SEMEIOTICS, GOOD SYMMETRY, AND THE LOGIC OF PROPOSITIONS

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Semeiotics is the study of all signs and sign processes. The logic of propositions is a fundamental part of symbolic logic. If one gives central emphasis to the role of SYMMETRY, when great care is put on SHAPE designing what it takes to construct a special set of sixteen ICONIC signs, then it is possible to bring to the logic of propositions an approach that not only SIMPLIFIES and CONSOLIDATES.

This approach, with its emphasis on symmetry, also receives major assistance from the ALGEBRA OF ABSTRACT GROUPS. It lends itself to the construction of as many as a tabletop of hand-held models. It brings elementary logic into the classroom of our secondary schools. It has practical implications for digital design, mirror logic, and optical computers. It bumps into and finds much in common with crystallography. What follows is both a small piece and a key piece from this approach.

Let us look at the 2-valued logic of two sentences, no more than that. Notice that (1) is a MASTER EQUIVALENCE, one that is sufficiently abstract, one that has enough algebra in it, so that

$$(1) \qquad (\underparen{\dot{A} \mathbin{\dot{*}} \dot{B}}) \equiv (\underparen{\dot{A} \mathbin{\dot{*}} \dot{B}})$$

it covers 4096 ATOMIC EQUIVALENCES. Notice also that (1) contains fourteen nodes of substitution, seven on each side of the equivalence sign, and that (1) brings three subsystems into the same system, itself able to receive one by one all of the 4096 unique substitutions in this CUSTOM-CONSTRUCTED larger system.

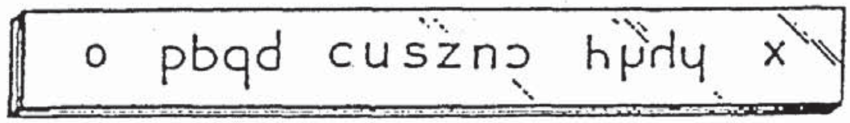
There are four nodes in the first subsystem, namely. two A's and two B's. These nodes cover the two PROPOSITIONAL variables.

There are two nodes in the second subsystem, namely, the two asterisks. These nodes cover the two CONNECTIVE variables. More specifically, each one stands for all of, and exactly any of, the sixteen binary connectives (and, or, if then, etc.).

There are eight nodes in the third subsystem, namely, six over-dots and two under-arcs. These nodes cover the eight OPERATION variables. Each dot stands for the PRESENCE or ABSENCE of (N)egation, and each arc stands for the PRESENCE or ABSENCE of (C)onversion [(A,B) or (B,A)]. Each side allows any of the sixteen combinations of dot-dot-dot-arc to operate on (A * B).

The figure at the top of the next page shows a ruler-sized FLIPSTICK, one that has been patented and one that has another set of letter-shapes (LSs) on its back side, in see-through orientation. It can be flip-flip-rotated and across the center.

mated. All of the LSs belong to a special set of sixteen TOPOLOGICAL ICONS that possess some carefully determined



ALGEBRAIC-GEOMETRIC PROPERTIES. The LSs stand for the sixteen binary connectives. These signs will be substituted for the asterisks in (1), as needed, and in keeping with any of the 4096 atomic equivalences, in order to honor the equivalence sign.

The LSs carry all combinations of four STEMS, from zero (o) to four (x). Put the truth table for (A,B) at the quadrants (TT, TF, FT, FF), in the same positions and just like the plus-minus signs in (x,y) coordinates. For example, because the d-letter has one T-stem in the upper-right TT-quadrant, N(A d B) stands for Not(A and B). (A h B) stands for the Sheffer stroke (FTTT).

Now, along the lines of what happens in crystallography (by way of the symmetries of a square), introduce ALL COMBINATIONS OF FOUR SYMMETRY RULES, along with the algebra of abstract groups. The four symmetry rules, called FLIP-MATE-FLIP AND FLIP (f-m-f and f), are in a class very much by themselves. They treat all of the LSs in the same way. NA FLIPS from left to right, N* always MATES with the LS symmetrically across the center of the flipstick, NB FLIPS from top to bottom, and C FLIPS diagonally from upper-right to lower-left. N(A an-d B) mates to (A h B).

NA, N*, NB, and C, acting alone, activate 2-groups; (NA,NB), a Klein 4-group; (NA,N*,NB), the 8-group (C2 x C2 x C2); (NA,NB, C), the octic group (D4); (NA,N*,NB,C), the 16-group (D4 x C2).

This approach lends itself to the construction of a whole family of hand-held symmetry models. In effect, (1) is a master equivalence that subordinates 4096 atomic equivalences, which is ALL AND EXACTLY ALL of the WELL-FORMED EQUIVALENCES that can be written under this algebraic form, including De Morgan's laws and many of the standard tautologies, when the asterisk on the right is determined by all of the subsets of substitutions that can be made at the other thirteen nodes. The marvel is that it takes only four symmetry rules (f-m-f and f) to cover all of these determinations, when THE RULES FIT THE CALCULATIONS.

References (See next abstract for more diagrams of the models).

Clark, Glenn (In Press). New Light on Peirce's Iconic Notation for the Sixteen Binary Connectives. In <u>Studies in the Logic of Charles S. Peirce</u>, Nathan Houser, Don D. Roberts, and James Van Evra (eds.), Bloomington: Indiana University Press.

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