Planar Cayley Graphs

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A *group* the full catalog of the symmetries of an object in the classical sense, that is, Felix Klein's Erlangen Program. A non-algebraic approach to groups is via the Cayley graph, which may be regarded as a drawing of the group which not only encodes the symmetries that are being described, but also exhibits them as an abstract object.

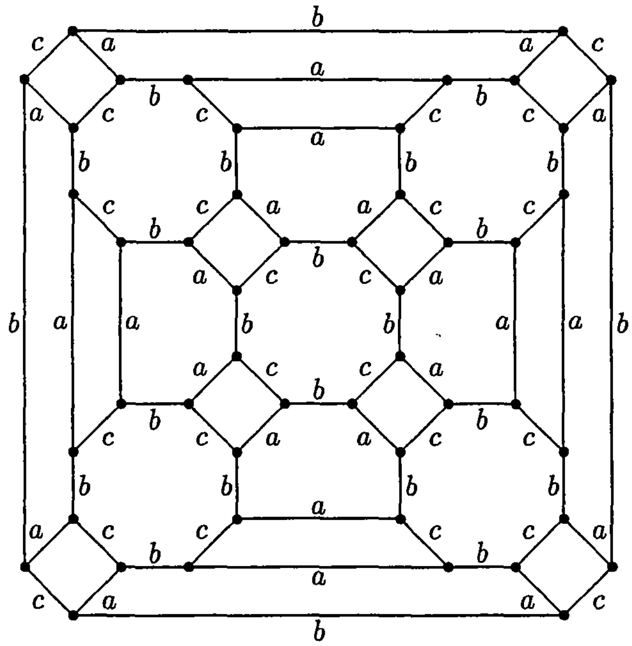

Figure 3: The Cayley graph of the symmetries of the cube.

The Cayley graph may actually be drawn upon a physical object with symmetry, so that the potential of symmetric decoration depends only upon how the graph may be drawn. The fundamental region determined by the embedding clearly demarks for the artist the areas which are open to decoration while achieving the desired symmetry. This is the technique which has made the wallpaper groups an essential tool in design, allowing the construction of large but manageable solid state circuits, as well as large scale communications networks.

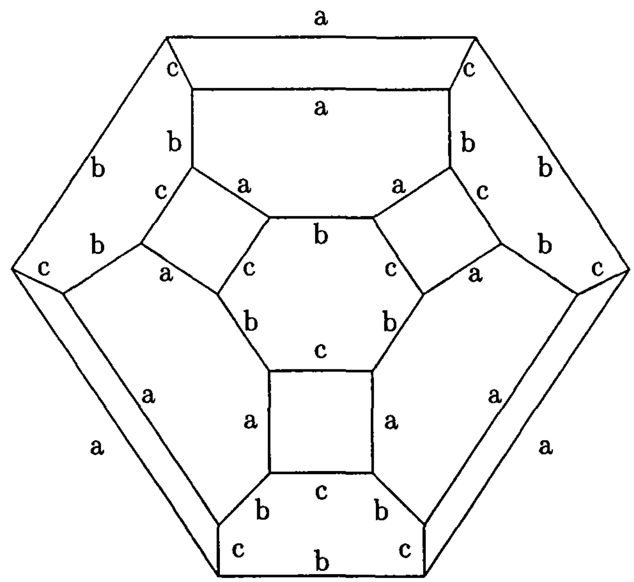

Figure 4: The Cayley graph of the symmetries of the tetrahedron

Taking the plane as a basic space, we examine the possibility of embedding Cayley graphs, and discover new symmetries which exhibit a fractal (hyperbolic) nature. These fractal symmetries achieve greatest regularity when the plane is punctured and allowed to bend into the third dimension.

Also, we will examine the existence of broken fractal symmetries, in which the symmetric points of view may only be achieved by allowing the object to be repeatedly disassembled, and reassembled.

In this talk we shall describe and classify all Cayley graphs, finite and infinite, which can be drawn in the plane without edge crossings and show in each case how to exhibit the geometric nature of the symmetries and how the symmetries may be used for answering routing questions in communications networks and for an examination of their reliability as well as for purely artistic purposes.

References

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