

ORDERING KNOTS

Slavik Jablan

The Mathematical Institute
Knez Mihailova 35, 11001 Belgrade, Yugoslavia

The possibility to study knots from the mathematical point of view was for the first time proposed by C.F.Gauss. Gauss formulated the "crossing problem", by assigning letters to the crossing points of a self-intersecting curve and trying to determine "words" defining a closed curve. J.B.Listing represented knots by their projections (diagrams) and made an attempt to derive and classify all the projections having fewer than seven crossing points. The complete derivation of non-isomorphic knot projections having fewer than ten crossings was completed by P.G.Tait and T.P.Kirkman. Kirkman's geometrical system for the systematic derivation of knot projections, closely connected with the enumeration of polyhedra, represented at the same time the geometrical method for the classification of knot projections.

In the 30-ties, after the appearance of the first knot invariant, discovered by J.W.Alexander, the knot theory was established as the part of topology, completely losing connection with its roots - geometry. In K.Redmeister's book "Knotentheorie" (1932), each knot is represented by one projection, (randomly?) chosen from several possible ones.

Today, with the development of computers, the enumeration of knots and links is very similar with the situation occurring in different unordered structures: prime numbers, polyominoes etc., giving no chance for any classification.

Following the "geometrical" line (Kirkman-Conway-Catidron), we will try to present the consistent (geometrical or graph-theoretical) approach to the derivation and classification of knots and links. In particular it is obtained the combinatorial formula for the number of rational knots and links ($2^{n-4} + 2^{\lfloor n/2 \rfloor - 2}$) and for the number of generating rational knots and links. They are given different particular combinatorial results for "prismatic" and "polyhedral" knots and links, as well as the complete derivation and classification of alternating knots and links with fewer than 13 crossings.