

SYMMETRY IN MAGNETISM

Martha Pardavi-Horvath
 Institute for Magnetism Research
 The George Washington University
 Washington, D.C. 20052
 U.S.A.
 pardavi@seas.gwu.edu

Magnetism is a collective phenomenon. One can visualize electronic devices operating based on a single electron, having sizes on the order of atomic dimensions, - magnetism needs at least about one million magnetic atoms to be noticeable. And still, magnetism is based on the properties of single electrons¹. Each electron is carrying its own magnetic angular momentum S , a totally quantum-mechanical entity: the *spin*. Depending on the number of "magnetic electrons", different atoms carry different magnitudes of magnetic momentum. In some elements (Fe, Co, Ni), compounds and alloys the electronic spins interact, exchanging information about their direction via the Heisenberg *exchange interaction* between spins S_i and S_j :

$$E_{\text{ex}} = -J \sum_i S_i \cdot S_j$$

where J characterizes the strength of the interaction. The result is the collective *ordering* of the spin magnetic moments. Depending on the sign of the exchange interaction, there are several possibilities for ordering of the electronic spins, as illustrated in Fig. 1: parallel ordering of the spins of an elemental magnetic metal, like iron gives rise to **ferromagnetism** (Fig. 1a) -with a permanent macroscopic magnetic moment $M > 0$. In the everyday language this kind of symmetry is called "*magnetism*". However, Nature is much more creative: there is a possibility for antiparallel ordering, with a resulting zero total magnetization $M = 0$. This is the case of **antiferromagnetism** (Fig. 1b). If a material (alloy, compound) has more than one kind of magnetic constituents, they might order antiferromagnetically, and the resulting magnetic symmetry is the **ferrimagnetism** (Fig. 1c).

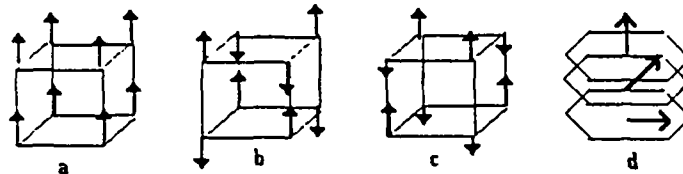


Figure 1 Magnetic ordering: *a* - ferromagnetic, *b* - antiferromagnetic, *c* - ferrimagnetic, *d* - helimagnetic (the arrows represent the magnetic moments of the atoms in a hypothetical elementary crystal lattice).

The type of the symmetry, the order in the arrangement of the individual spin magnetic moments is determined by the crystal symmetry of the given magnetic material. The interaction of spins with the internal crystalline fields creates the magnetocrystalline anisotropy energy, determining the directions

illustrated on the case of the most simple domain structure of a magnetically uniaxial (however, crystallographically cubic) thin single crystalline film of a magnetic garnet crystal (based on YIG: $\text{Y}_3\text{Fe}_5\text{O}_{12}$, thickness=5 μm , $A=3\times 10^{-7}$ erg/cm, $K=-2\times 10^6$ erg/cm³, $M_s=10$ G). The microphotographs are taken in polarized light, and the contrast is based on the interaction of polarized light with magnetic materials (*Faraday effect*). The average periodicity of the pattern is 10 μm . The black and white contrasts correspond to the "up" and "down" directions of the magnetization in the individual domains.

Fig. 2. illustrates the demagnetized state of the crystal, i.e. the $M=0$ in $H=0$ state. Even in this simple case, there are numerous possibilities to get zero magnetization, i.e. equal area of black and white stripes.

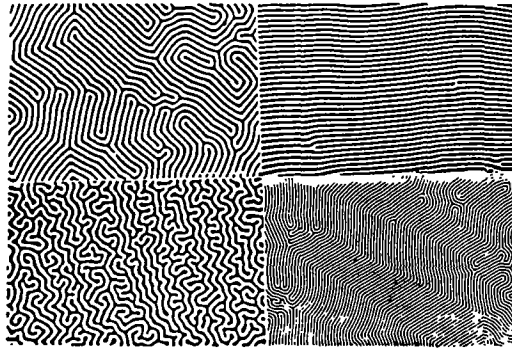


Figure 2 Various equilibrium demagnetized magnetic domain structures ($M=0$ in $H=0$) of an epitaxial YIG garnet film in polarized light

Applying a magnetic field to this magnetic garnet, the energy of the system changes and the domain structure will change, according to the new state of equilibrium. Depending on the magnitude, homogeneity and frequency of the applied magnetic field, we can observe the "bubble" domains, shown in Fig. 3a, or in a microwave magnetic field the circular domains of Fig. 3b.

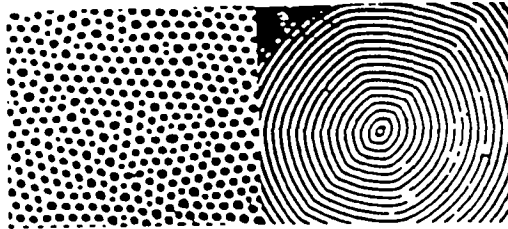


Figure 3 Circular "bubble domains" in an applied dc magnetic field (a), circle domains in a microwave field (b)

The epitaxial garnet crystals are among the most perfect single crystals ever made. They are applied in several high technology devices and sensors, equipments for optical and microwave communication, control and measurements. The perfection of the crystals is reflected in the domain patterns of Fig. 2. These crystals are grown epitaxially on (111) oriented substrates. According to this direction, there is a 3-fold anisotropy in the plane of the crystal. The magnetic domain structure is a sensitive detector of this anisotropy, as it is illustrated in Fig. 4. Starting from demagnetized state (a), then applying a magnetic field, the 3-fold symmetry of the (111) plane of the crystal is revealed.

of the magnetic moments ($\sin \theta$) with respect to the crystal axes:

$$E_m = K \sin^2 \theta$$

where K is a quantum-mechanical constant of spatial anisotropy. A delicate balance of other possible interactions (thermal, elastic, magnetoelastic, electromagnetic, etc) results in a wide variety of symmetry patterns in the arrangement of the magnetic moments of individual atoms in the solid. The magnetic moments of the rare earth metals show the most astonishing variety of the magnetic symmetry: the spiral, helicoidal, non-collinear symmetry of the magnetic structure (Fig. 1.d).

The observable magnetic symmetry is always a state of minimum energy, i.e. it is the state of equilibrium. Any change in the parameters of the system (temperature, stress, electromagnetic fields, etc) will change this magnetic order. The equilibrium state as a manifestation of the principle of the minimal energy is a general property of matter, it is the basis for stability. Magnetism is not an exclusion from this principle. The equilibrium state possesses a certain symmetry, as illustrated in Fig. 1. However, this principle governs not only the symmetry on the level of the arrangement of atomic magnetic moments, it also determines the magnetic symmetry of macroscopic bodies.

The principle of the minimum of the *total energy* of a magnetic body of *finite dimensions* would require that in the absence of any externally applied magnetic field, H , the total magnetization should be zero. In the case of a ferromagnet (or a collinear ferrimagnet) this seems to contradict to the symmetry based on the parallel arrangement of magnetic momenta. However, the principle can be still satisfied, if not on the atomic "*microscopic*", but on a higher "*mesoscopic*" level¹.

The distribution of the magnetic moments in a macroscopic crystal is the result of the compromise between the parallel arrangement of spins, required by the exchange interaction and the requirement of zero magnetization in zero applied magnetic field. The result is that the total magnetization breaks down into smaller areas, each of these areas, - called **magnetic domains** -, has all the magnetic moments ordered ferromagnetically inside the domain. At the same time, the total magnetizations of individual domains point in different direction, resulting in a zero average magnetization. The *magnetic domain structure* - the mesoscopic level magnetic symmetry - depends on the microscopic material parameters (electron structure of the atoms, exchange and anisotropy energies, crystal structure), determined by quantum mechanical principles, but it depends on the *size and shape* of the magnetic material too, providing an interesting example of the unity of quantum and macroscopic properties. These equilibrium magnetic structures can be calculated from the Hamiltonian of the system (i.e. by minimizing the total free energy). However, the integral-differential equations, leading to the domain structure, do not have a trivial or simple solution in most of the cases. For a three-dimensional body the analytical solution is generally not available. For thin magnetic films with relatively simple properties, the analytical solution is feasible and it is in good agreement with the symmetry of the observed domain structures.

In the following, the rich variety of the possible solutions to the energy minimization problem will be

¹ In fact, the *mesoscopic* level is the one, which should be called *microscopic*, because it is observable under a microscope

However, this symmetry is not perfect, indicating that the substrate is inclined with respect to the $[111]$ surface normal.

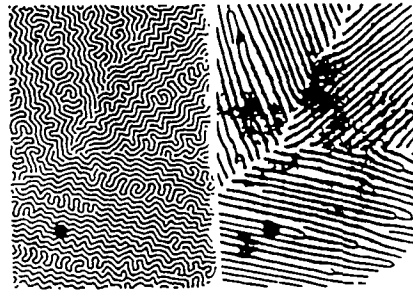


Figure 4 *Demagnetized domain structure (a) and domain structure in applied magnetic field (b) of a garnet film on a misoriented (111) substrate*

In a material with non-uniform properties, crystalline or other defects, the energy of the system will be non-uniform too. There are local minima of the energy, and the symmetry of the magnetic domains follows the path of the minimum energy. This is the basis of the application of these materials in defectoscopy. In a specimen with strong spatial inhomogeneity and nonregularity of the properties, several kinds of domains may coexist, as shown in Fig.5.

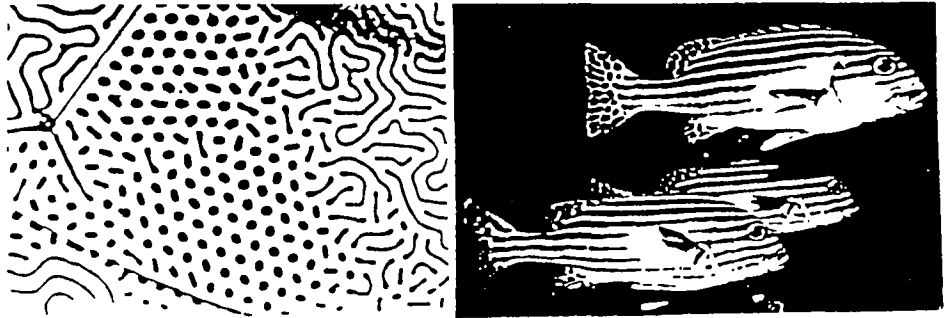


Figure 5 *Coexisting stripe and bubble domains in non-uniform materials.*

References

1. See e.g. in *Magnetism*. Edited by George T. Rado and Harry Suhl, Volume III, Academic Press, 1963.