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Lunda-designs of ancient Egyptian motifs and Celtic knots

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Lunda-designs are black-and-white designs with attractive local and global symmetries. I discovered them in the context of analyzing a generalization of a type of figure traditionally drawn by Chokwe story tellers in the sand to illustrate their tales, fables and proverbs (cf. Gerdes, 1990). The Chokwe live predominantly in northeastern Angola, a region called Lunda. Hence the name Lunda-design. In this paper, I present and analyze Lunda-designs that can be generated by some ancient Egyptian motifs and Celtic knots. Particular attention will be given to their symmetries.

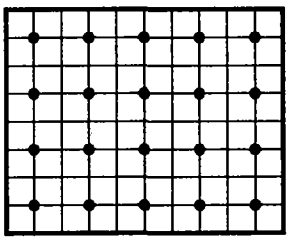


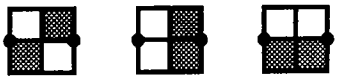
Figure 1: Rectangular grid of dimensions 5 by 4

Let us consider a rectangular grid of the type illustrated in Figure 1. A Lunda-design [2] is defined as a black-and-white coloring of such a rectangular grid that satisfies the following two local (two-color) symmetry properties:

(i) Along the border each grid point always has one black unit square and one white unit square associated with it (see the example in Figure 2a);

(ii) Of the four unit squares between two arbitrary (vertical or horizontal) neighboring grid points, two are black and two are white (see the examples in Figure 2b).

A possible border situation
a



Possible situations between two horizontal neighboring grid points
b

Figure 2

From this definition it follows that Lunda-designs have a global symmetry, characterized by the phenomenon that:

(iii) in each row (and in each column) there are as many black unit squares as there are white unit squares.

Figures 3a and 4a present two ancient Egyptian motifs which appear engraved on scarabs. Both are monolinear designs composed out of one smooth closed line that crosses itself five or four times. If we draw these motifs in a 2x3 rectangular grid (Figures 3b and 4b) and color the successive unit squares through which the curve passes alternately black and white, staring in the lower left corner with a white unit square (see Figures 3c and 3c), we generate the Lunda-designs presented in Figures 3d and 4d (grid points marked) and 3e and 4e (grid points unmarked). Although both Egyptian motifs have the same symmetries (horizontal and vertical axes of symmetry), their Lunda-designs display different symmetries: whereas the first has a vertical axis of one-color symmetry, the second has a vertical axis of two-color symmetry (a reflection in this axis interchanges black and white unit squares).

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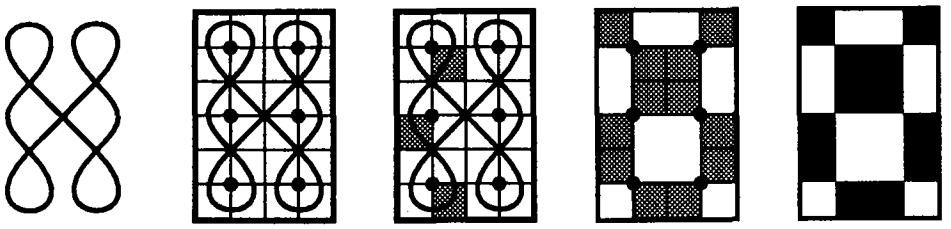
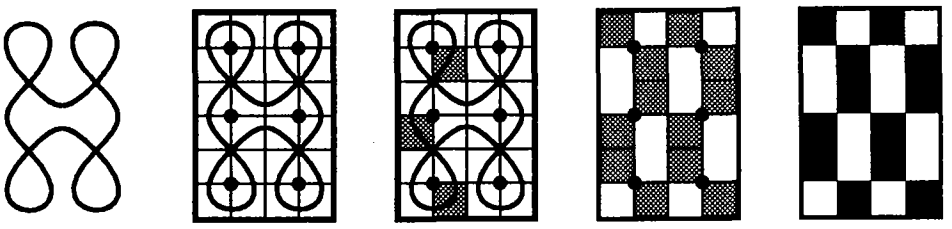
2 For generalizations of the concept of Lunda-design, see Gerdes, 1996, 1997, 1998.



a b c d e

Figure 3



a b c d e

Figure 4

The monolinear motif displayed in Figure 5a decorates an ancient Egyptian vase, has apparently no global symmetry. However, if we draw its topological equivalent in a grid (Figure 5b and c), it generates the Lunda-design in Figure 5d that has a vertical two-color symmetry axis. Moreover, its lower square (Figure 5e) has both horizontal and vertical two-color symmetry axes and diagonal one-color symmetry axes. We may say that the Lunda-design reveals hidden symmetries of the original line drawing.

The Lunda-design (see Figure 6b) of the Celtic knot in Figure 6a shows hidden vertical one-color and horizontal two-color symmetry axes. The Celtic knot in Figure 7a generates a Lunda-design with a horizontal two-color symmetry axis (Figure 7b). Both the left square of this Lunda-design (Figure 7c) and the remaining last two columns have double two-color symmetry. The Celtic knot design presented in Figure 8a is invariant under a half turn about its center. The colors of the attractive corresponding Lunda-design (Figure 8b) are reversed under a half turn. This Lunda-design has several interesting local one- and two-color symmetries as the details in Figure 9 illustrate.

References

Davis, Courtney (1991), *Celtic Designs and Motifs*, Dover, New York

Gerdes, Paulus (1990), On ethnomathematical research and symmetry, *Symmetry: Culture and Science*, 1(2), 154-170

__ (1995), *Une tradition géométrique en Afrique — Les dessins sur le sable*, L'Harmattan, Paris / Montreal, 3 vols.

__ (1996), *Lunda Geometry — Designs, Polyominoes, Patterns, Symmetries*, Universidade Pedagógica, Maputo

__ (1997), On mirror curves and Lunda-designs, *Computers and Graphics*, Oxford, 21(3), 371-378

__ (1997a), *Ethnomathematik dargestellt am Beispiel der Sona Geometrie*, Spektrum Verlag, Berlin / Heidelberg / Oxford

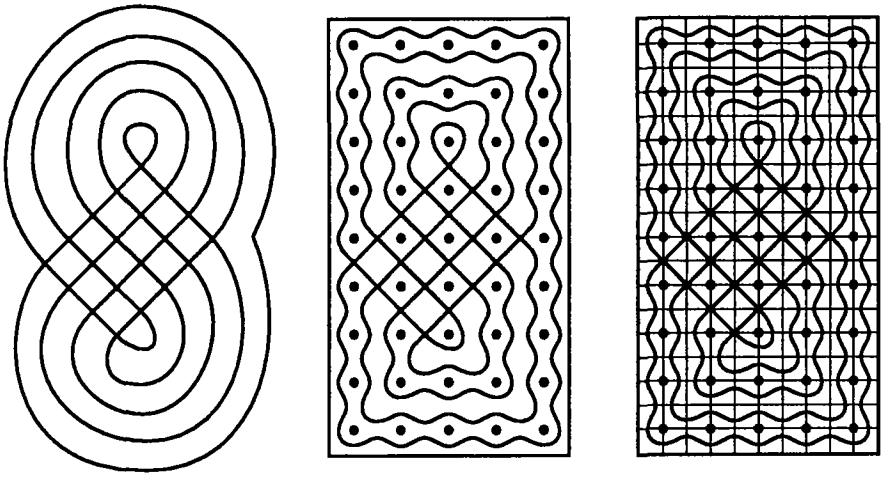
__ (1998), On Lunda-designs and some of their symmetries, in: *Symmetry: Art and Science* (in press)

Jablan, Slavik (1995), Mirror generated curves, *Symmetry: Culture and Sciĕnce*, 6(2), 275-278

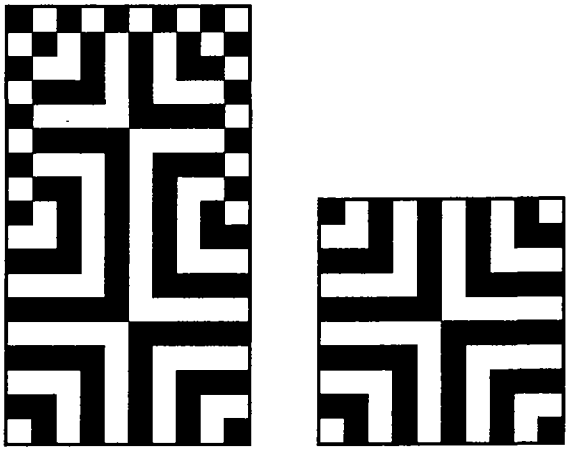
Meehan, Aidan (1991), *Celtic design: Knotwork, The Secret Method of the Scribes*, Thames and Hudson, London

____ (1995), *Celtic Design: The Dragon and the Griffin, The Viking Impact*, Thames and Hudson, London

Petrie, W. Flinders (1930), *Decorative Patterns of the Ancient World*, University College, London



a (drawn after Petrie, Pl. XLI) b c

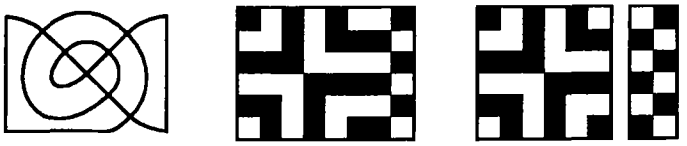


d e

Figure 5

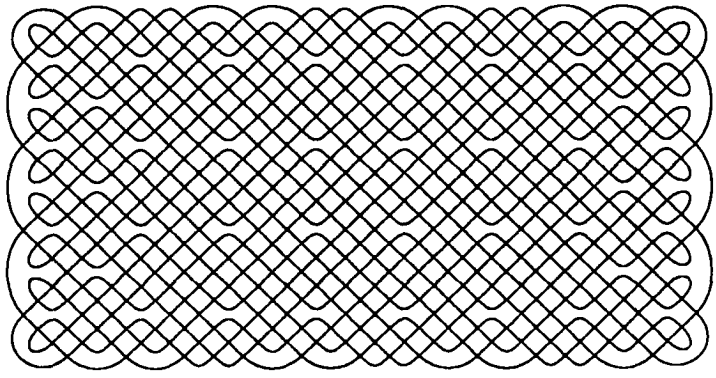
a (drawn after Meehan, 1995, p. 96) b

Figure 6



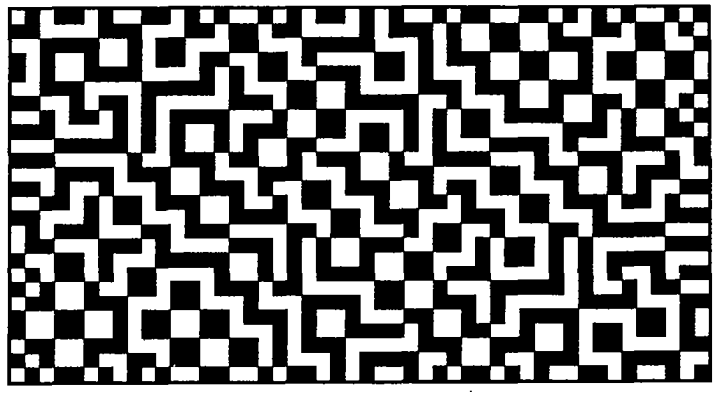
a (drawn after Meehan, 1991, p. 122) b c

Figure 7



(drawn after Davis, p. 21)

a



b

Figure 8

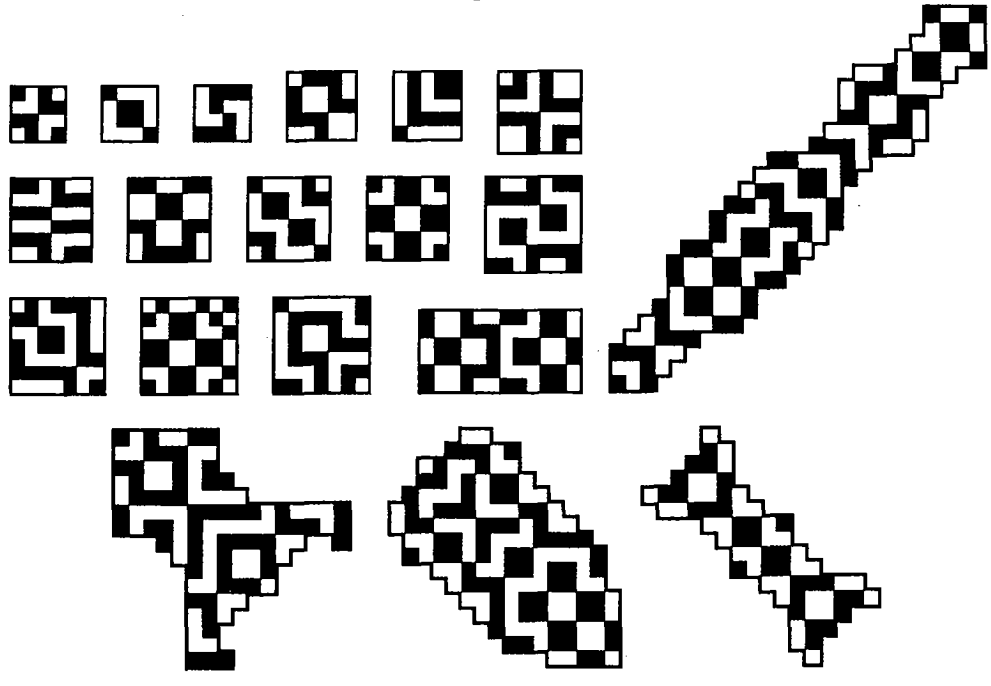


Figure 9