

SYMMETRIC SURPRISES FROM VIRTUAL ANTS

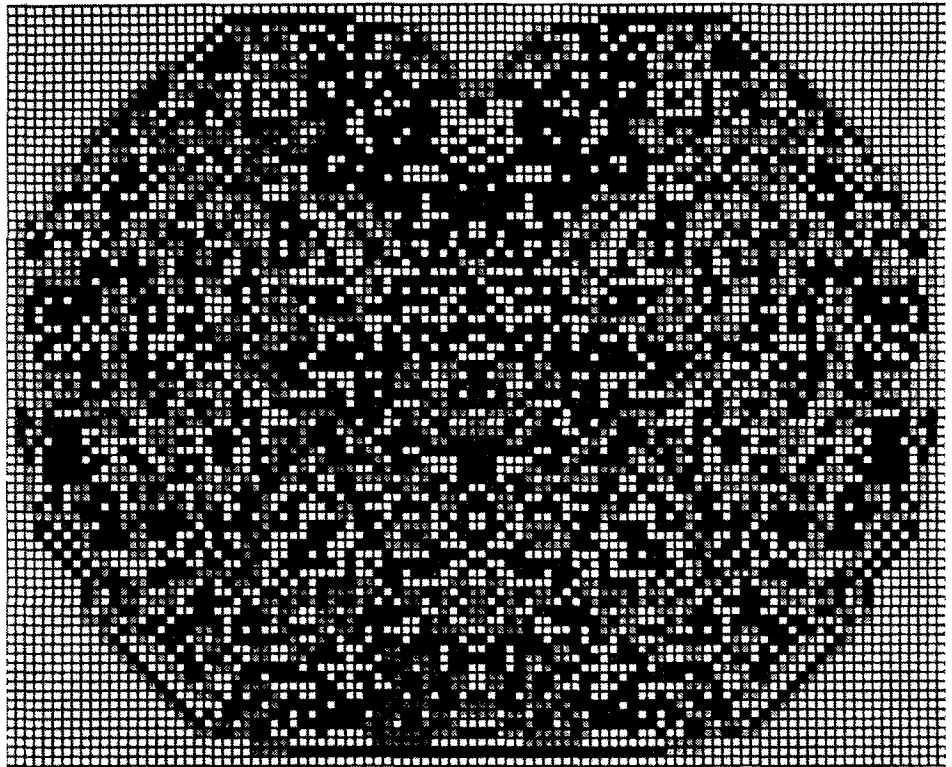
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Virtual ants are imaginary creatures inhabiting a simple but infinite world that they explore and modify. Their lifestyle is a humble one: when an ant comes to a location in its universe, which is in one of a finite number of possible states, it alters the state of that location and it makes either a right turn or a left turn before continuing on to a new location. In its travels it exhibits no freedom at all; its course of action is completely determined by its environment.

Despite being such a simple-minded creature, an ant will often create intricate structures that exhibit symmetry. This sort of symmetry is not that of an (idealized) growing snowflake, which remains completely symmetrical from beginning to end; rather it is a *recurrent* symmetry that is repeatedly destroyed and re-created. Moreover, the structures created by the ant grow larger and larger, so that the symmetry of the growing structure requires that the ant be capable of introducing and maintaining resemblances between the way things look at one location in its world and the way things look far away, where the notion of “far away” increases to infinity with the passage of time.

How is it possible for the ant to do this, given that it has neither memory nor the ability to plan? The answer, discovered by German computer programmer Bernd Rümmler, is that the “surface-level” rules that govern the ant are best understood in terms of a deeper structure that enables one to see that in between the moments at which the structure is symmetrical, the



symmetry is not completely destroyed but has merely “gone underground”, encoded by some hidden properties of the ant-universe. This deeper structure is most clearly exhibited through the use of marked squares known as Truchet tiles. Interestingly, Rümmler’s discovery was facilitated by a *lack* of up-to-date computer equipment.

The picture on the preceding page represents the appearance of the universe after one of these ants has been exploring an (initially blank) universe for one million steps. The location of the ant itself is not shown, but if you look hard you’ll notice that the configuration is not truly symmetrical; as it happens, the ant is located in the small spot where the asymmetry is concentrated. In general, the asymmetry is more dispersed; it just happens that we have caught the ant at an instant when it is about to recreate total symmetry (at time one-million-and-four).

I will demonstrate a computer program that simulates the virtual ant and the universe it inhabits, and that helps one to see how the ant manages its impressive feats of half-chaotic, half-orderly architecture.

References:

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