

QUASICRYSTALS FOR ARCHITECTURE
The Visual Properties of Three Dimensional Penrose Tessellation

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Penrose tessellation refers to non-repeating patterns made up of only two elements. Although the two-dimensional Penrose tessellation (made up of fat and thin rhombii) has been known for ten years, the three-dimensional case (made up of fat and thin rhombohedra) is only a few years old. This new way to construct three-dimensional lattices has remarkable visual and structural properties, leading to applications in the physics of atomic structure as well as architecture and environmental sculpture.

A non-repeating pattern is an apparent paradox; one thinks that if there is a pattern then there must be repetition, and if there is no repetition there can only be randomness. Penrose tessellation is something inbetween: there is positional order, meaning that given one unit the positions of the others are generated; small regions are repeated elsewhere in the pattern; there are rotations which leave the pattern essentially unchanged, in that unit cells are still oriented in one or another of just a few directions. And yet, the patterns are not periodic-- like an irrational number there is no regular repeat of sequence. (Steinhardt, 1986)

Quasicrystals are now made in physics labs, much to the surprise of solid state physicists who used to think that atoms must either be arranged in a well-ordered regular crystalline lattice or in a highly disordered glass arrangement. Quite by accident D. Shectman and his collaborators discovered in a rapidly cooled sample of an aluminum-manganese alloy properties of both metallic crystal structure and glassy random structure. Moreover these samples, later duplicated by others, had the fivefold (pentagonal) symmetry that had been disallowed for patterns until the discovery of quasicrystals. Once large samples of pure quasicrystals can be made, their electrical and chemical properties can be studied, possibly with quite startling results. (Nelson)

The history of quasicrystals is the development of more and more powerful mathematical techniques to generate them, techniques that allow more and more of their subtle symmetry to emerge.

The Matching Rules Technique, invented and used by Roger Penrose, is a local operation of marking the 1-d boundaries of the 2-d units in such a way that when they are assembled, mark to mark, a non periodic tessellation is guaranteed. This local operation can make no predictions about the position and orientation of units far from the area being worked, and thus can lead to the erroneous assumption that the pattern is more random (less constrained) than it really is. Investigations of the infinite pattern as a whole were accomplished by Penrose by showing that the matching rules imply a system for breaking the units apart into smaller self-similar units in such a way that another, more numerous non-periodic arrangement is made. (Gardner)

The Projection Method, invented by N. de Bruijn in 1981 requires the construction first of a cubic, thus periodic, lattice, generally in twice the number of dimension of the desired tessellation. This cubic grid is projected, sliced, and projected a second time to obtain the tessellation. (de Bruijn) Although this method requires, for example, the construction of a four-dimensional grid to obtain a two-dimensional Pen-

rose tessellation, there is no need to visualize the four-dimensional grid, and so it is possible to scale up to higher dimensions in a straightforward though cumbersome way. The four-dimensional Penrose tessellation has now been investigated by Elser & Sloane with the projection of an eight-dimensional hypercube. (Elser & Sloane) The projection method is also useful in solid state physics for the easy calculation of the diffraction patterns of particles shot through quasicrystal samples.

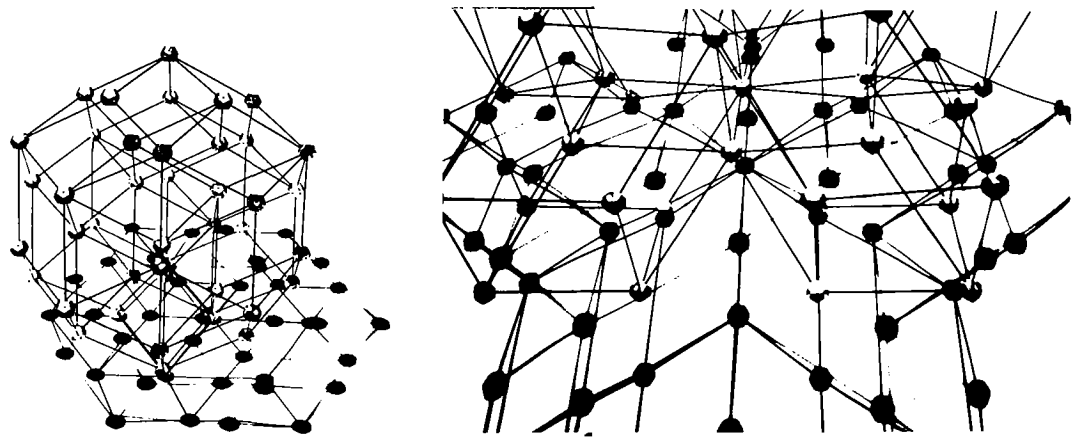
The Generalized Dual Method introduced by de Bruijn developed especially by P. Steinhardt in 1985 is by far the most powerful technique yet devised for the study of quasicrystals. First a grid dual (a dual is a distillation of a pattern into its basic structure) to the final tessellation is constructed, and then the dual grid is filled in with unit cells. This method can generate a larger class of non-periodic patterns including those with arbitrary orientational symmetry and those in any dimension; it can easily generate large patterns; and it provides a more complete description of the patterns. With this method we see how the long-range orientations are intrinsic to the structure, and we have a

technique to predict the sequence of position of units. Moreover it is the only method that generates four zonohedra (the four medium-sized groupings of the two unit cells of the three-dimensional Penrose tessellation). They are: the fat rhombohedron, the rhombic dodecahedron, made up of two fat and two thin rhombohedra, the rhombic icosahedron, made up of five fat and five thin rhombohedra, and the rhombic triacontahedron, made up of ten fat and ten thin rhombohedra. It is only these zonohedra that are the full three-dimensional analogue of the Penrose tiling in that only these medium-sized assemblies have matching rules and inflation and deflation capabilities that force non-periodic expansion, although each zonohedron can be resolved (subdivided) into its composite fat and thin rhombohedra. (Steinhardt et. al. 1985/6) I have discovered matching rules using two fat and two thin blocks that can sometimes generate these zonohedra. If they can be perfected so that the zonohedra, and thus the entire tessellation, are inevitably created, then it would be possible to create a computer program that simulated the growth of quasicrystals using local information only.

Using the powerful dual method, I have written computer programs which generate, rotate and slice 3d quasicrystals, and which demonstrate the visual behavior of these structures when seen from different angles. They have icosahedral symmetry which means that they look like they are made up of squares, triangles, rhombi, and pentagons, respectively as they are rotated. It is thrilling to see this structure transmute before your eyes, in real time, becoming one thing and then another, dissolving cells at one place and recreating them elsewhere, becoming at one moment a dense thicket and at the next a transparent lacework-- and all the while knowing that the structure is not really changing, that only a rotation of a fixed, rigid, structure is being observed.

Experiencing these programs supports an original application of this geometry to architecture and environmental sculpture. Rays of light from the sun are parallel, and cast shadows in isometric projection like the two-dimensional projections of three dimensional quasicrystals. Thus it is possible to build structures which visually behave like the computer program I have written. For example, consider Buckminster Fuller's geodesic dome in Montreal. This structure casts a shadow which is an intricate triangular net, and as the sun moves across the sky the triangular net shadow shifts across the floor. If the dome were made up of quasicrystal elements, tessellated according to Penrose matching rules in three dimensions, the shadows would be seen not so much as shifting but as transforming from a pattern of triangles to one of squares, to one of a 2-D Penrose pattern, to one of pentagons, and back again to one of triangles and hexagons. The same effect could be obtained to a lesser degree with a quasicrystal space frame (a flat slab) or a barrel vault or even a spherical cluster is seen from many angles, if for example it were hanging in a atrium space where viewers could be underneath and well as on balconies. (Computer generated engineering studies of these structures is being undertaken to study their structural properties.)

Like other large structures built with a new, and still obscure technology the effects would seem miraculous, like St Chapelle, or the Eiffel Tower, or the Grand Coulee Dam must have been to its first viewers, and still is to some extent for us. To build a crystal cage for the capture of a deity, to build a tower so much higher than any other structure, to fill in a mountain so that the vast American landscape can be pressed into the service of human beings, these are not only magical feats of mathematics and engineering, but in addition they tell us something profound about the values of the people who built them. In the case of the quasicrystal, even on a much smaller scale, we could be allowed the satisfying experience of seeing in complexity and apparent confusion a structure, a symmetry, that is elusive, subtle, flexible yet amazingly powerful. And that image of complexity mastered is something for which our culture yearns, now more than ever.



Models of Globe and Space Frame with Shadows

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