SYMMETRY ?
MOIRÉ vs OTHER QUASI-RELATED MOIRÉ PATTERNS

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Abstract
This paper deals with the observation of quasi-moiré patterns generated when two equal 2D transparencies containing any type of structures, like geometrical figures or even photographic pictures, are coplanarly superimposed. These type of patterns are called quasi-moiré patterns, because they obey the Transverse Translation rule of moiré patterns, but not the Symmetry rule.

Introduction

Moiré fringes are visually observed when two periodic (like a comb) or non-periodic (like a bar code) 2D grids or dots arrays are coplanarly superimposed and rotated. Moiré fringes pattern obeys two main rules: 1°) Transverse Translation. It means that moiré fringes moved perpendicular to the direction on which one of the grids is displaced, preserving the sign of the displacement, and 2°) Symmetry. It means that if one of the grids is turned down and coplanarly superimposed and rotated again to the other grid, moiré pattern is still observed. This paper deals with the observation of quasi-moiré patterns generated when two equal 2D transparencies containing any type of structures, like geometrical figures or even photographic pictures, are coplanarly superimposed as in grids or dots arrays cases. Visual observations using i) non-perpendicular biperiodic grids, ii) periodic (x) – random (y) grids, iii) completely random grids, iv) and fractal grids were analysed. Examples of these types of 2D transparencies are: i) an array of alphabetic ordered Greek letters in successive lines, but introducing some displacement between letters from one to the next line, ii) a CD magnified area, on which tracks are 1.6 μm periodically spaced, and recorded signals look as a random array, iii) a magnified metallographic picture showing the grain structure, and iv) any natural or prepared fractal picture, respectively.

* Work partially financed by CONICET (PID-BID-CONICET 1116/91 grant) and UNLP (FCE-3711/97).

Observations

When two identical transparencies T_1 and T_2 of the grain structure of a metal are coplanarly superimposed and the upper one is rotated an angle $+\alpha$ against the other, the quasi-moiré pattern observed consists of circles centred at the point of rotation. The wider the rotation angle is, the smaller the radius of the entire circular quasi-moiré pattern results. This means that the observation of the circular quasi-moiré pattern depends on the degree of resemblance between details in both transparencies. As it is obvious, its centre is located where the degree of resemblance is higher. Also, if the rotated transparencies are relatively displaced along the $\pm$ *x*-axis or the $\pm$ *y*-axis, the circular quasi-moiré pattern moved along the $\pm$ *y*-axis or $\mp$ *x*-axis, respectively. Then, it means that the Transverse Translation rule is verified. But if one of the transparencies is turned down and coplanarly superimposed again onto the other and rotated any angular value, the circular quasi-moiré pattern is not observed at all. So, the Symmetry rule does not hold any more, because the degree of resemblance between transparencies is zero. For this reason this pattern was called quasi-moiré pattern, because it obeys the first main rule on Transverse Translation, but not the second one related to the symmetry. As an example, Figure 1 shows a picture of the circular quasi-moiré pattern obtained by superposition and rotation of transparencies T_1 and T_2.

There are two more experimental observations using the same type of transparencies that can be explained in the insight of the degree of resemblance concept. Superimposing an original transparency on a scaled replica of itself can perform the first experiment. Both operations result in the observation of a radial lines quasi-moiré pattern, whose centre of dispersion is placed in a point where the degree of resemblance between details in both transparencies is higher. The area covered by the radial lines quasi-moiré pattern depends upon the scaling factor between transparencies. The higher the scale percentage is, the smaller the radius of the radial lines quasi-moiré pattern results. Of course, in this case the Transverse Translation rule is reduced to a Longitudinal Translation rule, as it occurs when a Ronchi linear grid is perpendicularly displaced to its traces onto a scaled replica of itself. Let be a scaled up, i.e. + 10 %, transparency onto the original one. A relative displacement between both transparencies along the $\pm$ *x*-axis or the $\pm$ *y*-axis will produce the longitudinal translation of the radial lines quasi-moiré pattern along the $\mp$ *x*-axis or the $\mp$ *y*-axis. On the other side, let be a scaled down, i.e. - 10 %, transparency onto the original one. Then, a relative displacement between both transparencies along the $\pm$ *x*-axis or the $\pm$ *y*-axis produces the longitudinal translation of the radial lines quasi-moiré pattern along the $\pm$ *x*-axis or the $\pm$ *y*-axis. Summarising this observational facts, it results that all-relative transparency displacements will produce the higher degree of resemblance at the centre of the radial lines quasi-moiré pattern. However, the symmetry rule does not hold as in the case of the circular quasi-moiré pattern. Figure 2 shows the radial lines quasi-moiré pattern observed by the superposition of transparency T_1 on its 10% scaled replica.

The second experiment can be performed after the observation of the radial lines quasi-moiré pattern by rotating one of the transparencies against the other. In this case, the radial lines quasi-moiré pattern turns to multiple logarithmic spirals quasi-moiré pattern, whose centre is placed in the point of the centre of the radial pattern, that is, where the degree of resemblance between details in both transparencies is higher. The arms of the spirals are developed according to the sign of the rotation angle and the sign of the scale factor between transparencies. For example, clockwise arms are developed when the upper transparency is rotated - α, and vice-versa, counterclockwise arms are observed with + α rotation angle. In this case, the Transverse Translation rule results as a vector composition between the circular quasi-moiré pattern rule and the radial lines quasi-moiré pattern rule. So, it means, for example, that displacements of one transparency along the $\pm$ x-axis will produce translations of the spirals centre along lines that are inclined with respect to the x-axis according to the sign and value of the scale factor and the sign and value of the rotation angle α between transparencies. Besides, the Symmetry rule does not hold in any case. Figure 3 shows the multiple logarithmic spirals quasi-moiré pattern observed by rotating transparencies at Figure 2.

Degree of resemblance and correlation

Finally, a comment related to the quantitative evaluation of the degree of resemblance between transparencies is presented. If the structure contained in the transparency could be expressed as a mathematical function $f(x,y)$, the degree of resemblance can be defined as the correlation function $f * f$, according to the expression:

$$f(x,y) * f(x,y) = \iint_{-\infty}^{\infty} f(u,v) f(x-u, y-v)\, du\, dv ,$$

as in the case of moiré patterns generated by the superposition of periodic grids.

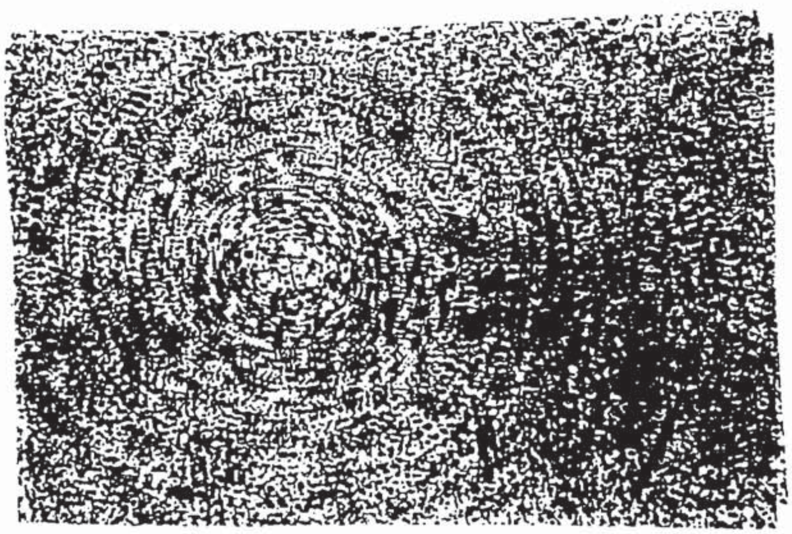

Figure 1. Picture of the circular quasi-moiré pattern obtained by superposition and rotation of two identical transparencies, T_1 and T_2, of a micrometallogram.

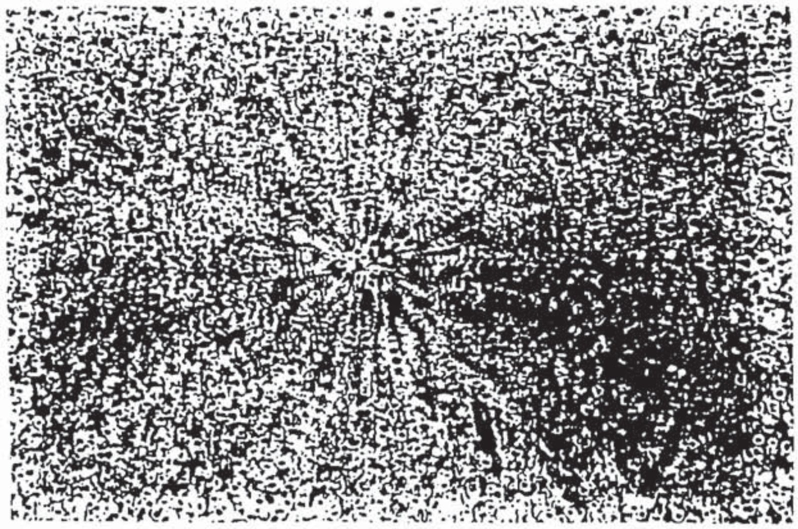

Figure 2. Picture of the radial lines quasi-moiré pattern observed by the superposition of transparency T_1 on its 10% scaled replica.

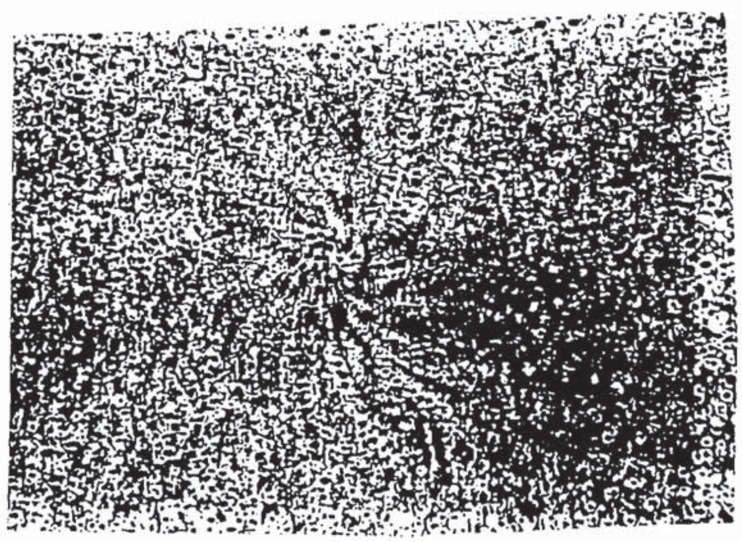

Figure 3. Picture of the multiple logarithmic spirals quasi-moiré pattern observed by rotating transparencies at Figure 2.