SYMMETRY VISUALIZED ON A NEW EDUCATIONAL AID:
THE DRAWING BALL SET

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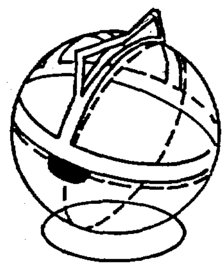
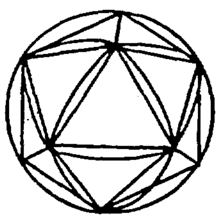
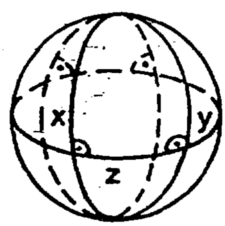
The idea that led me, to this device came from a particular area of mathematics teaching, namely, axiomatization on the spherical surface. In order to teach elements of spherical geometry, I had to construct the spherical equivalent of the well-known drawing board, draft paper, ruler, and compass. Part of the result is shown in Figure 1. The ball is about the size of a football, and empty inside; the ring or torus is the holder of the ball; and the ruler has two scaled tracer edges.

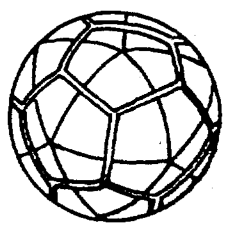


What is the set good for?

Not before it came into real existence, did I realize the fact that its application areas are far beyond my original expectations, ranging from geometry and algebra to geology and astronomy, navigation, crystallography, etc.

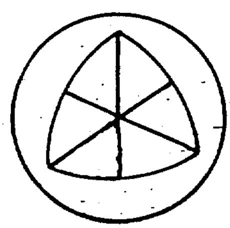


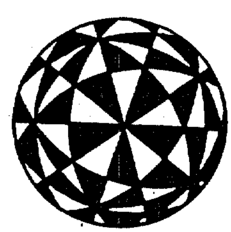




I often wonder why the sphere seems a treasury of symmetrical patterns for many of the fine arts and the natural sciences. My explanation, though a bit occult it may sound, is that the sphere, no way inferior to the plane, has to display the perfection of the infinite plane on the finite spherical surface.

When you try to adapt a problem in plane geometry to the surface of the sphere, and draw a sketch with all the corresponding spherical elements, it is quite often that the problem seems to defy solution. Then, by making use of the symmetry on the sphere, you complete the sketch with full great circles, opposite points, and dual constructions, and the solution falls in your hands.





Consider the following problem: given a spherical triangle; how can we construct the great circle passing through an apex, and cutting the triangle into halves of equal area? The next question is: How can we prove that these three area bisectors are concurrent? Both of these questions could be answered by completing the sketch on the ball. The graph in Figure 6 contains all the vertices of the five Platonic solids. As such, it plays an important role in constructing the spherical projections of regular and semi-regular, or Archimedean, solids. However, it also proved indispensable to the discovery of the three-dimensional structure of the polio virus.

Many products of folk-work make use of symmetrical patterns on the sphere, as, for example, the famous Japanese Temari Balls, which unite the exactness and symmetry of geometric constructions with the lovableness of a child's toy.