

SEQUENCE OF JULIA-LIKE SETS PRODUCED BY A NUMERICAL RECURRENCE TECHNIQUE

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We look at a particular symmetry pattern that portrays smooth frontiers invading on a fractal support into a basin of attraction. The formerly connected basin is split up to a Feigenbaum sequence of 2, 4, 8, 16, etc. parts.

In order to end up with such a structural form, we start with an iterative mapping somewhat similar to the complex logistic one,

$$x_{n+1} = x_n^2 - y_n^2 + ax_n + b ; \quad y_{n+1} = 2x_n y_n + c . \quad (1)$$

x_n and y_n denote the variables with index $n = 0, 1, 2, \dots$, while a , b , and c represent constants. Due to the term ax_n , equation (1) is not analytic any more. On the basis of that map, basins of attraction, i.e., filled-in Julia-like sets, as well as Mandelbrot-like sets can be calculated, reflecting self-similarity and further types of fractality (Rössler et al., 1986, 1990; Röhricht et al., 1987; Peinke et al., 1987, 1988, 1991; Parisi et al., 1991; Klein et al., 1991).

Figure 1 is obtained from a series of Julia-like sets. At first, the initial conditions were chosen as

$$-1.5 \leq x_0 \leq 1.5 ; \quad -1.5 \leq y_0 \leq 1.5 \quad (2)$$

and divided up into 600×1200 points. Each of those in the plane of the picture stands for a pair of initial conditions (x_0, y_0) and was iterated 500 times with double precision (FORTRAN). Only such points that fulfill the condition

$$x_n^2 + y_n^2 \leq 10 \quad (3)$$

were marked by dots. This numerical procedure was performed for the parameter values $a = 0.3$, $b = 0.1225390$, and $c = 0$. The next step implies the application of the same procedure with $b = 0.1225392$ (a and c remain constant). But, this time, non-divergent points marked before were now erased. That process was then repeated three times, always increasing the last digit of the value of b by 2 and inverting the color of the previous

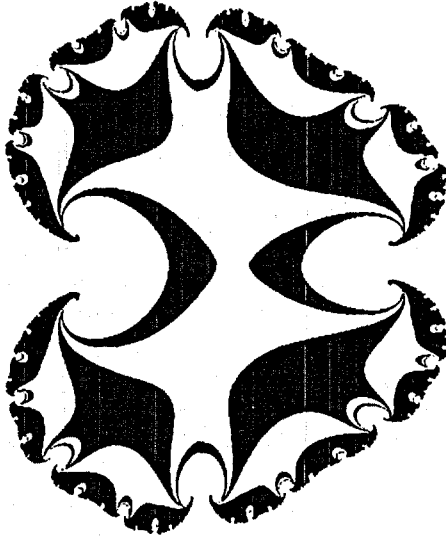


Fig. 1. Feigenbaum sequence of Julia-like sets. For details, see text.

step. With the help of this technique, it is possible to visualize how a Julia-like set vanishes under variation of a control parameter (here, b), corresponding to a transition over a borderline of an appropriately chosen Mandelbrot set.

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