

SEMANTIC ASPECTS OF SELF-SIMILARITY IN MUSIC

(Harmony in Non-linearity)

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1. There is a great number of self-similar process of fractal type in Nature and Society. Recently self-similarity has been discovered in Music too [1]. Available modern mathematical description does not explain the presence of fractal processes everywhere, it does not discover their sense, semantics and dynamics. We believe it is necessary that we should expand the strategy of research, including the subject factor in the investigated phenomenon. Then genesis, sense and semantics of self-similarity in Nature will be apprehended by us in the context of more general universal processes, through the reflexion of our own activity, that leads us to creation of fractal-similar structures. And research of the object in the context needs special methodology, mentioned below.

2. As a model of a fractal, "created" (but not "set by the formula" $X_{n+1}=X_n^2+C$, ref. [5]) we took a short and "not complicated" classical musical composition, whose methodical line has self-similar characteristics. It is the Prelude C-dur by F. Chopin. Melody of Prelude has a characteristic of discrete fractal, saving a complicated tracery for all reduction. Frequency of incidence of note-intervals of its melody line almost perfectly corresponds to $1/f$ distribution [2] — temporal analogue of fractal's form. It is well-known that methods of mathematics state only the result, fixing its quantitative side. With such approach sense and semantics of self-similarity formation leave "of screen". But modern musical theory does not either describe sense and semantics of these processes, for here we deal with structures that are not fixed by notation [3].

3.1. There is a problem situation, which requires a revision of the research strategy. It is necessary to include the perception (mentality) factor in a considered phenomenon of self-similarity. In this connection we have offered the new methodological approach to a work of art ("problematic-semantic approach" in our terms). By means of a special method, arisen on this methodological basis ("method of reconstruction" in our terms), according to the text (part) a Metatext (work of art as a Whole) is restored [3].

3.2. Shortly the essence of the new approach is following. The work of art is initiated by the initial contradiction (opposition = statement of a problem) and presents by itself (work of art) a process of transition from an opposition to a complementary state (elimination of the contradiction). In the general logical form [4] given process

represents a cycle, where first stage is a producing of contradictions-oppositions (statement of a problem) . This first stage is interpreted as a disjunction $A \vee \bar{A}=I$. The second stage is an oppositions synthesis, elimination of the contradiction (solution of a problem) and interpreted as a conjunction $A \wedge \bar{A}=I$. From the point of view of the given approach, sense a work of art lies in the solution of a stated problem, i.e. is INTELLECTUAL ACT!

3.3. This bistage cycle (intellectual act) can be interpreted as Universal Model of the Whole formation (as a work of art), in the basis of which a Principle of HARMONY according to Pythagor ("unity in duality") lies. Subject is interpreted as an INTELLECT.

3.4. An important consequence of our approach is that it "displays" a semantic field — CONTEXT — a network of virtual distant links connections between elements, said network being not fixed in the very music text, but playing enormous role in perception.

4.1.1. The research of Chopin's Prelude C-dur from a position of the proposed methodology gives in brief following results. The central problem of the Prelude is already started in the syntagma G-A. The paradigm of the initial C-dur (basic tune key) assumes the stability of G (V-th step of the tune) and the unstability of A (VI-th step). In the process of development this ratio varies to opposite one unstable G and stable A. The paradigm of a collateral tune key A-moll, of the "shadow" C-dur double (they have a common scale) is generated. In the end of the Prelude these paradigmas are combine to a Metaparadigma, in which the specified contradictions are settled. Each tone of the initial syntagma G-A is read out at once on two paradigmas instead of one initial, i.e. becomes a nonlinear (two-dimensional) object in the tone space.

4.1.2. Precisely the same transition $X_0 \rightarrow X_0^2$ has taken place in the following spaces:

a) interval ("interval doubles"

$$\begin{array}{c} \boxed{[G A]}, \\ [C C] \end{array}$$

b) chord ("chord doubles"

$$\begin{array}{cc} \boxed{[G A]} ; & \boxed{[G A]}, \\ |E E| & |F F| \\ [C C] & [D D] \\ & [H H] \end{array}$$

with association of complexes "doubles" in a Whole (all complexes are combined by the common initial syntagma G-A).

4.1.3. All pitch system of the Prelude has passed in new, more complex condition — the Metasystem transition of type $N_0 \rightarrow N_0^2$ has taken place at all levels of the FORM.

4.2. In other words, the base spaces of tone, interval, chord (one-, two-, three-dimensional spaces) are united in direction to greater

$(N_0 \rightarrow N_0^2)$ dimensionalness at all levels of the form, what creates Whole as the superspace with a common System of coordinates.

5. We shall generalize this "totally self-similar" sound-pitch process in Chopin's Prelude in the form of a model and bring it into correlation with the existing mathematical model, set by the following formula:

$$X_{n+1} = f(X_n) = X_n^2 + C \quad [5].$$

5.1. In a musical model the system of coordinates is not set but created!

5.2. The passage $X_0 \rightarrow X_0^2$ is not set but created. Apart from that, the musical model possesses two non-trivial properties which are missing in a mathematical model.

5.3. Because of the spectrum of similar passages $X_0 \rightarrow X_0^2$ stipulated by similar conjunction $A \wedge \bar{A} = I$ at different scale-hierarchical levels, there occurs the system of "inserted fractals" with smoothly changing dimensions (in the real time of sounding of the Prelude it self!). It is variable dimension (V. D.), the first non-trivial property of a musical model. As compared to V.D. the traditional fractal dimensions is a separate instantaneous "cut" in a general dynamic process of transformation of the dimensions.

5.4. Sound processes in different layers of the Prelude are synchronized by a certain component uniting spaces with different measure into the Common System of Co-ordinates and providing projection of the superior dimensions to the inferior ones (for instance, projection of a polyphony substance to the monophonic melody). This second non-trivial property of the model — we shall call "trans-dimensions" (T.D.). It is remarkable owing to the fact that it gives birth to Time or, to be exact, to the sense of Time as a correlation of different processes in different metrics and scales.

5.5. And, on top of all, a musical model shows the genesis and semantics of self-similarity as a dynamic multi-leveled process, the basis of which at all levels is a similar version — the synthesis of oppositions $A \wedge \bar{A} = I$ ("solution of the problem" = "intellectual act" = Harmony according to Pythagor"). This synthesis is possible only in non-linear medium with competitive systems.

6. The results of comparison of the two models of self-similarity (the musical one and the mathematical one) reveals fuller completeness of the musical model because it includes not only mathematical (statistic) aspect (ref. to p. 2) but also the semantic one. Because of the application of a special methodology [ref. to p.3], we introduced the Subject (INTELLECT) into the researched phenomenon, which factor brought us to the new multi-leveled model of self-similarity. Therefore, the way from Music to Mathematics (inversion of the traditional strategy of research) is more promising because it allows to include the Subject's Factor into the researched phenomenon.

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