SYMMETRY IN COLOR ORDER SYSTEMS

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Color order systems are conceptual or material structures that organize colors according to some criteria. Along the history of color research, hundreds of such models have been proposed by many theorists and scientific corporations. In the long list of these color systems, we find linear arrangements, circular or polygonal schemes, three-dimensional organizations, and even models with more than three dimensions. Most of the present color systems are three-dimensional models that allow for the orderly unfolding of the three usual color variables: hue, lightness, and saturation. As the criteria for the scaling of these variations are different and, most importantly, as the conceptions of color these models take into account are also diverse (color light, color pigment, color stimulus, color sensation), the geometric shapes that represent these systems are manifold. However, most of them respond to some principles of symmetry which depend on the organizational criterion or are bound to it.

In this paper, I call the symmetry related to the shape of the color models "external symmetry", and the symmetry related to the color scales inside the models "internal symmetry". It is argued that color order systems have evolved from external to internal symmetry, and that both kinds of symmetry cannot go together in the domain of the orderly representation of color perception.

The first color diagrams that appear in the middle ages usually relate colors to some other things. This is the case of the fourfold system by Byrtferth de Ramsey, with the four cardinal points, four elements, four ages, four seasons and four colors, and the three color rings by Joaquín de Flora, with the Divine Trinity: Father, Son, and Holy Spirit (Fig. 1). In these diagrams, symbolism and symmetry play a key role in the relationship between colors and the other represented concepts or things.

Fig. 1. a) Fourfold system with colors by B. de Ramsey, c.1080. b) The Trinity with colors by J. de Flora, XII century.

The first color wheel seems to have been devised by Robert Fludd in 1626. A succession of seven colors with black and white at the extremes are organized in a circular diagram (Fig. 2a). In 1704 Newton publishes his color circle, comprising seven colors that he observed in the spectrum of white light. These colors are separated by unequal intervals that are proportional to the seven notes of the diatonic musical scale (Fig. 2b). The chromatic circle by Moses Harris, published in 1776, organizes color relationships in a symmetric fashion, opposing the primary color triad red/yellow/blue to the secondary color triad orange/green/purple (Fig. 2c). This is the direct ancestor of the color circles or the sixfold color stars used by the painters from 1800 to our century (Fig. 2d). In between these six colors (three primary and three secondary ones), six other intermediate colors are placed that are considered as tertiary ones. Except for Newton's circle, all the other diagrams have bilateral and rotational external symmetry.

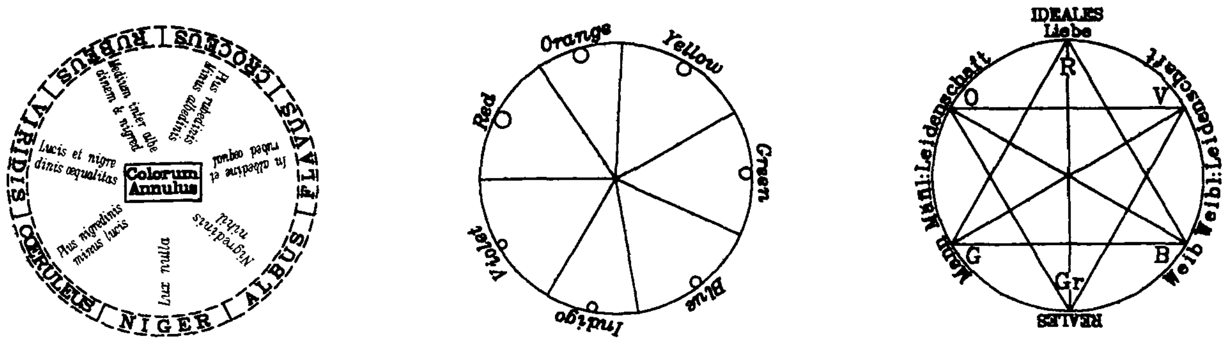

Fig. 2. Color circles: a) Robert Fludd, 1626. b) Isaac Newton, 1704. c) Moses Harris, 1776.

Already in the second half of the XVIII century some theorists realized that all colors could be obtained by mixture of three specially chosen colors, and thus color triangles appeared. But these triangles lacked black and white. By placing an axis perpendicular to the triangle, the scale of grays between black and white is related to the chromatic colors, giving origin to a three-dimensional model for color organization. The first of these color organizations appears around 1770, with Tobias Mayer's double pyramid and Lambert's pyramid (Fig. 3a). The next evolutionary step is the appearance of color spheres, such as the one by Philip Otto Runge (1810). The twelve colors of the chromatic circle are placed on the equator of the sphere, white is at the north pole and black at the south pole, and the axis that connects these poles contains the gray scale. We have the greatest amount of symmetry here (Fig. 3b). At the end of the XIX century, various color cones appear that instead of representing the organization of surface colors or pigmentary colors represent the organization of color lights, such as the model proposed by Bezold (Fig. 3c).

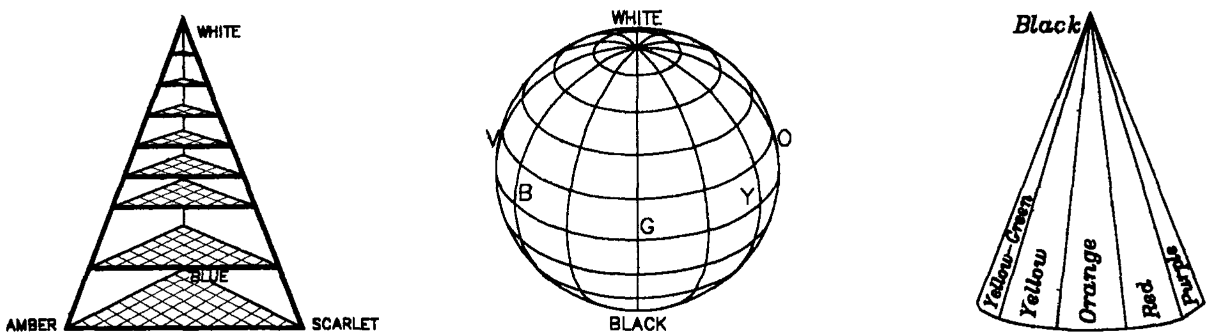

Fig. 3. a) H. Lambert, color pyramid ,1772. b) P. Runge, color sphere, 1810. c) W. von Bezold, color cone, 1874.

In 1917, Wilhelm Ostwald devised a double cone in which colors are organized and notated according to three variables. The circle that connects both cones at their bases contains 24 hues. In order to have equal perceptual steps, the succession of grays with white at top and black at bottom is organized in a logarithmic scale of reflectances, according to the Weber-Fechner law that establishes that the sensation varies proportionally to the logarithm of the stimulus (Fig. 4).

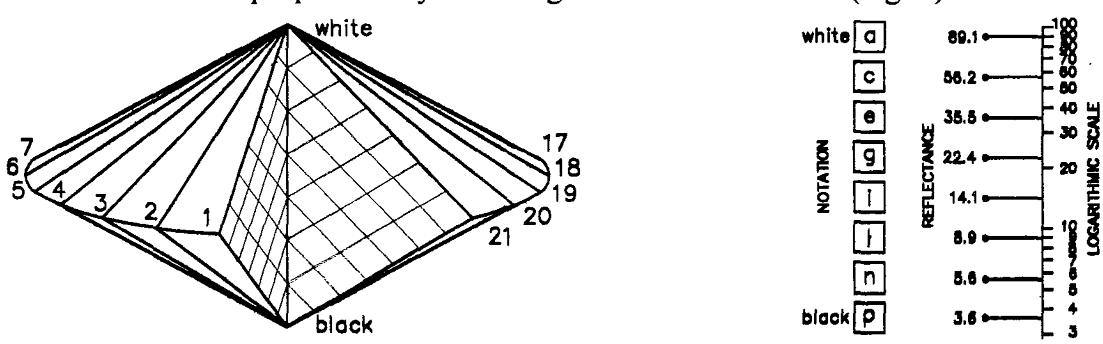

Fig. 4. Wilhelm Ostwald's double cone and logarithmic scale of grays.

Very soon, some theorists realized that the Ostwald model was somehow inaccurate, because in order to have more homogeneous color gradations yellow should be nearer to white and violet nearer to black. Thus, the chromatic circle was tilted to accomplish this purpose, the horizontal plane of bilateral symmetry of the double cone being lost. The solid of Pope (1949) illustrates this evolutionary step (Fig. 5a). However, it should be noted that Kirschmann had already published a tilted double cone in 1895, before Ostwald's symmetrical double cone (Fig. 5b). Considering the internal color relationships, all these models show the best examples of antisymmetry, because symmetrical points are occupied by black and white and the complementary hues.

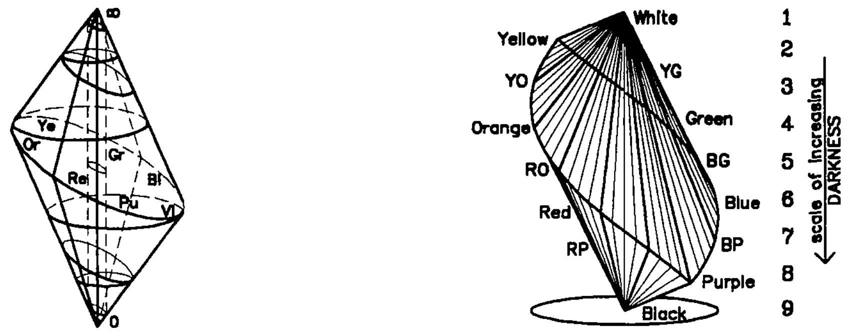

Fig. 5. Tilted double cones: a) August Kirschmann, 1895. b) Arthur Pope, 1949.

When actual color samples were produced in order to represent every point and every step within a color solid, color spheres, symmetrical double cones and regular tilted double cones proved unsuitable to contain equal color intervals within their boundaries. The Munsell atlas and color solid is the result of this fact, and in spite that the principle of organization is similar to the previous models the external shape is completely irregular and non symmetrical (Fig. 6).

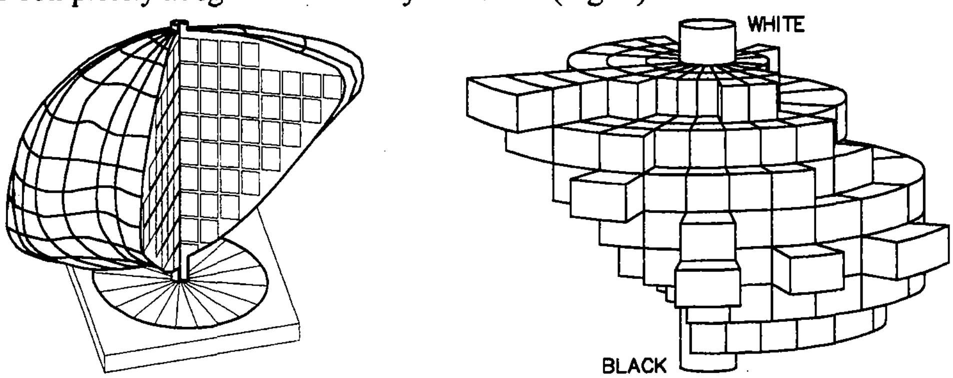

Fig. 6. Two representations of Albert Munsell's color solid.

The first conclusion that seems to derive from this process is that to be regular and symmetrical in their internal relationships (that is, with intervals between colors perceived of the same size throughout the model, holding a kind of translatory symmetry), the color space must have irregular and non symmetrical external shape. The particular evolution of the system devised by the Commission Internationale de l'Eclairage (CIE) gives support to this argument. The point of departure is the Maxwell equilateral triangle, with three spectral lights at the vertices (red, green, blue), mixtures of two lights on the sides, and mixtures of the three lights towards the center, where white is obtained (Fig. 7a). But the mixture of two spectral colors does not give a third pure spectral color, so that in order to include all spectral colors a curve was traced that is external to the triangle (Fig. 7b). Not to have negative numbers to deal with, this shape was included within the boundaries of a larger triangle with three ideal primary stimuli (X, Y, Z) at its vertices (Fig. 7c). This triangle was then transformed into a right-angled triangle in order to define colors by means of two coordinates, x and y (Fig. 7d). This is the CIE chromaticity diagram of 1931.

In 1942 MacAdam proved that the CIE space was not uniform in terms of color differences. According to his data, equal color differences in the CIE triangle, instead of being represented by circles of equal size, are represented by ellipses of different size and orientation (Fig. 8a). Thus, new transformations were necessary to obtain a uniform chromaticity diagram. With this purpose, MacAdam developed in 1944 a surface that is not planar any longer (Fig. 8b). In 1949, Stiles proposed his own surface of uniform chromaticity (Fig. 8c). Skipping some intermediate steps we arrive to 1997, when Socolinsky and Wolff produce a surface that most accurately represents a uniform chromaticity diagram according to MacAdam's original data (Fig. 8d). Evidently, from the initial equilateral triangle to this complex surface, a high amount of external symmetry has been sacrificed for the sake of internal symmetry and regularity in terms of color differences.

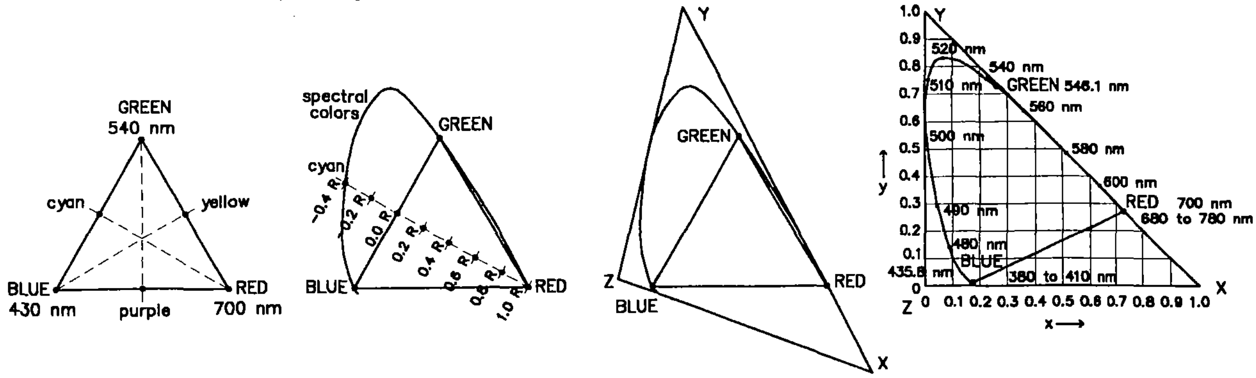

Fig. 7. CIE system. a) Maxwell equilateral triangle. b) Spectral colors placed on a curve outside the equilateral triangle. c) Scalene triangle with the three ideal stimuli X, Y, Z. d) Right-angled triangle with the coordinates x, y.

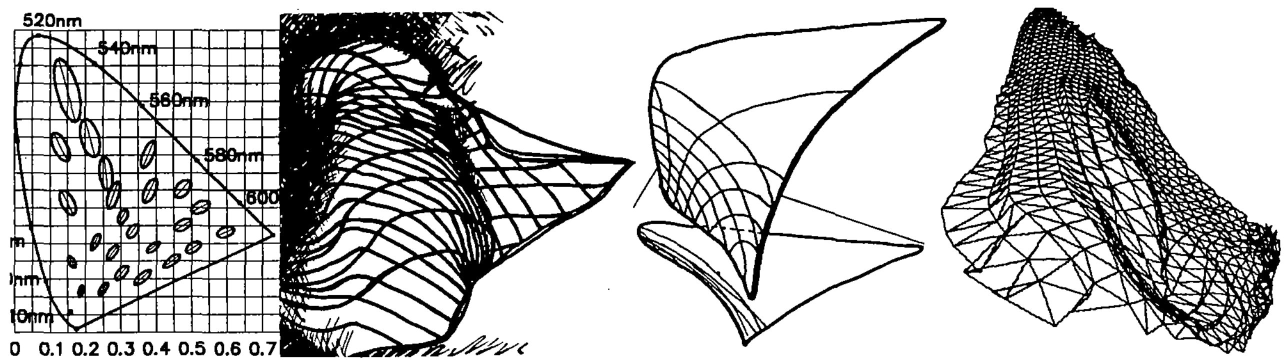

Fig. 8. a) MacAdam's ellipses of chromatic discrimination. b) Surface of uniform chromaticity by MacAdam. c) Surface of uniform chromaticity by Stiles. d) Embedding of the CIE diagram by Socolinsky and Wolff.

We have seen that the shape defined by the limits of chromatic discrimination of the human visual system is non symmetrical. This means that the visual system is non isotropic with respect to color perception, that is to say, the axes of chromatic perception extend in unequal lengths as measured in different directions.

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