SHAPING MIND BY SHAPING SPACE –THE TETRAHEDRON AS PARADIGM

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In a previous paper (1) I have tried to introduce symmetry as an overarching educational principle, as an organizing principle of active shaping of the mental development as well as an explanatory principle of scientific understanding of nature It thus was meant to relate structures of mind with structures of nature, as criticistic or constructivistic concepts about the relations between man and the world around him would postulate.

In particular, symmetry arguments were understood as a key concept in causal analysis of processes of self-organisation in nature, as an explanatory paradigm of modern science from its very beginning, instead of an only recent, even revolutionary concept, as is currently frequently proclaimed· For this understanding the meaning of „natural laws" in general, of „atomism" in particular is that structures in nature are generated in a natural way by means of those qualities by which the building elements interact causally (Thermodynamical calculations on states „far from equilibrium", with the result of self-organisation as an astonishing seeming contradiction to the Second Law may certainly count as an alternative, but perhaps not very adequate path to explain structures as caused by atomic interaction.)

In the abovementioned paper I have based this view on the role of symmetry on rather general arguments; in this note I shall try to illustrate it by more concrete, mainly chemical considerations As is usual in education, in doing so no really new insights in a scientifc sense will be gained, but rather a basical understanding of the inner context of otherwise well known scientific results The basic idea is: A set of atoms organizes itself to well defined structures instead of disorder (as the Second Law might seem to demand), because those atoms do not just move independently in a statistical manner, but do interact by means of forces between each other. The structures resulting depend from the symmetries of the atomic fields.

The paradigmatic symmetry of chemical interaction is that of a tetrahedron, because, following Bohr's Theory and Pauli's Principle, the most stable configuration of bonding electrons is that of one pair of s-electrons and three pairs of p-electrons. By means of quantum mechanical exchange forces these two kinds of bonding electron pairs average out to result four completely equivalent pairs of electrons, at least in the most „symmetric" case of the carbon atom, and these four equivalent bondages when distributed symmetrically in space result a tetrahedron So we will concentrate on this case, but we should keep in mind - and will demonstrate by some characteristic examples - that the manifold variety of different chemical laws and interactions giving specific varieties of structures may be understood as variations of the tetrahedral structure, which to their side are the result of specific laws of the concerned atoms.

If we couple three of these tetrahedra, the bonding angle between them will be about 103 degrees, and we will get a wide manifold of possible chemical structures if we make use of the possibility of rotations around the bonding axis and the arbitrariness of the free tetrahedral bonding directions that may be saturated by specific functional groups, the result of these combinations are the huge varieties of possible organic molecules that may be achieved in preparative chemistry by making use of the differences in energy levels that exist because of the influences of the functional groups in the neighborhood. We may now bridge the often stated gap between „molecular" and „solid state" chemistry by realizing that there exist two symmetrical positions of the tetrahedra that lead to two specific symmetrical configurations· two tetrahedra may be fixed to each other „ cross-wise" (peak opposite edge, and vice versa) or „parallel" (peak opposite peak, and edge opposite edge) (Fig 1). The first (left) will result an „undulating" hexagon, the second (right) a regular pentagon.

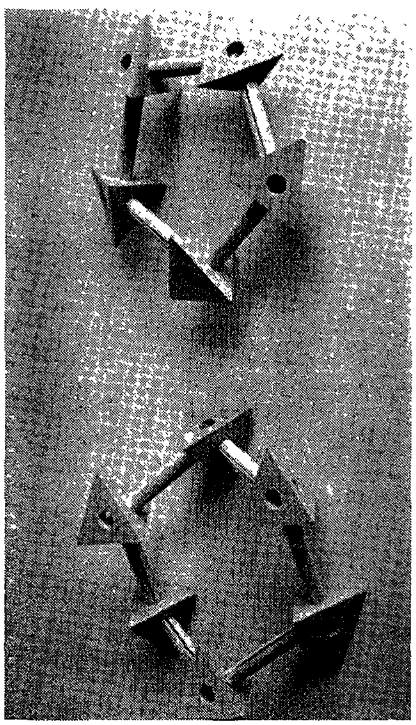

Fig 1 Symmetric Arr. of Tetrahedra

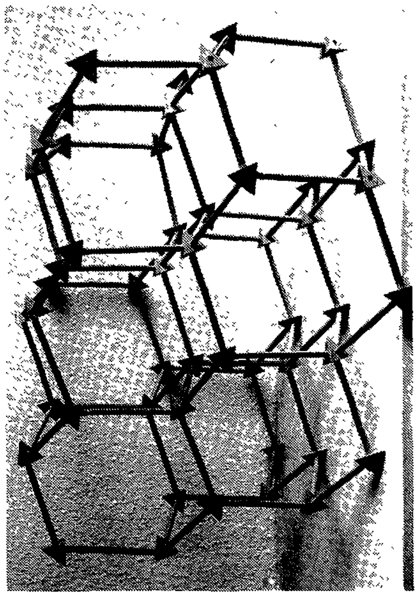

Fig.2 Diamond Lattice

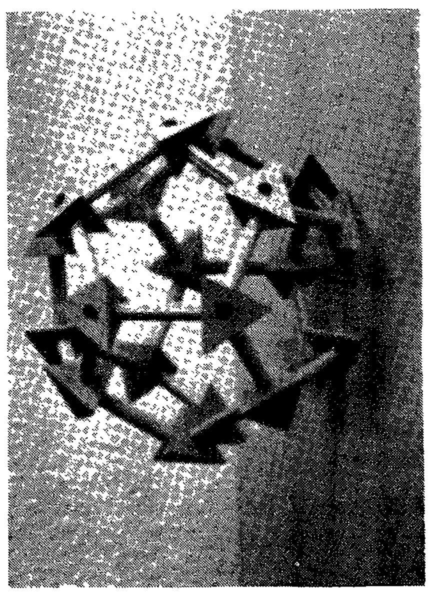

Fig 3 Dodecahedral Arr of Tetrahedra

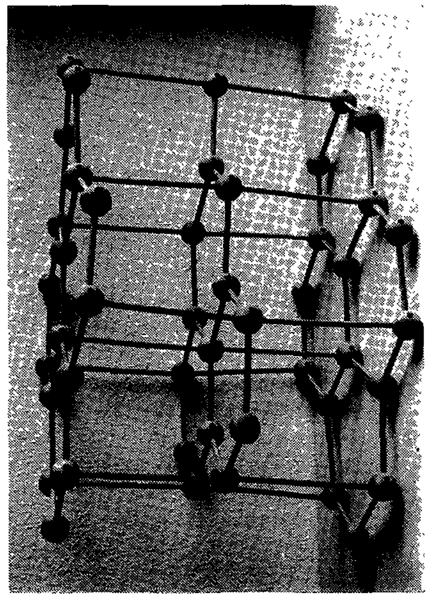

Fig 4 Graphite Lattice

It is possible to continue this building principle regularly into space by consequently symmetrical, namely „cross-wise", addition of tetrahedra, the result will be the well known „diamond" structure (Fig. 2), which, as is aslo well known, in nature is not restricted to diamond, but is also verified by a manifold of chemical interpretations of respective importance, as Ice, or Corund, or ZnS-type salt crystals, or A III/B V - semiconductors, and we will discuss some of them
The pentagon type „parallel" bonding also results a regular completion in space, namely a dodekahedron (Fig 3), but this, as is well known, can not be continued regularly into full space; it can not be arranged into a regular long range order, but will be arranged in true space „somehow", if such a building group really occurs in nature, as in fact is the case with water (if we follow Paulings „Chlatrate" suggestion) or with boron. The then sometimes appearing five-fold diffraction patterns have raised disturbance in the wider scientific public because it was felt as necessary to identify them with so called „quasi crystals". But we see, that neither their occurance is mysterious in the slightest sense, nor is their five-fold refraction signal: Just for the very reason that a long range order is not possible, this irregular long distance arrangement is averaged out on the diffraction pattern, and the short range order, i. e. in this case. the fivefold symmetry of the building group, must take command, if only its orientation is favored in a more than statistical sense; and again there is nothing mysterious about that.
On the contrary, the tetrahedral structures paradigmatically show the differences between the different investigative „views" onto a structure. Just the most representative atomistic bonding arrangement, that of the tetrahedron, cannot be expressed in space in terms of the mathematically favored coordinate systems, because it would need six (= 2x3), instead of four, bondages, esp. in the most symmetrical case, namely the cubic system. Nevertheless, the resulting „diamond" structure may be expressed crystallographically in terms of the cubic system, though its a very complicated „cubic" description, that makes use of four simple cubes per elemtary cell, set into each other Even worse, however, that this crystallographic description is a mere formal one, that by no means expresses the real chemical relation of the atoms, namely that they are arranged in a tetrahedral shape which is caused by the quantum mechanical laws of the atoms themselves. This situation described is widely typical for structural investigations, and the lecture will discuss in more detail these difficulties in science, and the attempts to overcome them
Finally, we will discuss the tendency esp. of carbon to change the tetrahedral structure (resulting those „undulating" hexagons) to structures of plain hexagons („honeycomb" - structure, as in Graphite (Fig. 4)) This occurs by preference of unsaturated to saturated bondage. What makes this change stimulating is that here again the characteristic mixture of hexagons and pentagons appears, but now as an expression of a formally unsaturated atomic interaction that nevertheless is energetically very stable. As in the tetrahedra caused by saturated bondage, the unsaturated hexagons compose plain, or nearly plain (esp. cylindrical) far range structures, while the pentagons take the part of involving edges in space The formal laws of this mixture give rise to the geometrical qualities of electric nanostructures, as the Hiroshima lecture of Sumio Iijima has pointed out, and to those of the Buckminster-Fullerenes At that point, my very basic educational considerations on tetrahedral structures border upon most important recent research, and i'd like to understand that as an ideal in education

Reference

(1) P Klein, On Symmetry in Science Education, in Symmetry - Culture and Science, vol 1 (1990), P 77-91

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