

SYMMETRY IN SOLUTIONS TO THE KNIGHT'S TOUR PROBLEM

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All my art is based on the way a knight moves on a chessboard, in particular, on solutions to what is known as the knight's tour problem: to move a knight on a chessboard so that all 64 squares of the chessboard are jumped on only once. A knight, one may recall, moves in a '1-2' or '2-1' L-shape.

I have discovered that the paths that the knight makes on the chessboard may exhibit symmetry themselves, or have symmetry hidden within. I'd like to share some observations I've made on one particular knight's tour, use it to explain how music can be generated from a knight's tour, and how a knight's tour can be identified by a unique 63-digit number. Using the 'music' approach and the 'numeric' approach may reveal other types of symmetry in a knight's tour that would not be revealed by observing its path:

Figure 1(a) is one solution to the knight's tour problem. This particular solution can be found in **A History of Chess**. It is said to be **re-entrant**, or **closed**, because position 1 and position 64 are a knight's move away from each other. It has the interesting property that the sum of any row or any column is the same, i.e., 260.

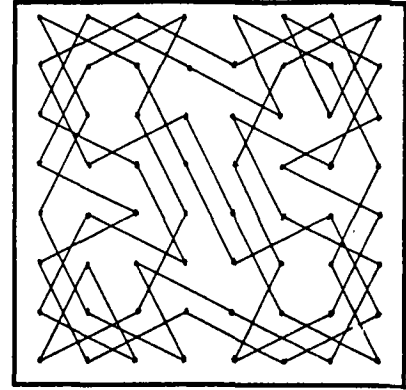
Figure 1(b) is the path of the knight. It is the result of connecting the centers of the numbered cells in sequential order. Notice that the path has 180° rotational symmetry.

Figure 1(c) is the result of connecting the odd numbers to the even numbers. Not only does this design have 180° rotational symmetry, but it has bilateral symmetry as well: a horizontal line of symmetry and a vertical line of symmetry. Notice that each quadrant is identical and each quadrant also has two lines of symmetry.

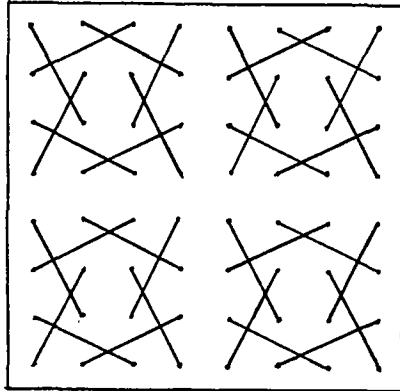
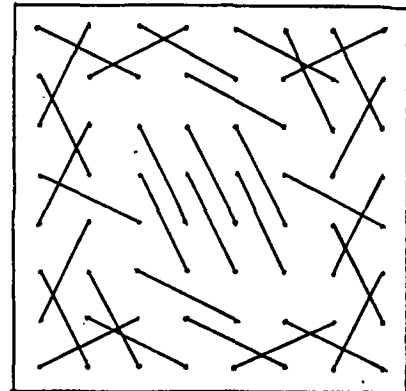
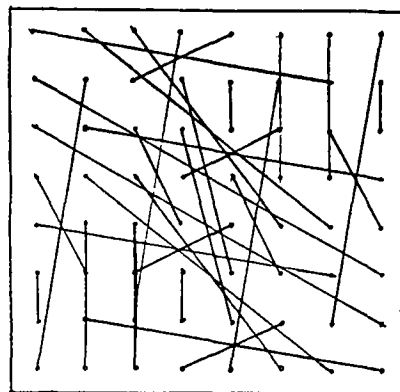
Figure 1

50	11	24	63	14	37	26	35
23	62	51	12	25	34	15	38
10	49	64	21	40	13	36	27
61	22	9	52	33	28	39	16
48	7	60	1	20	41	54	29
59	4	45	8	53	32	17	42
6	47	2	57	44	19	30	55
3	58	5	46	31	56	43	18

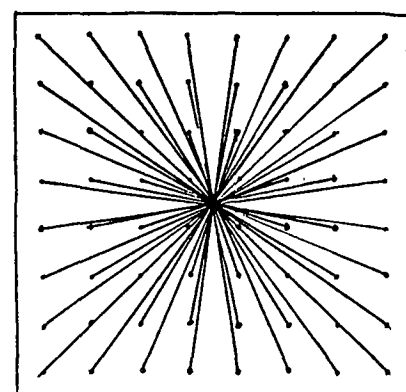
(a) Knight's Tour



(b) Knight's Tour Path

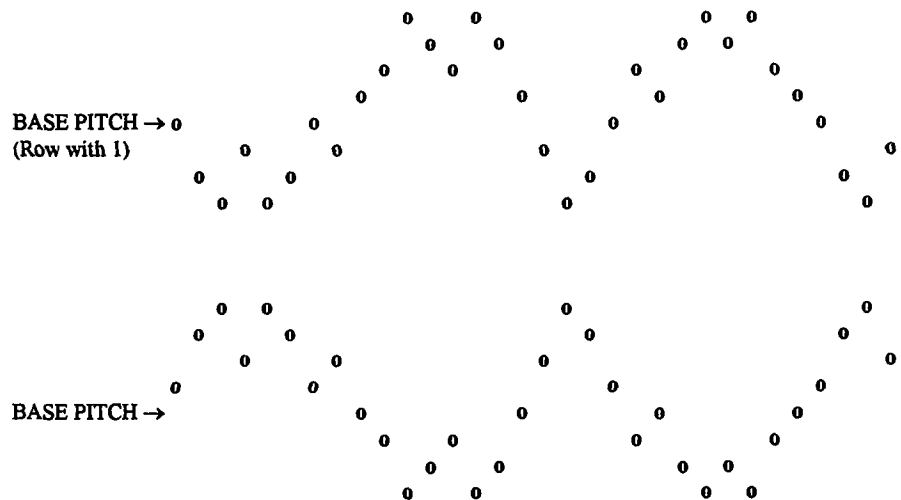
(c) Odd numbers to even numbers
1-2, 3-4, ..., 61-62, 63-64(d) Even numbers to odd numbers
2-3, 4-5, ..., 62-63, 64-1

(e) Extreme numbers connected



(f) First half to second half

Figures 1(d) and 1(e) both exhibit 180° rotational symmetry. Figure 1(d) is the result of connecting the even numbers to the odd numbers. Figure 1(e) is the result of connecting the end numbers, 1 and 64, and proceeding 'inward', i.e., 2 to 63, 3 to 62, ..., and 32 to 33. Figure 1(f) is the result of pairing the numbers 1 to 33, 2 to 34, ..., and 32 to 64. Not only does this design have two lines of symmetry but it also has 90° rotational symmetry. Another type of symmetry exists within this knight's tour that is not readily apparent. Consider the numbers in each row of Figure 1(a) as having the same pitch on the musical scale. If the Base Pitch is considered to be the row that 1 lies in, then each number can be regarded as being a note having a pitch which is offset from the Base Pitch. Using this approach, a 64-note musical composition results. If the 64 notes are placed side by side it is observed that the notes oscillate rather irregularly, first going 'down', then 'up', etc. Notice what appears, however, when notes 33-64 are placed beneath the first 32 notes:



The second half of the composition is a mirror image of the first half! Perhaps this could be called 'musical symmetry' (instead of a translation and reflection).

Another way to represent a knight's tour is to consider the moves that the knight makes as it jumps from square to square. The knight can make the following moves: (0) right one, up two, (1) right two, up one, (2) right two, down one, (3) right one, down two, (4) left one, down two, (5) left two, down one, (6) left two, up one, and (7) left one, up two. Using these operations, the knight's tour in Figure 1(a) can be uniquely identified by the 63-digit number.

450360276022723436775712135245070147246324663670723313565716014

A question that one can ask is 'Are there knight's tours that are palindromes when presented in this way?' That is, are there 63-digit numbers that read the same from right to left and left to right? Although the knight's tour discussed in this article does not have a 'palindromic' representation, perhaps there's a knight's tour waiting to be found that does. Perhaps it, too, may have some interesting properties.

Bibliography

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Weggel, Rob, personal correspondence, 1992, a Drexel University (Philadelphia, PA) student who used the 63-digit representation as output for a computer program he wrote that finds solutions to the knight's tour problem.