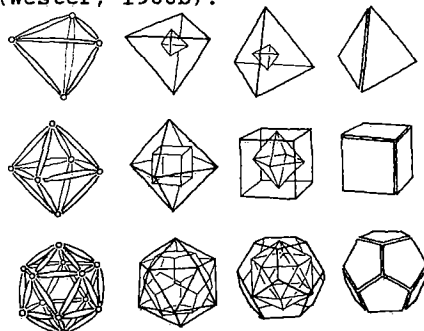


Fig.4

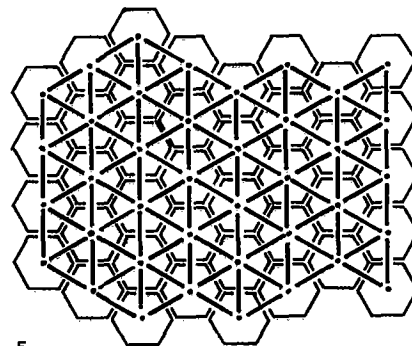
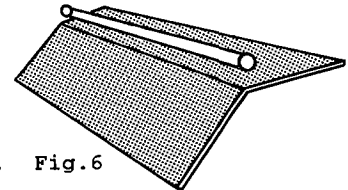


Fig.5

LATTICE	:	PLATE
In this case it turns out that the net consisting of		
triangular facets	:	3-way vertices
can only be stabilized by acting as a pure		
lattice structure	:	plate structure
and it will be just stable i.e. a statical and geometrical		
determinate structure. Hence no active element		
node or bar	:	plate or line of support
can be removed without losing the general stability of the		
structure. The inactive elements in the net		
the planes	:	the vertices
may be removed leaving the general stability unchanged.		
In this case the just-stable structure alone consists of		
nodes and bars	:	plates and lines of support
which give the structure the appearance of being totally		
open	:	closed
and the forces in the structure are		
concentrated	:	distributed
in lines and points	:	along lines and in planes

An example of dual polyhedral structures are shown in fig.3. The above considerations applied to the five Platonic polyhedra divides them into three groups, each with a stable lattice and a stable plate appearance, following exactly the well-known geometrical dualism (see fig.4). These relations may be extended to arbitrarily faceted polyhedra. A polyhedron, which is neither purely triangulated nor 3-way vertexed, may achieve stability by co-operation between the lattice part and the plate part of the structure, in such a way that so-called buffer forces may be transferred between bars and identically positioned lines of support (Fig.6). This effect means that a difference in axial force at the two ends of a bar results in the same difference between the shear forces at the two sides of the adjacent line of support. Fig.6



Dualism between exact geometry and statical analysis

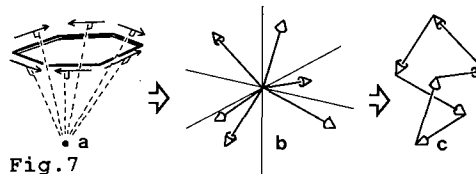


Fig.7

The dualism described so far at the level of topology and stability, may be extended to the level of exact geometry and statical analysis. The core in this relation is equalizing the equilibrium of forces to the equilibrium of moments (Wester, 1987a & 1987c). This exact dualism is based upon the simple fact that a spatial, closed vector polygon, where the vectors are one-way directed through the polygon (fig.7c), describes an unambiguous equilibrium of force vectors on a node (Fig.7b) as well as moment-vectors of the forces on a plate around a point outside the plane of this plate (fig.7a). In the latter case, fig.7b represents the moment vectors on origo. The two situations are so alike that it cannot be determined from the vector polygon alone (fig.7c) whether it describes an equilibrium of force vectors or moment vectors. It is further evident that the two systems cannot be mixed as the equilibrium requires that the whole polygon consists of either forces or moments. If these considerations are continued and put into equations, the whole thing ends up in very basic relations:

- A plane-point is polar to its dual point-point, i.e. the extension of their connection line hits origo, and the product of their distances to origo is an arbitrarily chosen constant.
- The ratio between an axial force and its dual shear force is a simple geometrical relation (the length of the bar over sinus to the angle between the vectors from origo to the nodes in the bar).
- The stiffnesses are related by equalizing the virtual work performed by the axial force in the bar and the shear force over the line of support. The dual ratio between the two is the square of the ratio for the forces.
- The dualism is valid for any 3-D plate and lattice structure.
- The "Euler-Number", the sign of the Gaussian curvature and the level of redundancy remain unchanged during the dual transformation.

Some more dual relations

LATTICE	:	PLATE
Node with N bars.	:	Plate with N lines of support.
N points in the same plane.	:	N planes through the same point.
Triangular mesh.	:	3-way vertex.
Right-hand sign rule for forces.	:	Left-hand sign rule for forces.
Visible deformations ie changing the length of its vector.	:	Invisible deformations ie rotation of a plane.
Invisible def. ie rotation of a node.	:	Visible def. ie changing the length of its position vector.

CONCLUSIONS

The present theory is seen to create a unification as well as a symmetry between the most basic, archetypal kinds of structure based on axial and shear forces. The correlations between them are general and may be used for any 3-D system. The symmetry is so basic that it fits on many levels, from general considerations of the nature of antagonism, over suitable pedagogical explanations on spatial general stability, over simple rules for design and general analysis of structures, e.g. several biological structures, up to an operational tool for numerical statical analysis. The theory has of course initiated a great number of interesting, only partly clarified, problems some of which seems to include great visual qualities.

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