Self-Dual Graphs

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Every line drawing (planar embedding of a graph) decomposes the plane into territories, or *faces*. If the embedding is on the sphere, then there is no distinguished exterior face. We can associate to an embedded graph a *dual graph* by putting one dual vertex in every face and drawing one dual edge crossing each edge. In the figure, the dual vertices are white.

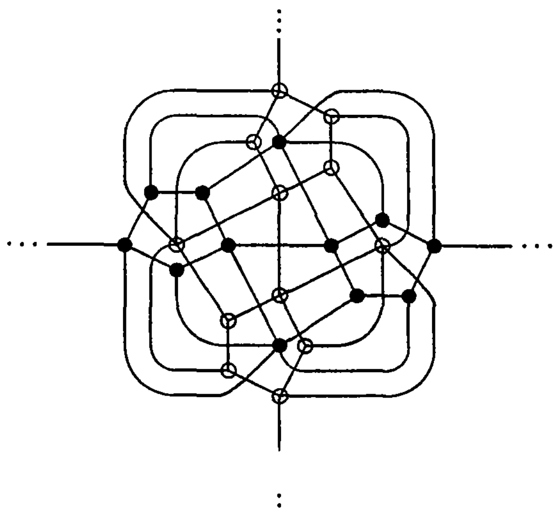

Figure 1: A self-dual polyhedron with its dual.

The dual nature of the construction is that the dual of the dual is the original graph. This abstract symmetry property is complemented profoundly in the case of self-dual graphs, in which the dual of the graph is already equal to the original graph, perhaps with a new point of view.

Self-duality gives new insight into some of Escher's works. With this extended concept of symmetry, we have the key to creating artwork with precise balance between color and form, a work, for example, in which a photographic negative would be indistinguishable from the actual picture of the work itself.

We have shown, that there are 24 distinguishable pairings of symmetry group and duality group that self-dual polyhedral objects could possess, leading to a classification of these self-dual objects in the spirit of the famous 17 wallpaper groups that influenced artists as well as designers.

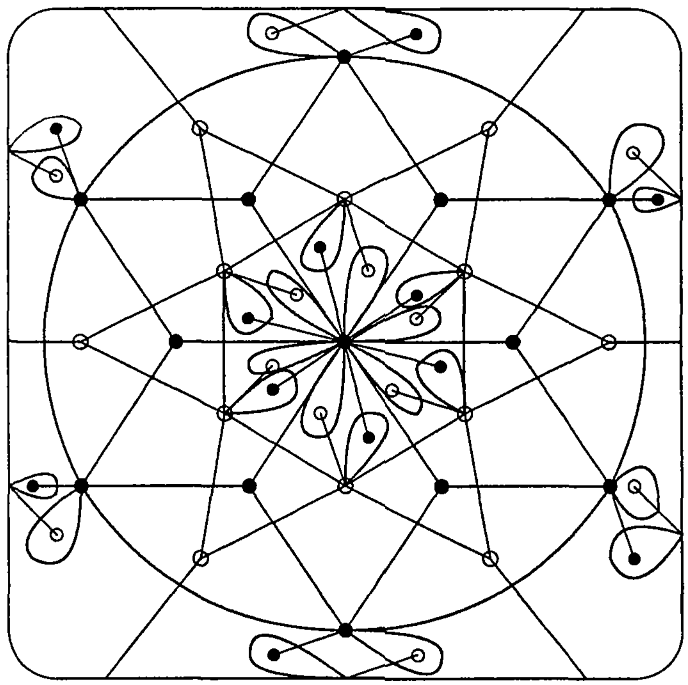

Figure 2: A graph with symmetry pairing $[6^+, 2] \triangleright [6]^+$

This workshop will first introduce participants to an elementary treatment of the mathematical background. We will explain the essential properties using our minimal examples which we will have at hand in form of drawings on cubes and other simple polyhedra.

In the second stage of the workshop participants will choose a medium to create their own self-dual objects to experience the excitement of their unusual qualities: turning them is equivalent to recoloring them! Suggested media are translucent spheres (balloons) or wooden (paper) cubes, and markers as well as other kinds of paint. For planar realizations, weaving patterns can be created and as more time consuming projects, one can consider the creation of reliefs and other sculptures. Also computer graphics is strongly encouraged.

We hope that participants might be able to share self-dual experiences of their own culture. The wallpaper groups are for example predominant in Islamic art, but presently we do not know if self-duality is apparent in the art of other cultures. We shall also discuss possible generalizations of this concept and search for applications not only in art but in areas of structural design and architecture.

References

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