ON RESTORING OF FRAMEWORKS BY INTERNAL STRESSES

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Let us consider the simplest pinned framework (Fig.1) in a plane, consisting of two bars p_1p_2, p_1p_3 and three hinges (joints) p_1, p_2, p_3, from which only p_1 is not pinned in the plane.

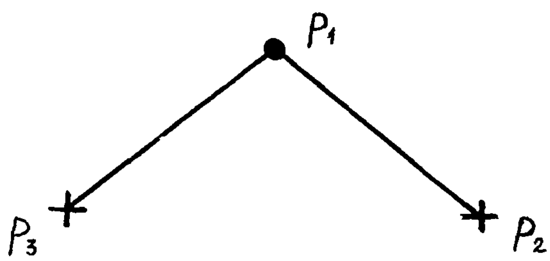


Figure 1

A non-zero set $\omega=(\omega_{12},\omega_{13})$ of real numbers, assigned to the bars of this framework, is called an internal stress or stress [2] if

$$\omega_{12}(p_2-p_1)+\omega_{13}(p_3-p_1)=0. \qquad (1)$$

The equality expresses an equilibrium of forces, applied by bars to the unpinned joint. Our framework obviously accepts stresses only when p_1 lies on the line p_2p_3. One can easily check, that if $\rho_2 \neq \rho_3$ are given, a framework p, accepting a stress ω, is uniquely defined by this stress. We say, that the framework p is restorable by its stress ω. The problem is to investigate such possibility in the general case.

Let $\mathfrak{P}=\mathfrak{P}(P,S)$ be a set of all frameworks with a given scheme S of bar connection and given positions P of pinned joints. A frameworks p from $\mathfrak{P}$ may accept internal stresses. If it is so, a set of all stresses, accepted by p, together with the zero stress, constitutes a subspace $\mathbf{W}(p)$ of the vector space $\mathbf{W}^r$ of all imaginable stresses (r is a number of bars in p). Really, the stresses - are solutions of a homogeneous linear equations system of the form (1). The question is: when a framework $p \in \mathfrak{P}$ can be restored by its stress subspace $\mathbf{W}(p)$? Or in the other words, - in which cases p is the unique in $\mathfrak{P}$ framework, having a stress subspace $\mathbf{W}(p)$?

Solving this problem, it turns out reasonable to put some a priori restrictions on the framework p. Namely, we shall consider p regular, i. e. all its bars are of non-zero length; and entirely stressable, i.e. having a stress non -zero on each bar. A complete answer to the question was obtained only for the frameworks, lying on a straight line. It depends merely on a scheme S and on P, not on a regular framework itself.

Even for the frameworks on the plane the problem is far from its solution. In the remaining part of the abstract I'll describe the situation in this case. At first, an important sufficient condition of restorability is known for an arbitrary dimension. It is the Connelly's spider web case [1]: if there exists a stress $\omega\in W(p)$ with all $\omega_{ij}>0$, then a framework p is restorable by $W(p)$. From the other hand a rather general condition of non-restorability of entirely stressable frameworks exists in all dimensions. Let us formulate its simplest case. Assume a set V of the joints of a plane framework p consists of two parts V_1 and V_2, such that all joints of V_2 are unpinned and all bars, going from V_2 to V_1, are ending either in a joint $p_1\in V_1$ or in a joint $p_2\in V_1$. A framework of this type is shown in Figure 2, where $V_1=\{p_1,p_2\}$, $V_2=\{p_3,p_4,p_5,p_6\}$.

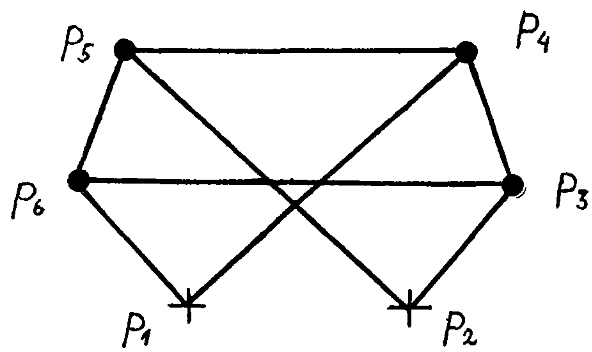
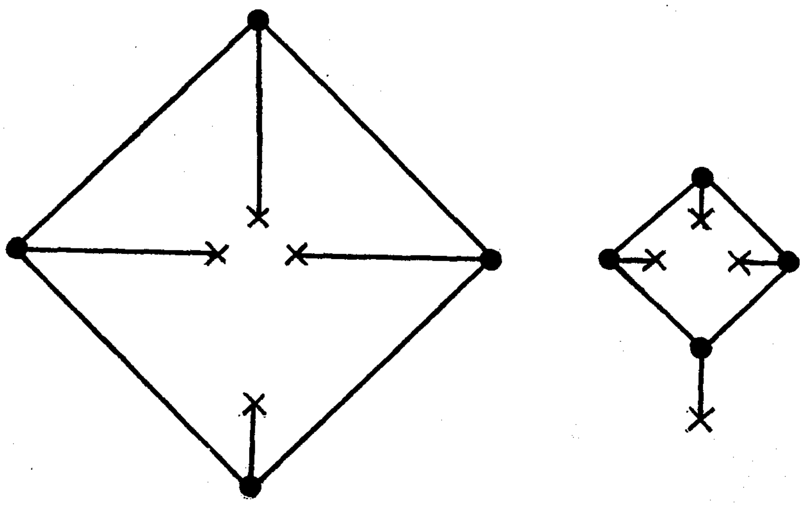


Figure 2

Suppose Ap be an image of such a framework p under arbitrary affine transformation A of the plane keeping joints p_1 and p_2 fixed. If p accepts internal stresses, then $W(p)=W(Ap)$. Thus if p doesn't lie on the line p_1p_2 it is non-restorable. This is strictly the case of the framework from Figure 2.

But a presence of such affine symmetry is not the only reason for failing restorability of entirely stressable regular frameworks. The following example looks somewhat surprising and asymmetric. Frameworks p^1 and p^2 in Figure 3 a), b) are belonging to the same set $\mathfrak{P}$. Both this frameworks are entirely stressable and have a one-dimensional stress subspaces. But the framework p^1 is restorable by its stress subspace, and p^2 is not!



a) b)

Figure 3

[1]. *Connelly R. Rigidity and Energy.// Invent. Math. 1982 v.66 1 p.11-33.*

[2]. *Crapo H., Whiteley W. Statics of Frameworks and Motions of Panel Structures, a Projective Geometric Introduction // Structural Topology.1982. 6.P.43-82.*