

SYMMETRY IN THE TRADITIONAL JAPANESE MATHEMATICS

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The aim of this paper is to show that SYMMETRY was a very important concept in traditional Japanese mathematics (in short, TJM) of the 18th and 19th centuries. SYMMETRY figures in the plane geometry are very useful for students, in the view of education of mathematics. It is very important for students to work both the right and left sides of the brain, instinct and calculation. Now, at first, I would like to talk about a short account on the traditional Japanese Mathematics.

Historical overview of the traditional Japanese Mathematics.

In far-East Japan of 18th and 19th centuries, there was a native traditional mathematics based on Chinese mathematics and independent from the west. In this period, there were no universities nor colleges but many private schools in Japan who took the policy of seclusion for outer world in about two hundred fifty years (1630-1867). And so, the native Japanese Mathematics was developed in this period. Many ordinary people enjoyed to solve the problems of TJM as creative recreation and lovers of TJM hung the votive wooden tablets (SANGAKU) in his near temples or shrines and, on the tablet they wrote the mathematics problems with figures in colored beautifully. The study of figures flourished in all over the Japan of 18th and 19th centuries. Japanese Geometry developed to find the relations of tangent circles, the relations of ellipses and circles touched externally or internally.

Mathematicians of TJM enjoyed solving the problems of mathematics and calculated intensively to discover the beautiful relations.

They left many joyful problems to us in the view of mathematics, geometry, art and mathematics education.

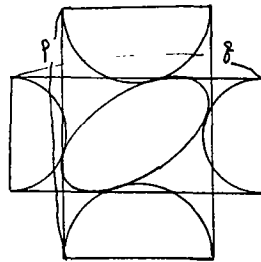
In those days of Japan, there were no universities nor colleges, so mathematicians had to make the efforts to introduce mathematics to ordinary people by themselves. All over Japan, there were and are many temples and shrines. They used the precincts of the temples and shrines where they hung wooden tablets of mathematics.

The tablets were to be beautiful and creative for ordinary people to have an interest in the problem. So, in SANGAKU problems, there were so many SYMMETRIC figures. For displaying mathematics problems to other people by means of the wooden tablet SANGAKU, the geometry was the best, since the problems on geometry are visual and beautiful. So many SYMMETRIC figures are drawn on SANGAKU since SYMMETRIC figures are beautiful and are able to let ordinary people have an interest in it.

This fashion of SANGAKU became popular and there were so many tablets all over Japan.

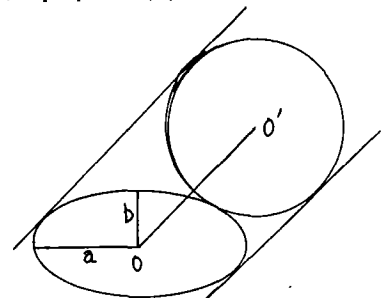
Now, I would like to introduce a SANGAKU problem concerning with an ellipse and four semicircles. Show that $p=q$ in (1).

(1)



(SANGAKU problem)

(2)

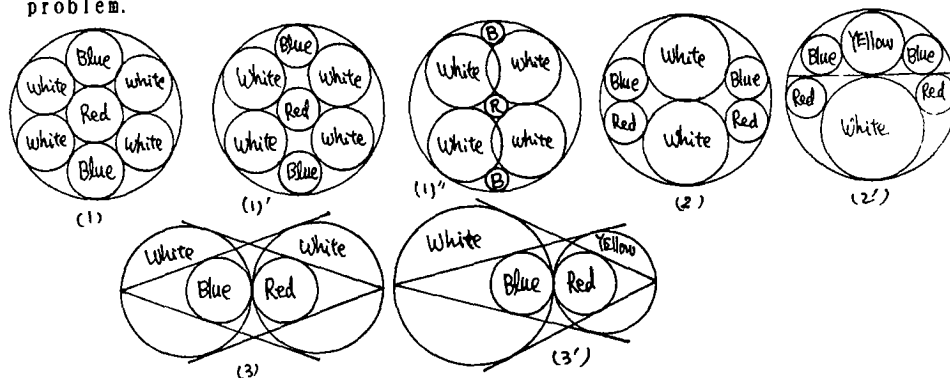


(Mathematics problem)

When a modern mathematician see the problem (1), then they maybe say that there are some needless lines in (1) and the problem is to be (2) and must ask the beautiful relation $OO' = a + b$ only where O and O' are the centers of the ellipse and circle externally touched on the ellipse, respectively. Indeed, I found the original solution of the problem which was the solution with the diagram (2).

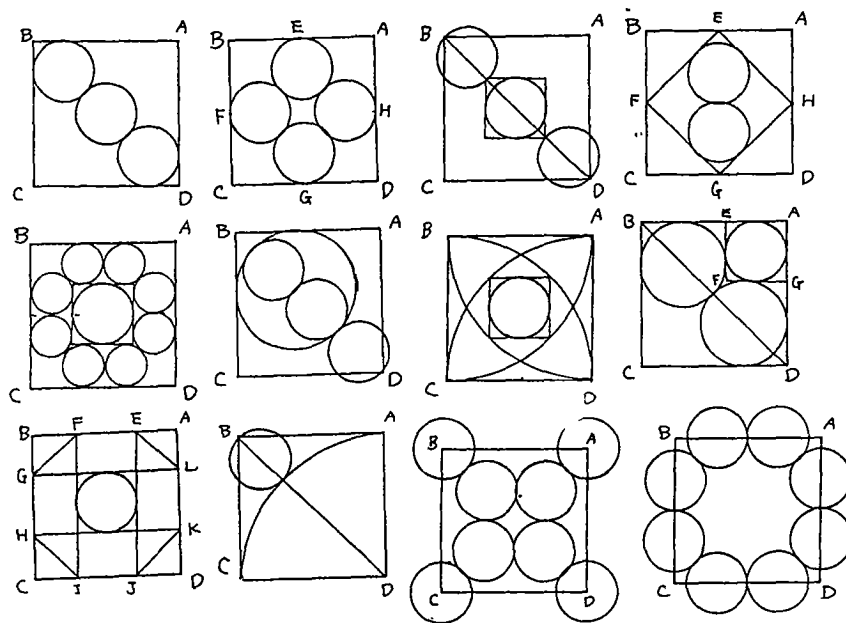
Why did the proposer draw some lines or rectangulars, in addition? The reason is simple. When ordinary people look the SANGAKU at a glance, he hoped that they had an interest in the problem so that he drawn the diagram (1). SANGAKU was a kind of art. Three SANGAKU problems with SYMMETRIC figures.

Show that Blue circles are congruent to Red circles in each problem.



Some elementary problems with SYMMETRIC figures quoted from the traditional Japanese Mathematics book.

The side a of the square drawn by thick line is given, then find the radius r of small circles.



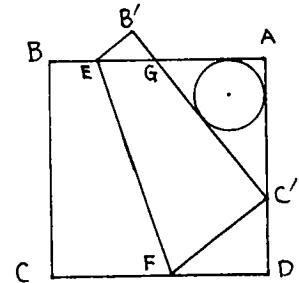
For each different problem, the answer is the same $r = (a/2)(\sqrt{2}-1)$, however problems are different.

How nice and excellent these problems are !

For students, problem figures of geometry are to be beautiful and creative. Beautiful and SYMMETRIC figures evoke an instinct in the brain of students and then the proving becomes a necessary calculation. The study of geometry makes use of both right and left hemispheres of the brain fully.

Folding a square paper problem.

Fold a square sheet ABCD such as C lies on the side DA. Conjecture which one of some segments AB, AG, BE, C'D and BG is the same as the radius of triangle AGC' by instinct. And then prove the rightness of your conjecture.



In Japan of the 18th and 19th centuries, mathematicians hung the tablets under the roof of big shrines or temples on which many original problems were written and challenged to other mathematicians. Lovers of TJM hung the tablets under the roof of shrines and temples near his neighborhood. On the tablet they draw the problems learned from his master. It is not the original but plagiarism. Almost all of the problems were on geometry and finding the volumes or length of curves of some figures. On one tablet, five or six problems were written with the last result only, without the solutions. The size is not constant. The most popular size is 180 by 90 cm. Some gifted people who had not so much mathematics knowledge and wanted to study mathematics were excited to see SANGAKU in the precincts of some shrines or temples shown in a glamorous way. And they developed an interest in mathematics. So, there were many attractive problems with SYMMETRIC figures on the tablets. It means that SANGAKU problems are very attractive for school students nowadays, too, I believe.