NEW APERIODIC INFLATION TILLINGS BY TRIANGLES

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Given a finite family F:={T_1,...,T_k} of prototiles T_kappa, a number eta with |eta| 1 and an inflation rule infl. That is to say: First expand every T_kappa by the factor eta and then split eta T_kappa into pieces, where every piece is congruent to some prototile. So eta T_kappa becomes a cluster infl (T_kappa) of F-tiles. This procedure can be iterated to produce arbitrary large clusters infl^n(T_kappa). By S(F,infl) we denote the species of all global tilings P such that every finite cluster of P has a congruent counterpart in some infln (T_kappa).

There are some very simple and rather natural inflation rules, where no P in the generated species is `vertex to vertex', even, if artificial vertices are permitted. Probably this explains, why the following tiling have not been studied in detail so far.

Consider F:={A,X}, where A is the rectangular triangle with edges of length 1, sqrt(tau), tau and X:=eta A with eta:=i sqrt(tau). The factor i indicates, that X is not homothetic to A but rotated by 90 degree. We define infl(A):=X and infl(X) by splitting eta X into a copy of A and a copy of X. This inflation has a unique inverse, so S(F,infl) is aperiodic. Further eta is a complex PV-number. And in fact: if we decorate every triangle by an `atom' (a Dirac-delta), the X-ray diffraction pattern (i.e. the Fourier transform of the autocorrelation function) of any global tiling is `strictly point' (consists of sharp Bragg peaks only).

The species S(F,infl) can n o t be defined by any l o c a l matching rule. But after colouring with three colours the new species S({A,B,C,X,Y,Z},infl*) possesses a perfect local matching rule.

Some more properties of these species will be described as well as some related species. They lead to the problem, how to generalize the notion of inflation properly. Even more general the striking question: What tilings should rightly be considered `h i e r a r c h i c a l' ?